

Computational Analysis of Gear Meshing Excitations Considering Surface Roughness and Oil Film Effects

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Abstract: Accurately predicting gear meshing excitation is crucial for high-performance transmission fatigue design and noise optimization. To address the frequently overlooked topography-lubrication coupling, this paper presents a loaded tooth contact analysis (LTCA) model integrating ground surface features and oil film stiffness. Utilizing the finite element substructure method, the model combines a non-uniform sinusoidal function for surface waviness with the Dowson-Higginson film thickness formula. Results show that higher grinding wheel speeds enhance surface quality, bringing time-varying meshing stiffness (TVMS) and static transmission error (STE) levels closer to ideal smooth surfaces. Conversely, increased axial feed rates or fluctuation amplitudes deepen surface textures, causing stiffness attenuation and a significant STE amplitude surge. Additionally, higher lubricant viscosity, pressure-viscosity coefficients, or driving gear speeds thicken the oil film, reducing equivalent stiffness and elevating global transmission error. Crucially, the roughness-oil film coupling causes the most severe stiffness loss. This study offers a precise theoretical foundation for predicting precision gear system contact characteristics.

Keywords: Gear Surface Topography; Oil Film Stiffness; Time-varying Meshing Stiffness (TVMS); Static Transmission Error (STE).

1. Introduction

As core power transmission components in aerospace vehicles, marine vessels, precision machine tools, and new energy vehicles, gear systems directly dictate system performance and reliability via transmission accuracy and dynamic characteristics. During gear meshing, time-varying meshing stiffness (TVMS) and static transmission error (STE) serve as critical excitation sources inducing internal vibration and noise [1]. Consequently, accurately predicting these meshing excitations is a prerequisite for anti-fatigue design and low-noise optimization. However, current research often overlooks the surface micro-topography and oil film stiffness correlation, particularly their combined impact on meshing excitations [2]. Addressing this, this paper establishes a comprehensive contact analysis model integrating realistic ground surface topography and oil film stiffness to reveal parameter variation effects on gear meshing characteristics.

2. Analytical Model

2.1. Oil Film Stiffness Model

In oil-lubricated gear pairs, an oil film exists between the two mutually compressing solid surfaces. The presence of this oil film alters the deformation and stiffness of the contact pair [3]. In the mechanical model, this viscoelastic fluid medium interposed between the solid surfaces is equivalent to a nonlinear stiffness-damping element connected in series at the contact interface. Its thickness variations and rheological properties continuously influence the equivalent contact compliance between the tooth surfaces.

The oil film stiffness is defined based on the definition of linear spring stiffness [4], as shown in Eq. 1.

$$K_{oil} = \lim_{\Delta\delta \rightarrow 0} \frac{\Delta w}{\Delta\delta} = -\frac{dw}{dh} \quad (1)$$

Where w represents the line load supported by the oil film in line contact configuration, with the unit of N/m; $\Delta\delta$

denotes the variation in oil film thickness, with the unit of m; and h indicates the oil film thickness, with the unit of m.

While various elastohydrodynamic lubrication (EHL) formulations exist, the Dowson-Higginson dimensionless minimum film thickness formula [5] is selected herein due to the thinner film characteristics of gear pairs compared to bearings. By utilizing the load-deformation relationship presented in Eq. 2, a simplified oil film stiffness calculation method is derived [6].

$$h_{min} = 2.65G^{0.54}U^{0.7}W^{-0.13}R \quad (2)$$

$$h_{min} = \frac{2.65\alpha^{0.54}\nu_0^{0.7}u_r^{0.7}R^{0.43}L^{0.13}}{E^{*0.03}p_j^{0.13}} \quad (3)$$

In the equation, W represents the dimensionless load parameter, U denotes the dimensionless speed parameter, and R indicates the equivalent contact radius. Furthermore, u_r represents the entrainment velocity, and G denotes the material parameter. For steel components lubricated with mineral oil, the value of G is taken as 5000. E^* represents the equivalent elastic modulus.

Eq. 3 serves as the formula for calculating the lubricant film thickness on each individual element, accounting for the discretization of the gear tooth contact line. In this equation, L represents the contact length, and p_j denotes the load applied to each element.

The literature expresses Eq. 3 as a function relating the line load to the film thickness. Subsequently, by differentiating with respect to the film thickness as the independent variable, a simplified expression for oil film stiffness is derived, as presented in the Eq. 4 [7].

$$K_{oil} = -\frac{dw}{dh_{min}} = 7.69LE^*R(2.65G^{0.54}U^{0.7}R)^{7.69}h_{min}^{-8.69} \quad (4)$$

Due to the extremely thin lubricating film on gear teeth, the lubricant properties in the contact zone differ from those at ambient conditions. Parameters for stiffness calculation are

adopted from Ref. [8], with an initial dynamic viscosity of 36.4 mPa·s and a pressure-viscosity coefficient of $1.85 \times 10^{-8} \text{Pa}^{-1}$.

