

Optimizing EGR and Air Mixing Uniformity in Intake Manifold through CFD Simulations

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Abstract. Optimizing exhaust gas recirculation (EGR) mixing in the intake manifold is a crucial step for reducing nitrogen oxides (NO_x) emissions in multi-cylinder engines while maintaining combustion stability. In this study, the computational fluid dynamics (CFD) method was used to systematically evaluate the impact of EGR introduction strategies (injection position, angle, and EGR ratio) on the uniformity of EGR and air mixing in the intake manifold of a four-cylinder engine. The results show key trade-off relationships and design principles. With a higher EGR ratio, distribution uniformity improves, but local mixing quality degrades. EGR introduction should be placed far from the plenum, as a longer flow path can enhance turbulent dispersion. Most notably, an against-flow injection has a significant impact on mixing. Compared with the vertical-flow and the co-flow, EGR can mix more thoroughly with air using against-flow injection. These research findings provide practical and feasible insights for designing efficient exhaust gas recirculation systems. The overall optimization approach is highlighted in the future EGR system design process (optimizing the injection strategy alongside different intake manifold structures).

Keywords: Exhaust Gas Recirculation (EGR); Intake manifold; Mixing uniformity; Computational Fluid Dynamics (CFD); Engine emission reduction.

1. Introduction

Under both the pressures of global climate change and the carbon peaking and carbon neutrality goals, emission-reduction technologies have become the key to enabling the continued development of the transportation sector [1]. Despite the rapid growth of related new energy technologies, internal combustion engines (ICEs) still play an irreplaceable role in transportation and industrial applications due to their high energy density and mature technology [2-4]. However, the use of internal combustion engines results in significant nitrogen oxide (NO_x) emissions, posing a serious threat to human health and the environment. To enhance engine efficiency while reducing emissions, exhaust gas recirculation (EGR), which recirculates a portion of the exhaust gas back into the cylinders, has been widely adopted [5]. The introduction of EGR increases the mixture's total heat capacity, thereby reducing the peak combustion temperature and significantly decreasing NO_x emissions [6,7]. Furthermore, the exhaust gas dilutes the intake air's oxygen content, resulting in milder combustion. In gasoline engines, this also helps reduce pumping losses and prevent detonation, thereby increasing thermal efficiency [8].

Although the Exhaust Gas Recirculation (EGR) technology has shown potential to reduce nitrogen oxides (NO_x), its actual effectiveness is often affected by the uneven distribution of EGR and the uniformity of EGR and fresh air mixing [9]. This unevenness is mainly due to the intake manifold's structure. When the EGR introduction strategy is not specifically optimized, incomplete mixing occurs, resulting in differences in exhaust gas recirculation concentration between cylinders [9,10]. This uneven distribution will result in less EGR effect than expected. A smaller exhaust gas recirculation (EGR) mixing ratio will also lead to less inhibitory effect of EGR technology on peak combustion temperature. This will increase thermal generation of nitrogen oxides in some local regions, thereby offsetting the benefits of overall emission reduction [10]. On the contrary, a higher exhaust gas recirculation (EGR) mixing ratio would increase the risk of unstable combustion, leading to a higher coefficient of variation of the indicated mean adequate pressure (COV of IMEP), incomplete combustion, and even detonation [10]. These effects not only increase emissions but also

adversely affect fuel economy and driving performance [10]. Therefore, optimizing the EGR introduction strategy is vital for achieving a uniform mixture, ensuring that all cylinders operate with continuous low NO_x emissions and stable combustion conditions.

Based on the above, this study aims to investigate the impact of EGR introduction strategies on the uniformity of the mixture. A simplified four-cylinder engine intake manifold model was built and used for computational fluid dynamics (CFD) simulation analysis. By adjusting the model and parameters, three factors affecting the EGR introduction strategy were studied: position, angle, and flow rate.

Overall, this study aims to determine the optimal introduction strategy for exhaust gas recirculation (EGR), thereby providing a theoretical basis and design guidance to improve the uniformity of EGR and the air mixture within the intake system, thereby further enhancing the emission reduction and efficiency of EGR technology.

2. Methods

Computational Fluid Dynamics (CFD) is a technique that uses numerical methods to solve the governing equations of fluid flow on a computer. Which can analyze and predict the flow field. Its core lies in solving the Navier-Stokes equations that describe the conservation of mass, momentum, and energy [11]. In this study, the CFD method was employed to simulate the mixing process of exhaust gas (EGR) and fresh air in the intake manifold [12]. CFD technology can provide detailed data across the entire flow field at a low cost, thereby efficiently parameterizing the study of different EGR introduction strategies and providing profound insights for optimizing design [13].

