

Comprehensive Study on Airfoil Optimization Methods

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Abstract. With the advancement of technology and the increasing integration of aircraft into daily life, there is a growing demand for enhanced wing optimization. In response, this study provides a summary and discussion of common research directions and achievements in airfoil optimization, with the aim of identifying optimal solutions. Based on a review of the literature and an examination of core airfoil theory, four key research directions are identified: data measurement, airfoil structure, functional diversity, and environmental adaptability. Recent advancements in artificial intelligence (AI) have been extensively integrated into data measurement processes, particularly through the application of AI models for simulating experiments across diverse scenarios. This integration has significantly enhanced data accuracy while reducing experimental costs. The study concludes that the primary challenge remains achieving high-precision measurement of airfoil data, while functional innovation represents a prominent and actively pursued research focus. With the increasing range of applications for aircraft wings, there has been a growing demand for airfoil profiles that can perform effectively across diverse and demanding scenarios. Examples include hovering technologies and the adaptability of wing materials to extreme environmental conditions.

Keywords: Airfoil Optimization; Bernoulli's Principle; Data Measurement Methods; Wing Design.

1. Introduction

In the past ten years, with the development of science and technology, aeroplanes, drones, helicopters and other flyable vehicles have gradually entered everyone's life. According to the World Civil Aviation Organization, the number of global flights reached 2.28 billion flights in 2021 which can be seen as aeroplanes have become a common long-distance means of transportation in people's lives. In addition, at the opening ceremony of the 2020 Beijing Olympic Games, 1,824 drones participated in an important performance, displaying the pattern through techniques such as hovering.

In summary, the aforementioned applications all utilize the special airfoil structure of wings to achieve flight. Consequently, research on wing profiles has become increasingly extensive. Based on the fundamental principles of Bernoulli's equation, it can be inferred that the airfoil design is the primary factor affecting wing lift. Thus, the optimization of wing airfoil profiles has emerged as a major focus of discussion.

This paper integrates and analyzes various airfoil optimization research approaches to derive optimal solutions for airfoil enhancement. Optimization is conducted across four dimensions: refinement of data measurement methodologies, advancement of fundamental airfoil structure, diversification of wing functionalities, and contextual adaptation for specific operational scenarios.

Refining data measurement techniques enables the acquisition of more accurate experimental data, thereby reducing the risk of irreversible distortions in subsequent research caused by systematic errors. Enhancements in fundamental airfoil structure built upon theoretical studies rooted in Bernoulli's equation, with the geometric definition of the airfoil itself representing a primary focus.

The functional diversification of wings reflects evolving demands in modern science and technology. As performance expectations continue to rise, wings are increasingly employed across a broader range of applications, requiring capabilities that extend beyond basic lift and descent.

It is also essential to account for varying environmental conditions. Examples include deployment in high-risk environments where human access is limited—such as disaster zones for rescue operations or remote topographic mapping. Under such circumstances, material characteristics including compressive strength, thermal stability, and abrasion resistance must be carefully considered.

2. Refinement of Data Measurement Methodologies

This paper introduces five methods for optimizing the accuracy of measurement data, catering to a wide range of optimization scenarios. These include Text-to-Airfoil Generation via FoilCLIP [1], Data-Driven Surrogate Modeling [2], ICP-Based Measurement Techniques [3], Multi-Fidelity Design Optimization [4], Numerically Stable RANS Solvers [5].

2.1. Text-to-Airfoil Generation via FoilCLIP

As illustrated in Fig.1, As for Text-to-Airfoil Generation via FoilCLIP. Recent breakthroughs in contrastive language-image pre-training (CLIP) and generative AI have significantly advanced cross-modal understanding and content generation. Integrating these breakthroughs, this study formulates a new framework that utilizes natural language processing (NLP) for aerodynamic shape optimization [1]. It bridges the gap between natural language and geometric design by establishing a complete, reversible correspondence. (e.g. “a low-drag supercritical airfoil under transonic conditions”) and parameterized geometric shapes characterized by CST parameters. By integrating a CLIP-inspired contrastive learning model for text-airfoil alignment with a semantics-driven decoder, our approach generates physically viable geometries. Experimental results confirm their efficacy in producing aerodynamically optimal airfoils from text and in text-based airfoil classification. This research democratizes the preliminary airfoil design process and showcases the promise of human-AI collaboration in aerospace engineering.