2.2. Surface Roughness Model

In the grinding process, the vibration behavior of the grinding wheel is a critical factor influencing the surface quality and machining accuracy of the workpiece. Although the traditional uniform sinusoidal function model can describe the periodic vibration of the grinding wheel to a certain extent, it fails to accurately reflect the complex waviness morphology caused by non-uniform wheel wear, uneven material distribution, or variations in dynamic characteristics under actual operating conditions. To simulate the vibration characteristics of the grinding wheel more realistically, a non-uniform sinusoidal function is introduced to characterize its waviness profile [9].

The waviness profile defined by the non-uniform sinusoidal function is formed by the superposition of multiple sine waves with varying amplitudes and wavelengths. The maximum amplitude of each sine wave is a fixed value, while the wavelength can be adjusted according to the actual vibration characteristics. This non-uniformity effectively captures the local variations of the grinding wheel surface waviness, and its mathematical expression can be represented as follows:

$$y(x) = \sum_{i=1}^n A_i \sin\left(\frac{2\pi f_i}{v_w} x + \phi_i\right) \quad (5)$$

In Eq. 5, A_i denotes the amplitude of the i -th wave, and f_i represents the vibration frequency, which is determined by dividing the grinding wheel rotational speed by 60. Additionally, v_w indicates the grinding wheel feed speed, while ϕ_i stands for the random initial phase of the i -th wave.

2.3. Loaded Tooth Contact Analysis Incorporating Oil Film Stiffness and Surface Roughness Effects

To accurately capture both macroscopic and local deformations, this paper establishes a gear Loaded Tooth Contact Analysis (LTCA) model by integrating the Finite Element Method (FEM) with contact mechanics.

The modeling procedure first involves constructing a gear finite element model and discretizing the meshing surface to extract the bending-shearing compliance matrix via the substructure method. Subsequently, deformation compatibility equations for the tooth surface elements are formulated based on force analysis and solved iteratively to determine the meshing stiffness, comprehensive mesh error, and Static Transmission Error (STE) [10].

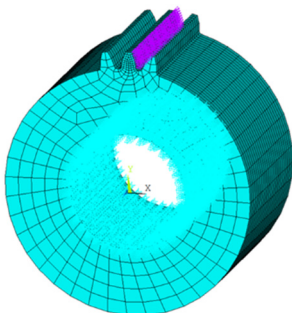


Figure 1. Finite Element Model of Spur Gear

As illustrated in Figure 1. First, the gear model is meshed using tetrahedral or hexahedral elements and analyzed in ANSYS via APDL commands. The bending-shearing compliance matrix of the target nodes is extracted using the substructure method, and a custom MATLAB program performs coordinate transformations to obtain meshing node parameters.

As illustrated in Figure 2, meshing initiates at point B₂ (driving gear root contacts driven gear tip) and proceeds along the transverse line of action to point B₁ (meshing end point). A meshing plane coordinate system is established with the face width B as the X-axis and the line of action B₁B₂ as the Y-axis [7].

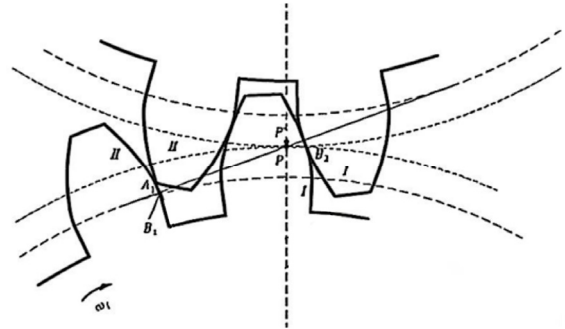


Figure 2. Schematic of the Gear Meshing Cycle

When accounting for surface roughness, the total initial gap at any meshing node includes the microscopic geometric gap from grinding in addition to macroscopic parameters. To ensure feature fidelity and algorithm convergence, MATLAB's griddata function with 'v4' biharmonic spline interpolation is employed [11]. This method generates a surface strictly passing through each original data point, preserving simulated peak-valley characteristics to ensure accurate excitation calculation.

Let the original roughness dataset be defined as $S_r = \{x_r, y_r, z_r\}$, where z_r represents the roughness amplitude in μm . The numerical calculation model discretizes the grid along the face width and tooth profile directions, with node coordinates in the computational domain denoted as $(x_{\text{mesh}}, y_{\text{mesh}})$. Utilizing the biharmonic spline interpolation algorithm, the discrete roughness data is mapped onto the computational nodes of the contact line.

The mapped roughness matrix can be expressed as follows:

$$E_r(i, j) = \text{Interpolate}(S_r, x_{\text{mesh}}(i, j), y_{\text{mesh}}(i, j)) \quad (6)$$

In Eq. 6, the unit is micrometers (μm), where i and j correspond to the j -th contact slice node at the i -th meshing instant, respectively.