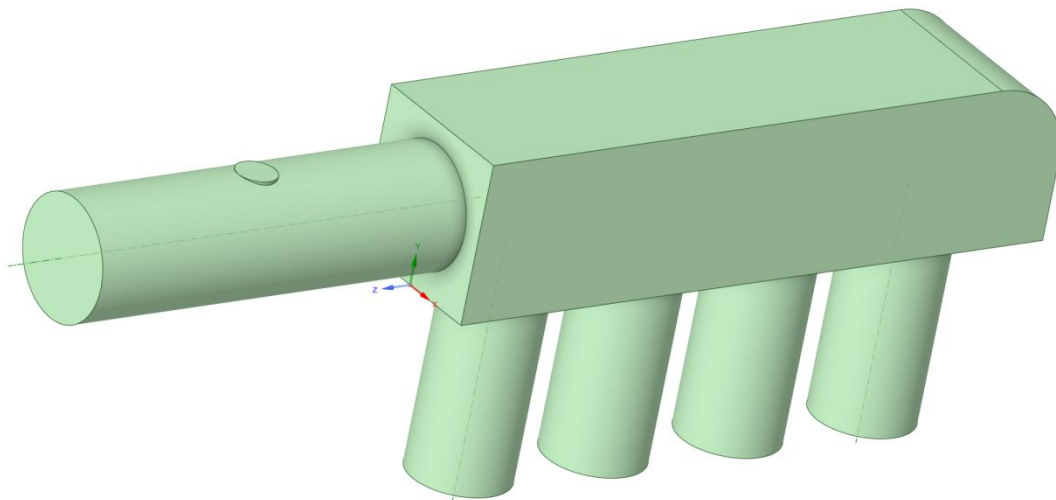


Fig. 1 Four-cylinder intake manifold CAD model

A 3-dimensional computer-aided design (CAD) model was built, consisting of an EGR delivery pipe connected to the fresh air intake pipe, an intake plenum, and four intake runners. The simulation area includes the intake pipe, the intake plenum, and four intake runners, as shown in Figure 1. A vital dimension restriction applies to the plenum, which is configured as a rectangular prism with an 80 mm × 80 mm cross-section and a length of 320 mm. These dimensions have been carefully selected to simulate the plenum dimension of a 2.0L engine. The intake pipe is modeled as a circular duct with an inner diameter of 60 mm. In comparison, the runners connecting the plenum to the individual cylinders are also modeled as circular ducts with an inner diameter of 50 mm.

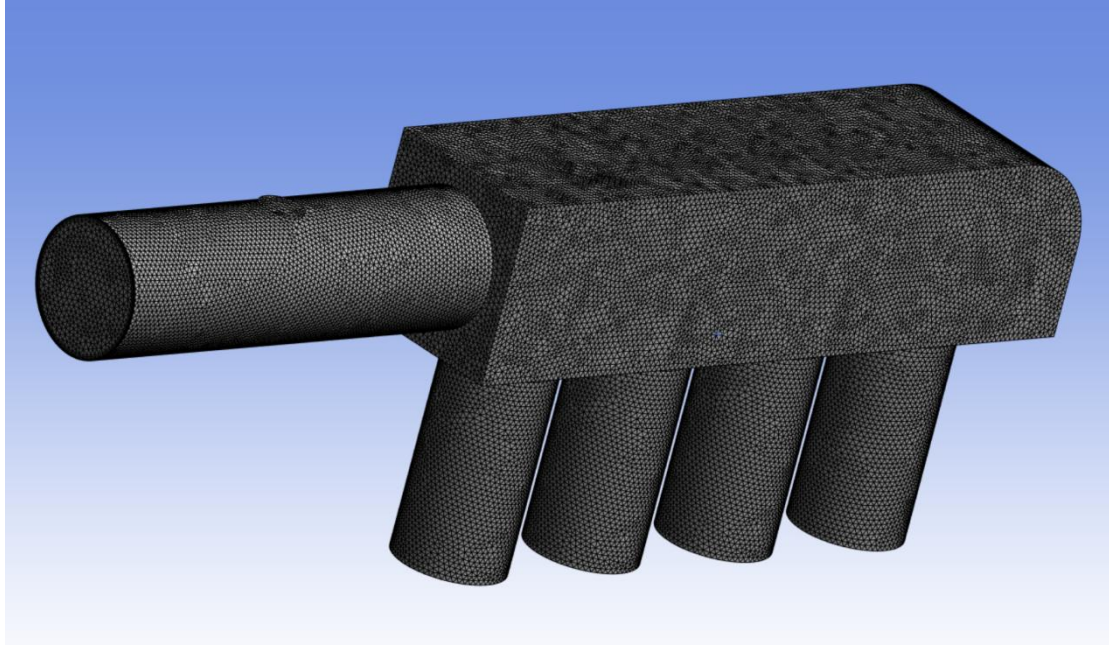


Fig. 2 Meshed four-cylinder intake manifold CAD model

The entire simulation area was quad-meshed with a 2.5mm element size, as shown in Figure 2. To precisely address the critical mixing process between the exhaust gas recirculation flow and the fresh air, the meshes around the EGR intake and its downstream area were refined to smaller unit sizes. In addition, inflation layers with a growth ratio of 1.2 were generated near all wall boundaries to more precisely simulate the physical phenomena near the wall surface. The first-layer height of the elements was determined based on the dimensionless wall distance. By reducing the y^+ value to less than 10, the influence of the viscous sublayer is eliminated. This was achieved by setting the initial height of the layer to 0.1 millimeters at the intake pipe wall, calculated based on the estimated flow velocity and fluid properties.