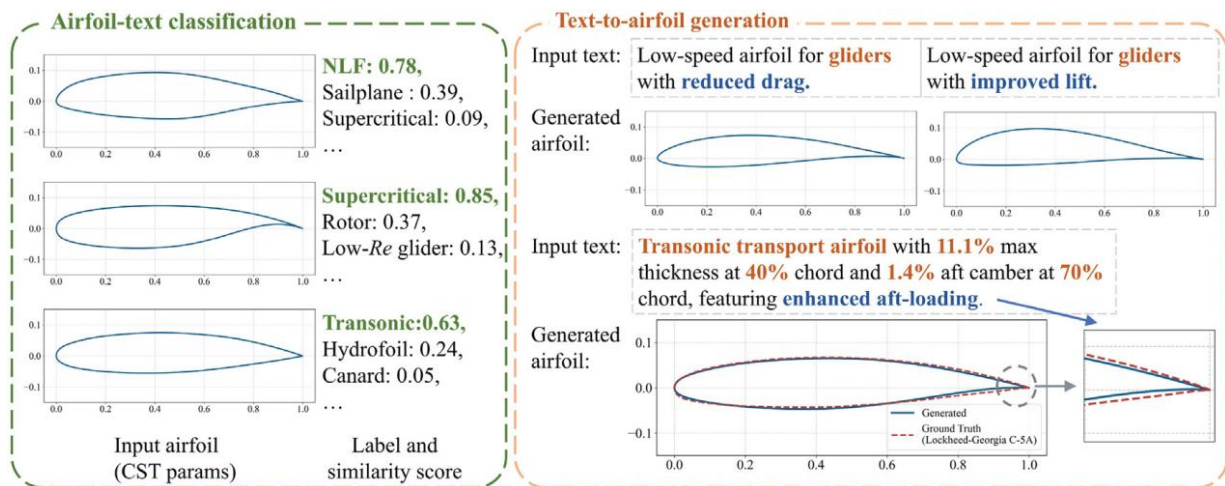


Fig 1. Schematic diagram of FoilCLIP's core bidirectional tasks [1].

2.2. Data-Driven Surrogate Modeling

Regarding Data-Driven Surrogate Modeling. Shape optimization of ship hulls and airfoils is crucial for enhancing performance and reducing environmental impact. This survey focuses on recent advances (since 2015) in data-driven surrogate (DDS) models for such optimization—a yet underexplored area [2]. It systematically reviews the four-step CFD-driven DDS framework, highlights the integration of surrogates with CFD, identifies research gaps, and explores future directions like machine learning. The study bridges traditional approaches with emerging innovations, offering insights for both academia and industry.

2.3. ICP-Based Measurement Techniques

Turning to ICP-Based Measurement Techniques. The approach enables a more precise and detailed assessment of wing geometry and its relationship to aerodynamic performance [3].

Validation using synthetic data demonstrates that the method can effectively identify both local and global geometric errors—such as camber, thickness variation, and shape deviations—with high sensitivity. Although the introduction of Gaussian noise revealed certain limitations in estimating leading-edge curvature, the overall accuracy remains acceptable for practical applications. The

method shows particular strength in distinguishing between different types of geometric errors and correlates well with aerodynamic performance metrics. It offers a valuable tool for quality control in manufacturing, providing manufacturers with the ability to ensure that produced airfoils closely match theoretical designs. This supports more efficient and cost-effective production while meeting strict aerodynamic functional requirements. Future work may focus on integrating noise reduction techniques to enhance robustness and extending the methodology to broader aerospace design and inspection applications.

2.4. Multi-Fidelity Design Optimization

In terms of Multi-Fidelity Design Optimization. This paper presents a computationally efficient optimization method for transonic airfoil design [4]. The approach employs a low-fidelity surrogate model based on modified physics to replace high-fidelity simulations during optimization. The shape-preserving response prediction technique enhances the low-fidelity solution, bringing its pressure distribution into agreement with the high-fidelity model. Applied to a 2D inviscid transonic flow case for lift maximization under constraints of wave drag and cross-sectional area, the method reduces the computational cost of high-fidelity evaluations by more than 90% compared to a direct approach.