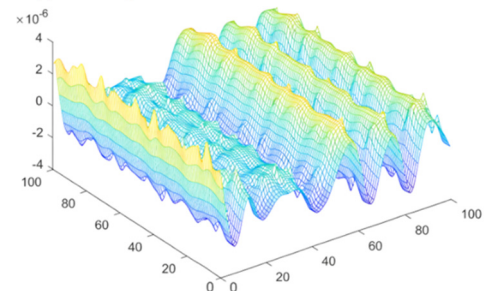


Figure 3. 3D Surface Topography Map based on Non-uniform Sinusoidal Interpolation

As illustrated in Figure 3, the plotting results generated by

the code demonstrate that the interpolated roughness surface preserves the stochastic features of the original topography. Consequently, these amplitude fluctuations are directly superimposed onto the geometric gap of the gear tooth surface.

Considering the oil film, gear meshing is modeled as the contact between elastic bodies. Upon applying an external load P , the oil film undergoes normal compression, bringing the discretized tooth surfaces into proximity. The resulting total elastic deformation consists of a linear macroscopic component (induced by overall bending, shearing, and friction) and a nonlinear local contact deformation at node i . Consequently, the deformation compatibility condition is formulated as [7]:

$$u_{bi}^{(1)} + u_{bi}^{(2)} + u_{ci} + \delta_o + \varepsilon_i - x_s - d_i = 0 \quad (7)$$

Where $u_{bi}^{(1)}$ and $u_{bi}^{(2)}$ denote the bending-shearing deformations of the driving and driven gear tooth surfaces at contact point i , respectively. ε_i represents the total initial gap at point i ; when accounting for roughness, ε_i is defined as the sum of the macroscopic initial gap ε_a and the microscopic geometric gap ε_r . Furthermore, x_s signifies the rigid body approach, δ_o denotes the normal deformation of the lubricating oil film, and u_{ci} represents the local contact deformation at point i .

Given that the normal deformations induced by bending and shearing exhibit a linear relationship with the applied load, $u_{bi}^{(1)}$ and $u_{bi}^{(2)}$ can be formulated as follows.

$$u_{bi}^{(1)} = \sum_{j=1}^n \eta_{bij}^{(1)} F_j \quad (8)$$

$$u_{bi}^{(2)} = \sum_{j=1}^n \eta_{bij}^{(2)} F_j \quad (9)$$

F_j denotes the load at contact point j . The sum of the bending-shearing compliance coefficients of the gear pair at the contact point is:

$$\lambda_{bij} = \eta_{bij}^{(1)} + \eta_{bij}^{(2)} \quad (10)$$

Combining the equations derived above, the deformation compatibility condition for the contact nodes is expressed as follows:

$$-\sum_{j=1}^n \lambda_{bij} F_j - \delta_o - u_{ci} + x_i + d_i = \varepsilon_i \quad (11)$$

In matrix notation, Eq. 11 can be rewritten as:

$$-[\lambda_b]\{F_j\} - \{\delta\} - \{u_c\} + x_s + \{d\} = \{\varepsilon_a\} + \{\varepsilon_r\} \quad (12)$$

In the matrix formulation, $[\lambda_b]$ denotes the bending-shearing compliance coefficient matrix, where the nodal parameters of the elements are extracted using the substructure method based on the previously established gear finite element model. The vector $\{F_j\}$ represents the load vector corresponding to each contact point, while $\{d\}$ indicates the residual gap vector. Additionally, $\{\delta\}$ denotes the normal oil film deformation vector calculated using Eq. 13, and $\{u_c\}$ represents the contact deformation vector derived from empirical formulas proposed in the literature. Finally, $\{\varepsilon_a\}$ and $\{\varepsilon_r\}$ signify the macroscopic initial gap vector and the microscopic geometric gap vector at each contact point, respectively.

$$\delta_i = F_i / K_{oi} = 1 / (7.69 LE' R (2.65 G^{0.54} U^{0.7} R)^{7.69} h_{min}^{-8.69}) \quad (13)$$

The total normal load P transmitted by the gears must consistently equal the aggregate load borne by the teeth at all times. Specifically, at any given instant and meshing position, the summation of the loads at all contact points distributed across the contact lines must strictly equal P . This load equilibrium relationship is mathematically formalized in Eq. 14.

$$\sum_{i=1}^n F_i = \{I\} \{F\} = P \quad (14)$$

The contact relationship governing the interaction between the load and the gap at each individual contact point satisfies the conditions presented in Eq. 15.

$$\begin{cases} \text{When } F_i > 0, & d_i = 0 \\ \text{When } F_i = 0, & d_i > 0 \end{cases} \quad (15)$$

The flowchart of the iterative solution procedure is illustrated in Figure 4 [7].

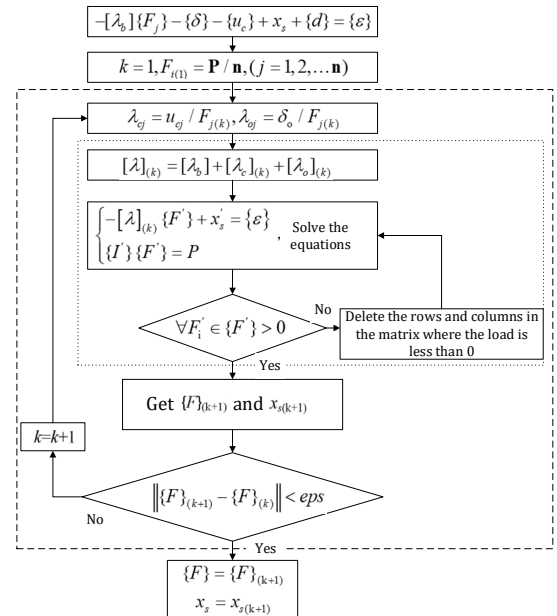


Figure 4. Iterative Solution Flowchart for Contact Equations

Upon completion of the iterative solution for the contact equations, the meshing stiffness and Static Transmission Error (STE) of the gear pair are derived. As illustrated in Figure 5, the stiffness interactions among all contact points are analogous to a system of springs arranged in parallel. Consequently, the total meshing stiffness of the gear pair at a specific meshing position is obtained by summing the stiffness values of all active contact points, as expressed in the following equation [12]:

$$k_m = \sum_{i=1}^n k_i = \sum_{i=1}^n \frac{F_i}{x_s - \varepsilon_i} \quad (16)$$

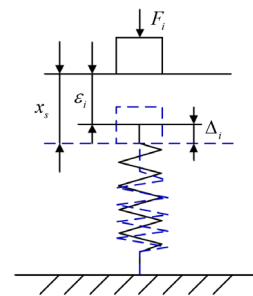


Figure 5. Deformation Relationship at a Single Contact Point

Based on the static equilibrium equation presented in Eq. 14, it follows that:

$$\sum_{i=1}^n F_i = \sum_{i=1}^n [k_i(x_s - \varepsilon_i)] = \sum_{i=1}^n k_i x_s - \sum_{i=1}^n k_i \varepsilon_i = k_m x_s - \sum_{i=1}^n k_i \varepsilon_i = P \quad (17)$$

Defining $e_m = (\sum_{i=1}^n k_i \varepsilon_i) / k_m$ and $C = P / k_m$, Eq. 17 can be expressed as:

$$k_m(x_s - e_m) = P \quad (18)$$

$$e_m = x_s - C \quad (19)$$

Where e_m is defined as the composite meshing error at the current meshing position, and C is referred to as the composite deformation at the current meshing position.

3. Influence of Machining Parameters on Spur Gear Meshing Excitations

3.1. Influence of Grinding Wheel Speed on Spur Gear Meshing Excitations

Utilizing the established loaded tooth contact analysis (LTCA) model, this section analyzes the meshing excitations of spur gears. To isolate the effects of surface topography, the oil film stiffness component is initially deactivated. The topography is characterized by non-uniform sine functions, with gear and grinding parameters detailed in Table 1 and Table 2. Adopting baseline settings of $2\mu\text{m}$ amplitude, 2000 rpm wheel speed, and 10 mm/s feed rate, the time-varying meshing stiffness and static transmission error are computed and analyzed across three distinct grinding wheel speeds.

Table 1. Gear Parameters

Parameter	Value
Number of teeth (Pinion) z_1	37
Number of teeth (Gear) z_2	106
Module m/mm	5
Face width B/mm	100
Addendum coefficient ha^*	1
Clearance coefficient c^*	0.4
Pressure angle $\alpha/(\circ)$	20
Elastic modulus $E_1, E_2/(\text{N}/\text{mm}^2)$	2.1×10^5
Poisson's ratio ν	0.3
Driving gear speed n_1/rpm	1000
Normal load F_n/N	20000

Table 2. Grinding Parameters

Parameter	Value
Grinding wheel speed n_s/rpm	2000
Grinding feed rate $v_w/(\text{mm}/\text{s})$	10
Amplitude $A/\mu\text{m}$	2

(1) Meshing Stiffness

Figure 6 illustrates the time-varying meshing stiffness of spur gears over a meshing cycle, comparing an ideal smooth surface against three grinding speed conditions. The horizontal axis represents the normalized transverse base pitch. Relative to the ideal case, surface roughness causes overall stiffness attenuation due to asperity-induced reductions in contact area and increased local compliance. Higher grinding speeds produce smoother topographies with lower residual heights, driving the stiffness to converge toward the ideal curve. Conversely, rougher textures at lower

speeds introduce larger gaps, significantly compromising the gear's deformation resistance.