2.5. Numerically Stable RANS Solvers

With respect to Numerically Stable RANS Solver. This study addresses the challenge of robustness in RANS solver-based gradient optimization for aerodynamic shape design, particularly under highly separated flow conditions that often lead to divergence or limit cycle oscillations [5]. Traditional reliance on averaging non-converged solutions tends to yield misleading gradients, resulting in poor optimization outcomes. To overcome this, an improved version of the BoostConv method is introduced, employing complex-step derivative technology to produce mathematically accurate gradients and the proposed technique innovatively decouples the modal decomposition of the complex-valued residuals into the primary field quantity (real part) and its spatial derivative field (imaginary part), thereby enhancing the stability of earlier BoostConv implementations. Evaluations using chaotic logistic maps and separated flow around a bluff body demonstrate that the gradients from fully converged solutions are significantly more accurate than those from average non-converged results. Starting with an 80%-thick bluff body, this method successfully optimizes the shape to an 18%-thick airfoil with high lift-to-drag ratio—consistent with the optimized NACA 0018, indicating a global optimum. In contrast, optimization using average grades fails to improve the design. This approach underscores the critical importance of convergence-accurate gradients for guiding optimizers into attached flow regimes where RANS models are more reliable.

3. Advancement of fundamental airfoil structure

This work focuses on three main aspects of airfoil structure optimization: simultaneous surrogate modeling and PDE-constrained optimization of airfoil geometry, aerodynamic analysis and aerodynamic optimization.

3.1. Simultaneous surrogate modeling

As for simultaneous surrogate modeling and PDE-constrained optimization of airfoil geometry. This work investigates the use of neural networks (NNs) as surrogate models for partial differential equation (PDE) solvers, leveraging their inherent differentiability to enable efficient gradient-based optimization of performance metrics under PDE constraints. Traditional PDE solvers map input parameters—such as airfoil shape, Reynolds number, and Mach number—to field variables like velocity, pressure, and temperature, and can optionally output performance quantities such as lift-to-drag ratio. While these field variables are sufficient in some contexts, users often seek to optimize performance metrics by adjusting parameters, as seen in applications like airfoil drag reduction, acoustic metamaterial design, horn geometry optimization, or multi-core electrical cable design.

Although optimization can be performed using gradient-free methods, these typically require a prohibitive number of direct evaluations when the number of parameters is large. Gradient-based optimization is more efficient, as it computes the gradient of the performance metric with respect to the parameters via the chain rule, often in a single computation. However, obtaining such gradients usually requires either solving adjoint equations or having a differentiable PDE solver—both of which often entail significant modifications to existing simulation codes.

Neural networks (NNS), by design, are automatically differentiable. The gradient of a loss function (e.g., the difference between predicted and target performance) with respect to the network parameters can be computed efficiently and used with gradient descent [6]. As universal function approximators, NNs can be trained to act as surrogate PDE solvers. Moreover, they can be constructed to simultaneously predict performance metrics and their gradients with respect to all input parameters.

In this paper, we explore whether NNs can serve as surrogate PDE solvers while exploiting automatic differentiation to solve PDE-constrained optimization problems—without the structural modifications required by conventional solvers. We focus on airfoil shape optimization due to its particular challenges for traditional grid-based PDE solvers. The approach presented here is general and can be extended to a broad range of PDE-constrained optimization problems.

3.2. Aerodynamic analysis

As for an aerodynamic analysis. The study of how topological changes in impermeable immersed bodies affect fluid forces has long been a complex and fascinating challenge in aerodynamics. The spatial distribution of an object—apart from flow conditions—significantly alters the nature of its interaction with the fluid (Hörner). Typically, the shape of a body can be described by a set of parameters that uniquely determine the flow of the field and its interaction with the object. Properties such as second-derivative properties, surface integrals, cross-sectional measures, and continuity constraints profoundly influence flow behavior, including critical phenomena like the transition process, turbulence development mechanisms, flow separation zones, and reattachment locations. These flow features, consequently, govern overall performance indicators like lift generation, drag resistance, and propulsive efficiency, all of which are critical for practical implementations.

Furthermore, understanding and controlling these relationships facilitate optimized designs that enhance overall efficacy, handling and stability characteristics, and energy utilization efficiency. The scientific community has devoted extensive efforts to developing shapes with high lift and low drag to improve the performance and efficiency of flight vehicles. These studies have accumulated substantial expertise and methodologies, providing valuable resources for airfoil development.