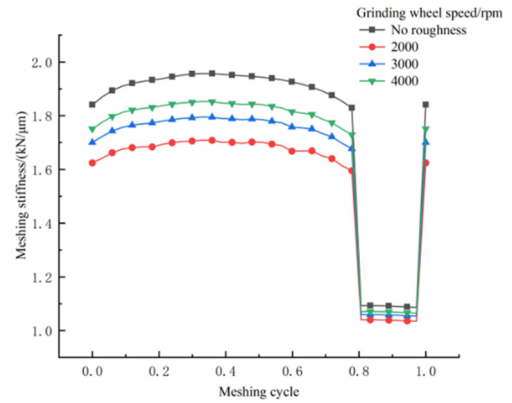


Figure 6. Variation of Spur Gear Meshing Stiffness under Different Grinding Wheel Speeds

(2) Static Transmission Error (STE)

Figure 7 presents the Static Transmission Error (STE) profiles—a critical NVH metric comprising geometric and elastic components—under ideal and various grinding conditions. The STE exhibits periodic behavior inverse to meshing stiffness and increases significantly with surface roughness. At the lower speed (2000 rpm), the reduced stiffness results in peak amplitudes and distinct micro-fluctuations. Conversely, increasing the speed to 4000 rpm enhances surface quality, shifting the STE curve toward the ideal trajectory and suppressing high-frequency oscillations. Consequently, higher grinding speeds effectively mitigate displacement excitations, thereby reducing system vibration.

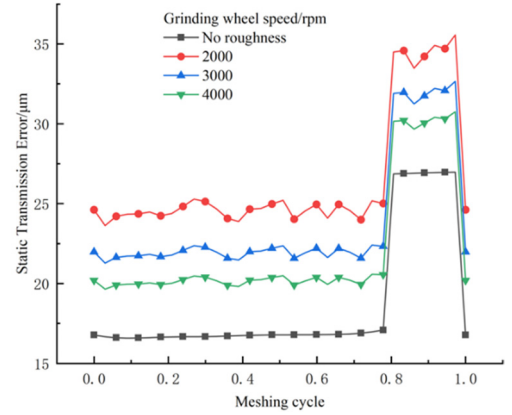


Figure 7. Variation of Spur Gear STE under Different Grinding Wheel Speeds

3.2. Influence of Feed Rates on Spur Gear Meshing Excitations

(1) Meshing Stiffness

Using baseline non-uniform sine parameters (amplitude $2\mu\text{m}$, wheel speed 2000 rpm, feed rate 15 mm/s), the meshing excitations are computed across three distinct axial feed rates. As illustrated in Figure 8, increasing the axial feed rate from 15 to 45 mm/s results in a marked degradation of the time-varying meshing stiffness. This reduction occurs because higher feed rates increase grain cutting thickness and residual heights, thereby deteriorating surface roughness. The resulting deeper textures introduce larger initial contact gaps at the interface, compromising local contact compliance and minimizing the system's resistance to deformation.

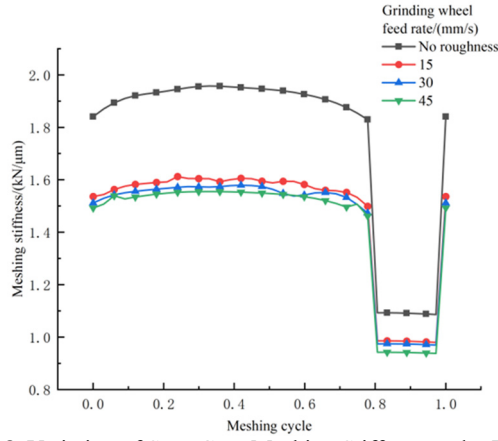


Figure 8. Variation of Spur Gear Meshing Stiffness under Different Grinding Feed Rates

(2) Static Transmission Error (STE)

As shown in **Figure 9**, the mean Static Transmission Error (STE) increases with feed rate, marking a transition from high-frequency, short-wavelength fluctuations to low-frequency, long-wavelength undulations. This shift is attributed to the spatial stretching of grinding vibration trajectories at higher feeds, which extends the texture wavelength. Furthermore, the superposition of roughness-induced geometric deviations and compliance-induced elastic deformations amplifies both the STE amplitude and its fluctuation period.

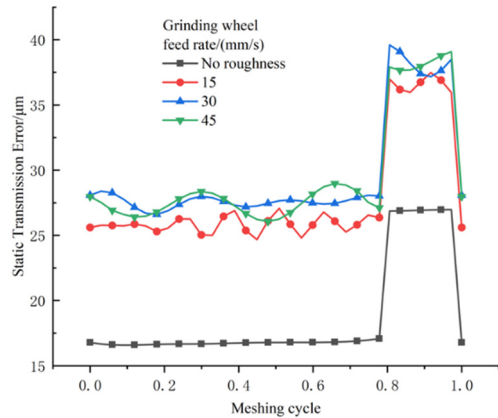


Figure 9. Variation of Spur Gear STE under Different Grinding Feed Rates

3.3. Influence of Roughness Amplitudes on Spur Gear Meshing Excitations

Adopting baseline parameters (grinding wheel speed 2000 rpm, feed rate 10 mm/s), meshing excitations are computed across three distinct roughness amplitudes relative to a 2μm reference. The resulting time-varying meshing stiffness and static transmission error profiles are analyzed.