Early aerodynamic design practice progressed from empirical approaches to inverse problem-solving strategies, using tools such as conformal mapping to adjust shapes according to target velocity distributions. These approaches, including multi-point expansion techniques, the development of sophisticated airfoils was facilitated, yet it demanded substantial domain knowledge to establish appropriate target properties. Derivatives such as the hybrid inverse approach provided constrained regional adjustment, though they exhibited limited adaptability for shape variation.

The transition toward direct optimization—the automated adaptation of geometric variables to fulfill aerodynamic goals—was initially constrained not only by the substantial computational expense associated with iterative flow simulations, but also by the proficiency of conventional design approaches, which had already yielded highly refined airfoils extensively utilized in industry. With the expansion of computational capabilities, direct optimization has emerged as both practicable and preferable. It facilitates the investigation of an expanded design domain, the integration of sophisticated constraints, and the explicit optimization of performance objectives, thereby creating new opportunities for precision and advancement in aerodynamic shape design.

Hicks et al. applied this approach to transonic airfoils, demonstrating promising results despite high computational demands [7]. To maintain design flexibility while reducing dimensionality, shape parameterization schemes based on polynomial combinations and convex functions were proposed, laying the foundation for more efficient optimization. Jameson later reframed airfoil design as a

control problem and extended Pironneau’s adjoint method to fluid dynamics, enabling a wide range of multidisciplinary optimization studies.

3.3. Aerodynamic optimization

As for aerodynamic optimization. We refer to the figure for illustration. A versatile bi-level surrogate modeling framework has been successfully applied to various robust design optimization challenges with moderate dimensionality and uncertainty, as documented in.

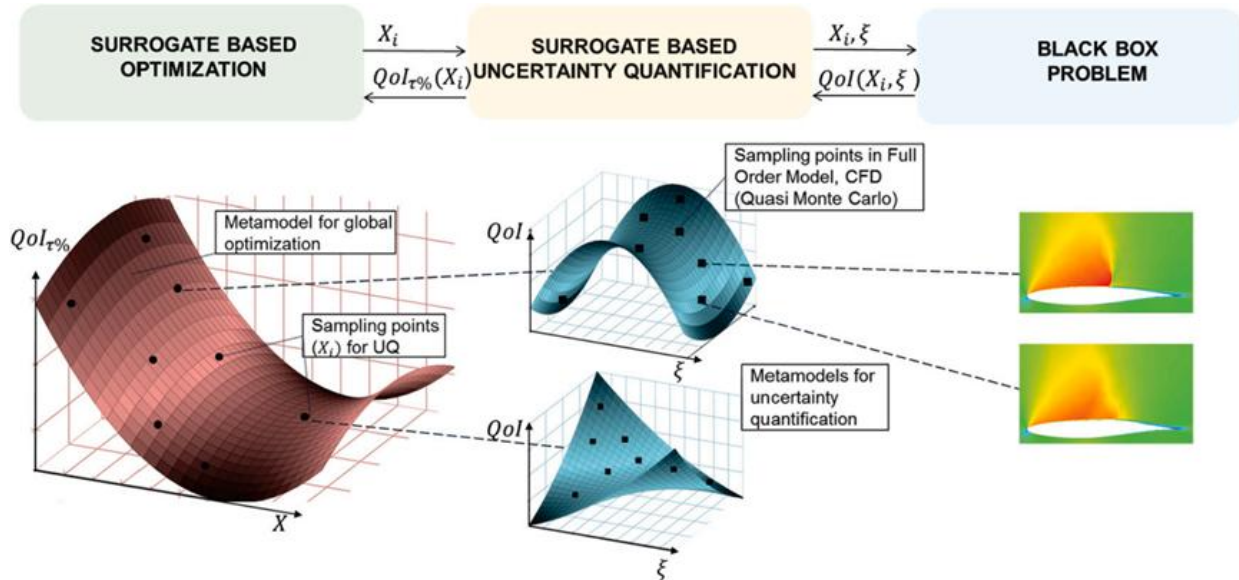


Fig 2. Aerodynamic optimization framework [8].

As illustrated in Fig.2, the framework consists of three components: statistical surrogate (left), stochastic variable substitution (center), and full-order model evaluation (right).

The proposed framework integrates two cycles: The overall framework consists of two nested cycles: the outer cycle performs optimization using a surrogate model (SBO), whereas the inner cycle carries out uncertainty quantification also via surrogate modeling (SBUQ). During each optimization step, only the design parameters are passed to the SBO surrogate (shown as the red response surface), which outputs statistical estimates of the performance metric. On the other hand, the SBUQ surrogate (illustrated as the blue response surface) takes fixed but uncertain design parameters as inputs and computes the performance metric through a black-box function.