(1) Meshing Stiffness

The fluctuation amplitude of surface micro-topography serves as a critical geometric indicator of roughness severity. **Figure 10** illustrates the impact of varying sinusoidal amplitudes on the time-varying meshing stiffness under constant grinding kinematic parameters. For comparative analysis, the stiffness profile of an ideal smooth surface is also presented.

As observed in **Figure 10**, grinding-induced micro-amplitudes increase contact interface deformation compared to an ideal surface, causing global stiffness attenuation that intensifies with larger amplitudes. Therefore, strict control of

texture amplitude is critical for system load capacity and stability.

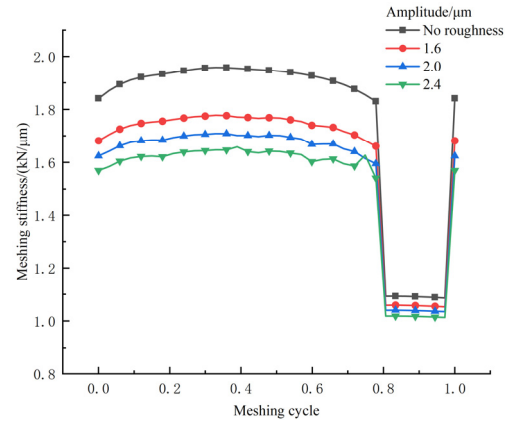


Figure 10. Variation of Spur Gear Meshing Stiffness under Different Roughness Amplitudes

(2) Static Transmission Error (STE)

As shown in **Figure 11**, the STE exhibits a significant positive correlation with roughness amplitude. Increasing the amplitude from 1.6μm to 2.4μm reduces stiffness, thereby amplifying elastic deformation and the mean STE. Furthermore, larger amplitudes alter local fluctuation characteristics, superimposing severe vibration components onto the STE profile compared to the smooth ideal curve.

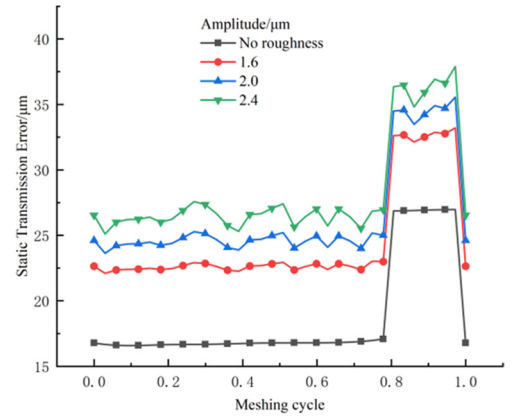


Figure 11. Variation of Spur Gear STE under Different Roughness Amplitudes

4. Influence of Oil Film Parameters on Spur Gear Meshing Excitations

4.1. Influence of Lubricant Dynamic Viscosity on Spur Gear Meshing Excitations

Establishing a baseline with a dynamic viscosity of 36.4 mPa·s, a pressure-viscosity coefficient of $1.85 \times 10^{-8} \text{ Pa}^{-1}$, and a driving gear speed of 1000 rpm, this study analyzes the time-varying meshing stiffness and static transmission error curves of the spur gear pair across three distinct dynamic viscosity values.

Table 3. Lubricating oil Parameters

Parameter	Value
Dynamic viscosity η / mPa·s	36.4
Pressure-Viscosity coefficient α / Pa^{-1}	1.85×10^{-8}
Driving gear speed n_1 /rpm	1000

(1) Meshing Stiffness

Figure 12 illustrates the time-varying meshing stiffness under four operating conditions. The results indicate that the lubricating oil film significantly attenuates meshing stiffness. Acting as a compliant element in series with the gear teeth, the oil film reduces the system's equivalent stiffness compared to dry contact. Increasing dynamic viscosity thickens the oil film, enhancing interface compliance and causing a progressive decline in overall meshing stiffness.

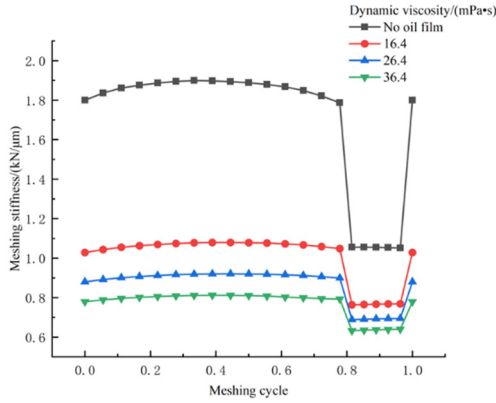


Figure 12. Variation of Spur Gear Meshing Stiffness under Different Dynamic Viscosity

(2) Static Transmission Error (STE)

As shown in Figure 13, STE exhibits a distinct upward trend with increasing viscosity, while the geometric fluctuation characteristics remain unchanged. Under constant load, the oil-induced stiffness reduction amplifies elastic deformation, directly elevating the STE amplitude. This confirms that while higher viscosity improves lubrication, it compromises transmission accuracy.