This gradient-free robust optimization framework is particularly advantageous in CFD simulations involving transition predictions such as those using the method or the DLR- model—where obtaining adjoint solutions is often infeasible [8]. Overall efficiency is further enhanced through adaptive sampling techniques that refine the regions near optimal solutions. Additionally, computational performance is improved via parallel evaluation of design of experiments (DoE) and by leveraging the inherent parallelism of the black-box CFD solver.

4. Different types of airfoil

Considering that wings come in different types, the optimization solutions required also vary. This paper takes three aspects as examples: large airfoil models, vortex generators, and UAV airfoils under atmospheric icing conditions.

4.1. Large Airfoil Model

The development of the Large Airfoil Model (LAM) represents an innovative approach to addressing aerodynamic challenges in airfoil design, requiring extensive datasets and corresponding modeling support. To this end, a novel probabilistic machine learning tool, the Airfoil Deep-learning-based Adaptive Prediction Tool (ADAPT), has been developed. By incorporating experimental

measurements and accounting for sensor inaccuracies, ADAPT provides predictions of airfoil pressure coefficient distributions with inherent uncertainty quantification. The approach employs deep kernel learning to conduct Gaussian process regression within a ten-dimensional embedded feature space, which is constructed using neural networks. This enables robust processing of irregularly sampled and noisy experimental data.

Concurrently, A major milestone has been reached with the introduction of the first large-scale open-source platform for experimental airfoil data—the Airfoil Surface Pressure Information Repository of Experiments (ASPIRE) [9]. This collection integrates historical datasets dating back a century with modern experimental studies, creating a distinctive repository of physical pressure measurements that addresses a key limitation of conventional databases, which depend heavily on computational results.

Tests conducted on three common airfoil shapes confirm that ADAPT delivers accurate forecasts of pressure patterns and key aerodynamic parameters across various operating conditions, achieving an enclosed mean absolute error (MAE_{enclosed}) of just 0.029. ASPIRE and ADAPT form the basis for a conversational airfoil evaluation system enhanced by large language models, which supports engineering design through intuitive language-based commands rather than structured technical inputs.

4.2. Vortex generators

As for vortex generators. Driven by rising oil prices, increasing energy demands, and growing environmental awareness, the wind energy industry has experienced rapid development in recent decades. Over the past thirty years, the size of new wind turbines has significantly increased—rotor diameters have grown from the initial 10–15 meters to the 236-meter offshore model recently introduced by Vestas. The design and development of such large turbines pose major challenges for engineers, who must strike a balance between lightweight, low-cost materials and aerodynamic performance.

With advances in design technology, modern wind turbines employ longer and thinner blade structures that endure higher static and dynamic loads compared to earlier generations. Novel design approaches achieve structural optimization by increasing the thickness of the inner mid-section of the blade—particularly through flatback airfoil designs—thereby reducing blade weight while improving aerodynamic efficiency. However, as blade radius increases, elastic deformation also becomes more pronounced, leading to greater dynamic loads on the turbine. These loads typically result from combined effects such as gusts, rapid pitch changes, yaw misalignment, wind shear, and aeroelastic torsion induced by rotation.

Dynamic flow conditions are generally classified based on the reduced frequency range in wind turbines. The reduced frequency is defined as $k = \Omega C / 2V$, where V is the relative velocity of the airfoil, Ω is the angular frequency, and C is the chord length. Leishman classified unsteady flow conditions in rotorcraft into four categories: steady flow at $k=0$; quasi-steady flow for $0 \leq k \leq 0.05$, where unsteady effects are weak and negligible; unsteady flow for $k \geq 0.05$; and highly unsteady flow for $k \geq 0.2$. In contrast, Pereira et al. suggested that unsteady effects should be considered in wind energy applications when $k > 0.02$.

Leishman further categorized sources of unsteady aerodynamic loads into periodic and non-periodic types. Non-periodic sources include wind misalignment, atmospheric turbulence, wake induction, and topological rotor effects, while periodic sources comprise yaw mismatch, blade-tower interaction, and wind shear [10]. Due to these phenomena, blades operate at varying angles of attack during rotation, leading to differential loading and influencing wake development and flow induction effects.

Dynamic stall, caused by rapid changes in the angle of attack, alters the state of the boundary layer over the airfoil surface. This delays the onset of stall beyond the static stall angle and further postpones flow reattachment, resulting in hysteresis effects that exceed those under static conditions.

Such behavior can expose wind turbines to extreme loads, potentially causing structural damage or significantly reducing fatigue in life.

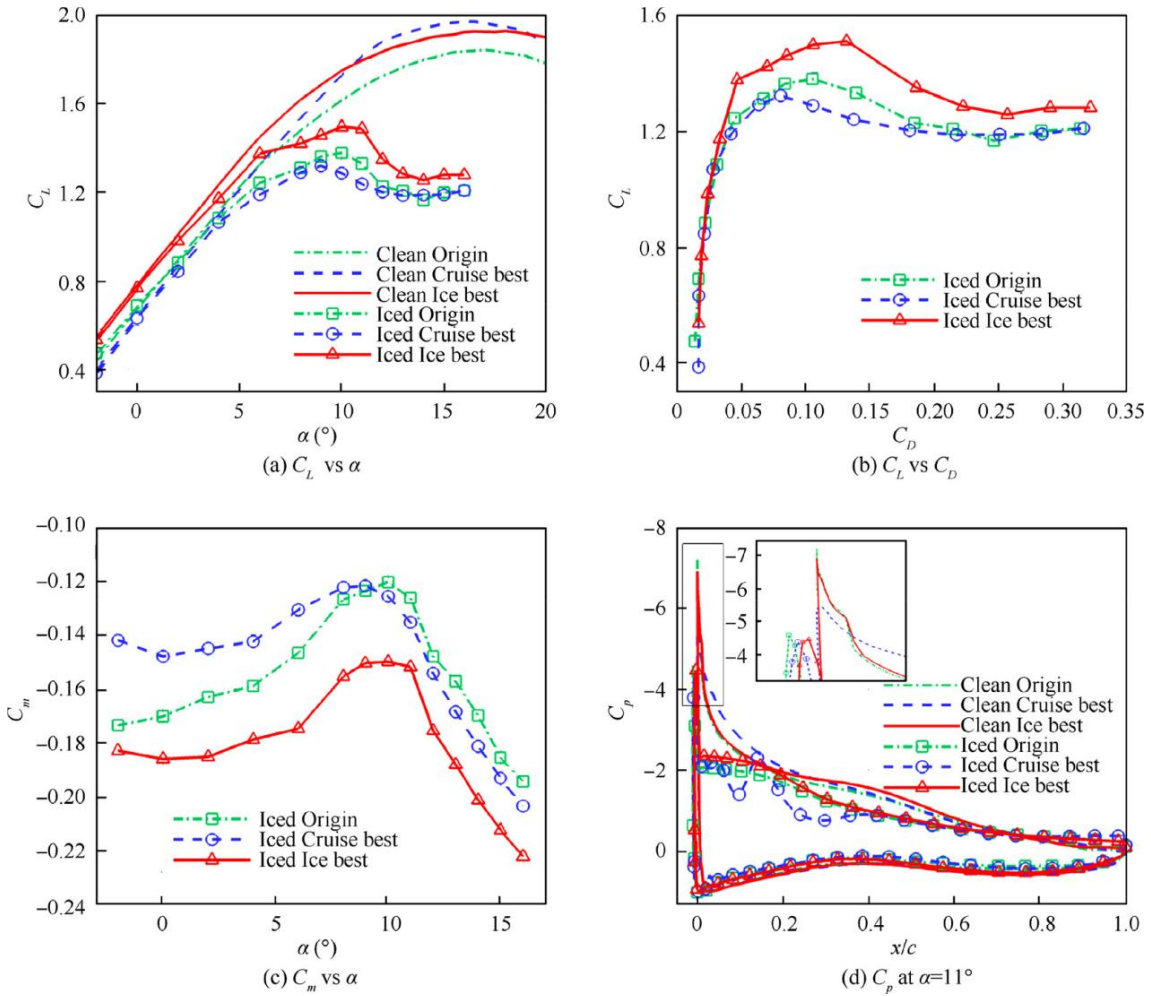


Fig 3. Comparisons of the force coefficients and pressure distributions of typical airfoils [11].