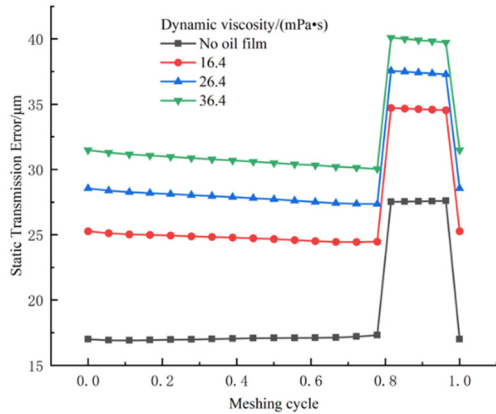


Figure 13. Variation of Spur Gear STE under Different Dynamic Viscosity

4.2. Influence of Lubricant Pressure-Viscosity Coefficients on Spur Gear Meshing Excitations

(1) Meshing Stiffness

Figure 14 illustrates the impact of varying pressure-viscosity coefficients (α) on the time-varying meshing stiffness of spur gears under constant operating conditions. The introduction of an oil film notably attenuates meshing stiffness. As α increases from 1.11×10^{-8} to $2.59 \times 10^{-8} \text{ Pa}^{-1}$, the stiffness curve exhibits a downward trend. This reduction arises because higher α values promote the formation of thicker oil films in high-pressure contact zones. Within the series stiffness model, the thickened film reduces deformation resistance, thereby significantly compromising the system's

overall meshing stiffness.

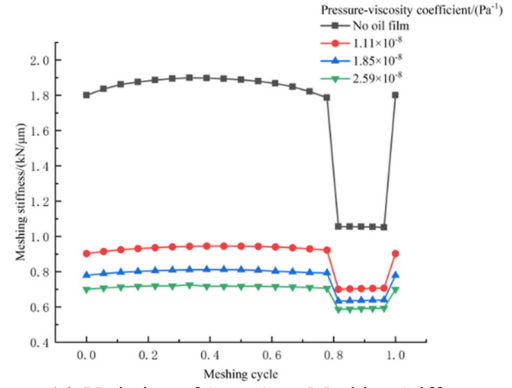


Figure 14. Variation of Spur Gear Meshing Stiffness under Different Pressure-Viscosity Coefficients

(2) Static Transmission Error (STE)

As shown in Figure 15, the Static Transmission Error (STE) exhibits a significant upward trend with increasing pressure-viscosity coefficients, inversely correlating with stiffness. This surge results from a superposition of geometric and mechanical factors: higher coefficients thicken the oil film, introducing additional geometric displacement, while the consequent stiffness reduction amplifies elastic deformation under constant load. Thus, while beneficial for film formation, higher coefficients compromise transmission stiffness and accuracy.

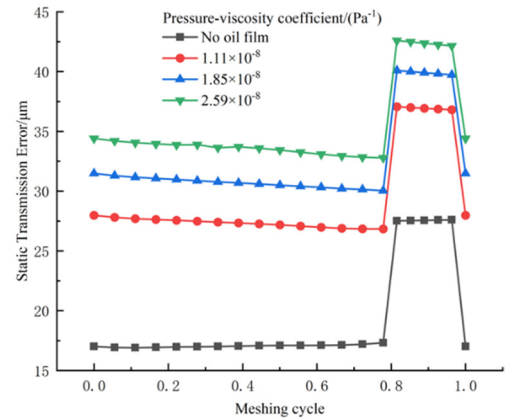


Figure 15. Variation of Spur Gear STE under Different Pressure-Viscosity Coefficients

4.3. Influence of Gear Rotational Speed on Lubricated Spur Gear Meshing Excitations

(1) Meshing Stiffness

Figure 16 illustrates the impact of driving gear speed on the time-varying meshing stiffness under constant load and lubricant conditions. As the speed increases from 1000 to 2000 rpm, the stiffness profile shifts downward, indicating progressive stiffness loss. This attenuation results from the enhanced hydrodynamic effect: higher speeds elevate entrainment velocities, significantly increasing central oil film thickness. In the series stiffness model, the thickened film acts as a compliant medium, thereby reducing the system's equivalent stiffness.

(2) Static Transmission Error (STE)

As shown in Figure 17, the Static Transmission Error (STE) exhibits a stepwise escalation with increasing speed, inversely correlating with stiffness attenuation. Compared to dry

contact, lubricated STE is significantly higher, peaking at 2000 rpm. This surge results from the superposition of geometric and mechanical factors: higher speeds thicken the oil film via hydrodynamic effects, introducing additional normal clearance, while simultaneously reducing contact stiffness, which amplifies elastic deformation under constant load. Consequently, speed-induced oil film dynamics are critical considerations in high-speed transmission design.

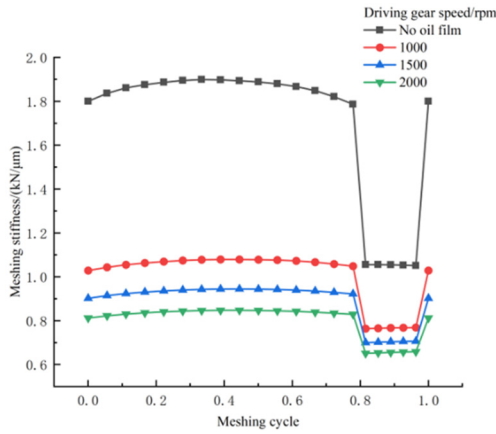


Figure 16. Variation of Spur Gear Meshing Stiffness under Different Driving gear speed

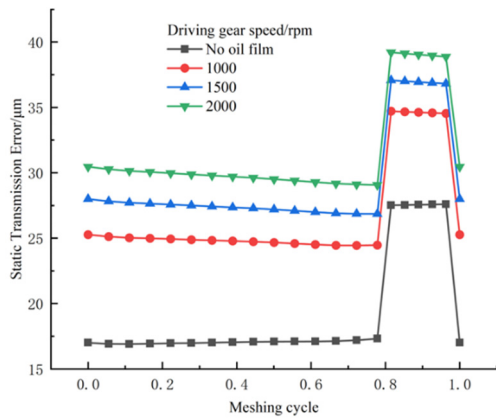


Figure 17. Variation of Spur Gear STE under Different Driving gear speed

4.4. Influence of Surface Topography and Oil Film on Spur Gear Meshing Stiffness

The complex interplay between surface topography and lubrication is often neglected in traditional approaches, which tend to analyze these distinct physical phenomena independently.

(1) Meshing Stiffness

To address this, four contact boundary conditions are compared using baseline parameters. As shown in **Figure 18**, Results indicate that while ideal Hertzian contact yields the highest stiffness, introducing either roughness or an oil film causes significant attenuation. When coupled, the superposition of micro-topography-induced compliance and hydrodynamic compliance further reduces stiffness to its minimum. This phenomenon results in a realistic meshing stiffness significantly lower than theoretical dry contact predictions.

(2) Static Transmission Error (STE)

As shown in **Figure 19**, Static Transmission Error (STE) exhibits an inverse correlation with meshing stiffness and a cumulative increasing trend. While the baseline condition yields minimal error, introducing roughness shifts the profile

upward. The intervention of an oil film causes a substantial surge by introducing geometric clearance and stiffness softening effects. Under coupled conditions, the STE peaks with superimposed micro-fluctuations. This confirms that both roughness and oil film are critical sources of transmission accuracy loss, necessitating a coupled model to prevent significant evaluation bias.

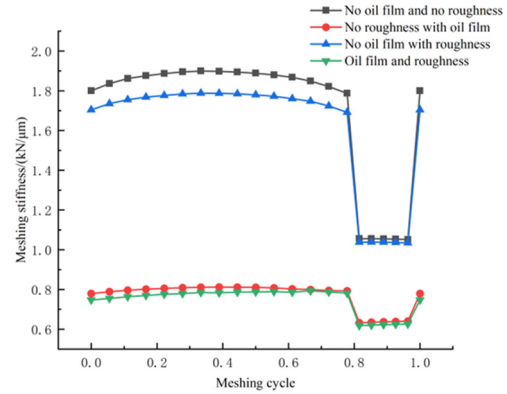


Figure 18. Variation of Spur Gear Meshing Stiffness under Different Contact Conditions

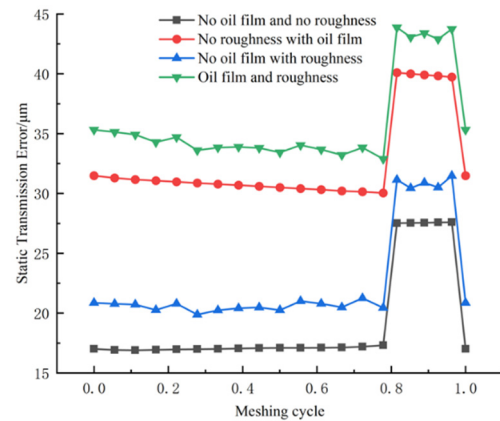


Figure 19. Variation of Spur Gear STE under Different Contact Conditions

5. Conclusion

A loaded tooth contact analysis (LTCA) model integrating surface topography and oil film stiffness was developed to investigate the mechanisms of spur gear meshing excitations.

Research indicates that higher grinding wheel speeds improve surface finish, driving meshing stiffness and Static Transmission Error (STE) to converge toward ideal smooth conditions. Conversely, increasing roughness amplitudes or axial feed rates deepens micro-textures, causing stiffness attenuation and a significant rise in STE amplitudes, with higher feed rates also elongating the STE wavelength.

Regarding lubrication, increases in dynamic viscosity, pressure-viscosity coefficients, or driving gear speed thicken the oil film. This enhanced film compliance reduces the overall meshing stiffness and escalates the global STE level.

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