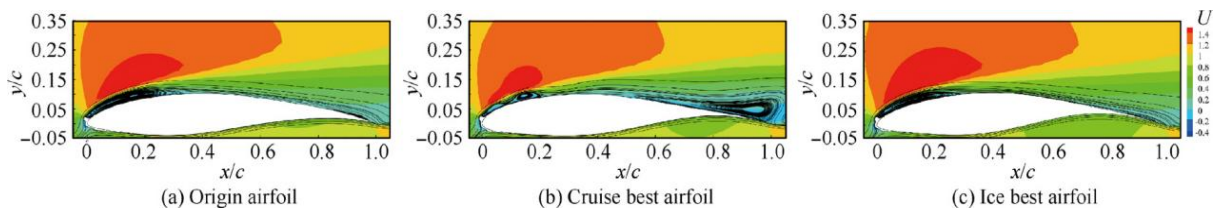


Fig 4. Flow fields of the three airfoils at $\alpha = 11$ [11].

4.3. Ice-accretion shape distinct

The "Best Cruise" configuration features an ice-accretion shape distinct from other cruise designs. Figure 4 compares force coefficients and pressure coefficient (C_p) distributions near the stall angle for iced airfoils. The curves in Fig. 3(a) illustrate the lift performance in both clean and iced conditions. Through optimization, both "Best Cruise" type airfoils—without ice—exhibit improved lift performance. The "Best Cruise" design in clean condition achieves the highest maximum lift coefficient, while the "Best Ice" design demonstrates a longer linear region in the lift curve. Under specific atmospheric icing conditions, the "Best Ice" airfoil maintains higher lift coefficients across all angles of attack compared to the other two designs. Furthermore, the iced "Best Ice" airfoil attains the largest stall angle, whereas the iced "Best Cruise" airfoil experiences the earliest stall.

Ice accretion significantly increases drag. As shown in Fig.3(b), at the same critical lift coefficient (CL), the “Best Ice” airfoil exhibits a lower drag coefficient (CD), contributing to reduced power requirements. The moment coefficients in Fig. 3(c) indicate that the iced “Best Ice” airfoil generates a higher pitching moment (Cm), with relatively small variations across angles of attack.

Figure 3(d) compares pressure distributions at $\alpha = 11^\circ$ for all three airfoils under clean and iced conditions. In the clean state, the “Best Cruise” airfoil shows a reduced peak suction pressure due to its larger leading-edge radius. After icing, the leading-edge geometry of all airfoils is altered into a non-streamlined form, resulting in similar peak suction values. The iced “Best Ice” airfoil exhibits the highest pressure plateau in the region $0 < x/c < 0.2$. In contrast, the “Best Cruise” design displays fluctuations in the pressure coefficient (Cp) due to an unsteady separation bubble simulated on its upper surface.

Figure 4 presents flow fields for the three representative iced airfoils, with contours indicating the dimensionless streamwise velocity U [11]. Separation bubbles form near both the leading and trailing edges on the upper surface. The iced “Best Cruise” airfoil exhibits an unsteady leading-edge separation bubble and a larger trailing-edge bubble, resulting in lift fluctuations and early stall. The presented lift coefficients are obtained via time-averaging. The “Best Ice” design maintains a relatively small separation bubble, which minimally influences the effective airfoil shape.

In summary, optimization focusing solely on cruise performance leads to degraded performance under icing conditions. The present bi-objective optimization approach demonstrates that both clean and iced performance can be improved simultaneously under certain atmospheric icing conditions. Advancement of fundamental airfoil structure

This work focuses on three main aspects of airfoil structure optimization: simultaneous surrogate modeling and PDE-constrained optimization of airfoil geometry, aerodynamic analysis and aerodynamic optimization.

5. Diversified applications in aerodynamics

As mentioned in the introduction, modern technological advancements require more diversified applications in aerodynamics. In response to this year's hot topics in scientific development, this paper explores topics such as "International Journal of Heat and Fluid Flow," "Aerodynamic Design Optimization of Helicopter Rotor Blades," and "Structural Topology Optimization."

5.1. International Journal of Heat and Fluid Flow

This study showcases the potential of deep reinforcement learning (DRL) in active flow control (AFC) applications for a two-dimensional NACA 0012 airfoil operating at a Reynolds number of 3000. By deploying a DRL agent capable of real-time dynamic actuation, the efficacy of this method in minimizing drag and enhancing aerodynamic performance is demonstrated [12].

Initially, the DRL method was designed to achieve significant aerodynamic drag reduction while maintaining lift performance through the introduction of additional regularization constraints. This resulted in a 43.4% reduction in drag; however, it also led to a 35.9% decrease in lift.

5.2. Aerodynamic Design Optimization of Helicopter Rotor Blades

This work introduces a methodology for the aerodynamic optimization of helicopter rotor blade profiles to improve hover efficiency. A novel geometric parameterization technique utilizing the Class Function/Shape Function Transformation (CST) is applied to generate airfoil sections, treating the contour geometry as a primary design variable [13]. The optimization system incorporates multiple interdependent modules developed by the authors. Key design parameters consist of twist distribution, taper ratio, taper initiation position, root chord dimension, and the coefficients defining the airfoil profile distribution function.

Aerodynamic constraints incorporate available power limits in both hover and forward flight, with the requirement to maintain a balanced state. As this work focuses exclusively on rotor blade

configuration in hover, the power required in hover is selected as the objective function for the optimization. Sensitivity analysis of the design variables indicates that airfoil shape significantly influences rotor performance.

The optimized rotor blade achieves a 7.4% reduction in hover power requirement and a 6.5% improvement in the performance figure of merit, representing a substantial advance in rotor design.

5.3. Structural Topology Optimization

As illustrated in Fig.5, As for the last kind. This study proposes a topology optimization (TO) strategy for the design of aircraft slat structures that considers both aerodynamic load-bearing capacity and the use of multi-layer composite materials [14]. The resulting structure exhibits two characteristic features: a topologically optimized cross-section and a functionally graded layout.

The optimized slat structure is filled with aluminum foam and encapsulated within a sandwich cladding, which is fabricated by laminating an aluminum honeycomb layer with composite face sheets. For comparison, the arrangement of the three-layer structure is also topologically optimized using an interpolation method. Boundary conditions incorporate dynamic shape, de-icing tubes, and two typical aerodynamic load cases. The proposed design is subsequently validated via bird impact simulation using smoothed particle hydrodynamics (SPH) [14].

Compared to the original metallic slat structure, the TO-based sandwich shell demonstrates significantly improved stiffness under aerodynamic loading. The optimized design achieves a 28.9% reduction in structural weight, absorbs 24.1% more impact energy in a typical bird strike scenario, and exhibits a 1.28% reduction in damaged area across a 2.88-meter-long slat. This design can be industrially manufactured using matched-mold tooling processes.

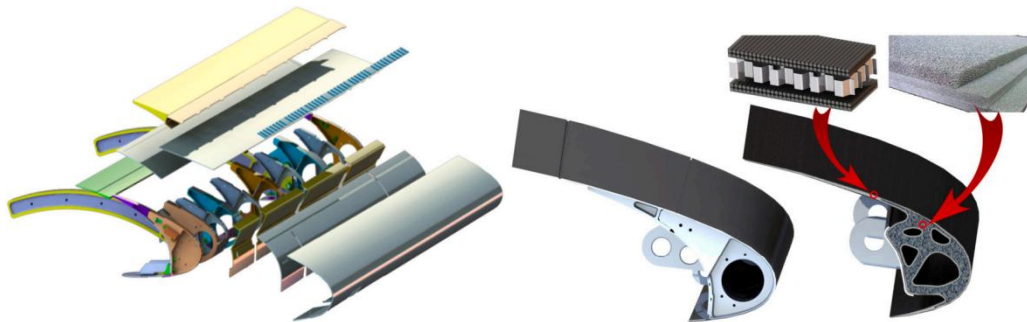


Fig. 5 Typical slat structure compared with the proposed topology optimized slat layout.[14].

6. Conclusion

This study summarizes several common optimization approaches and major research directions for airfoil design currently available on the market. The most critical aspects lie in the enhancement of measurement accuracy and the incorporation of AI-based simulation models. Based on the fundamental theory of Bernoulli's principle, it is evident that data precision is crucial for experimental research; hence, the majority of research teams regard this as a key breakthrough point. In addition to core theoretical foundations, some research teams have also integrated recent technological advancements that reflect evolving demands for airfoil performance—such as functional innovations and adaptability to extreme environments. In line with the objectives of this study, future research on airfoil design will be more targeted. This will enable subsequent research teams to identify research directions more rapidly and accurately, delve into primary challenges, and develop more comprehensive experimental frameworks.

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