$$y_1 = \frac{\rho \times u \tau}{Y + \mu} \quad (1)$$

$$y_1 = \frac{D \times Y^+}{0.17 \times Re^{0.875}} \quad (2)$$

The CFD simulation was operated using a pressure-based, steady-state solver, which is well-suited for simulating incompressible to low-speed compressible internal flows [13]. The heat transfer model was enabled in the simulation. Even after cooling, a significant temperature difference persisted between the recirculated exhaust gas and the intake air. Ignoring this would lead to differences between the simulation and actual results. The Shear Stress Transport (SST) k-omega model was used to achieve turbulence closure. The hybrid formulation is renowned for its accurate prediction of flow separation and its robust near-wall treatment, without requiring excessive mesh refinement, making it ideal for application to the model used in this study, which featured complex internal flows [11].

The mixing analysis in this study was achieved by activating the species transport model without reactions. Two distinct chemical species were defined. Fresh air was modeled as an ideal mixture of 23.2% oxygen and 76.8% nitrogen. And EGR gas was modeled as a perfect mixture of 18% carbon dioxide, 3% water vapour, 2% oxygen, and 77% nitrogen. The main intake inlet was assigned fresh air, while the EGR inlet was assigned EGR gas. Both set a mass fraction of 1.0. The total intake mass flow rate was set to 1.020 kg/s, simulating the operating condition of a 2.0L engine at 2000 rpm.

As shown in Table 1, to systematically investigate how factors influence exhaust gas recirculation (EGR) mixture uniformity, three key operating variables were studied: the position, angle, and flow rate of EGR introduction. The axial location of the EGR introduction point along the intake pipe to the plenum was differentiated. Three positions were evaluated to determine the effect of mixing length on the uniformity of the mixture entering the cylinders at 50 mm, 100 mm, and 150 mm from

the plenum inlet. The injection angle of the EGR stream relative to the primary airflow was differentiated. Three configurations were analyzed to determine how flow direction affects penetration and turbulent mixing: a 45-degree forward-inclined angle (directed towards the plenum), a 90-degree vertical angle (perpendicular to the intake pipe), and a 45-degree backward-inclined angle (directed away from the plenum). The EGR ratios were varied at three levels: 10%, 20%, and 30% of the total intake mass flow. This enables the analysis of the mixing uniformity under different dilution conditions. The combination of these variables facilitates a comprehensive understanding of their individual and interactive effects on EGR induction.

Table 1. Case configurations

	Introduce Position	Injection Angle	EGR ratio
Case 1	Middle	Vertical	10%
Case 2	Middle	Vertical	20%
Case 3	Middle	Vertical	30%
Case 4	Middle	With-flow	10%
Case 5	Middle	Against-flow	10%
Case 6	Near-end	Vertical	10%
Case 7	Far-end	Vertical	10%

3. Result & Discussion

The flow field and species distribution within the intake manifold under various EGR introduction strategies were successfully captured through the CFD simulations. By analyzing the results of mass flow rate, carbon dioxide mass fraction, and carbon dioxide uniformity at four individual runners, it was found that the EGR rate, injection position, and injection angle have significant effects on the distribution uniformity and mixing quality among different cylinders.

3.1. Simulations Overview

A notable phenomenon is that, across all simulation cases, there are significant differences in airflow distribution among the runners, with the total airflow at the downstream outlets (especially the last outlet) significantly higher than that at the upstream outlets (especially the first outlet). This phenomenon occurs due to the geometric asymmetry of this type of intake manifold. The mixture of fresh air and EGR has considerable flow kinetic energy, so it tends to flow along the path of least resistance. In this linear, asymmetric layout, this path is a straight line leading directly to the rear of the intake manifold. Therefore, at the end of the manifold, an "accumulation" or "packing" effect occurs, increasing the local static pressure there. This, in turn, increases the total airflow entering the final exhaust pipe. The flow bias observed in the study is actually consistent with the physical characteristics of the asymmetric intake manifold in the real world. In such a manifold, achieving a perfect airflow distribution is challenging.

In this study, the specific manifold flow channel geometry was employed for three main reasons. Firstly, it reflects the actual conditions in engineering applications. Although more optimized geometries are typically used in actual engine designs, the unevenness of fluid distribution may not be as noticeable. However, the flow problem caused by the intake manifold asymmetry is quite common. Secondly, this inherent uneven distribution can magnify the differences in simulation results between different EGR strategies. This makes it easier to compare differences and summarize patterns from the simulation results. Finally, since the core objective of this study is to compare and analyze different EGR introduction structures and strategies, the relative performance differences or improvements between these strategies remain valid and insightful, even when an asymmetric flow field is applied.

3.2. Effect of EGR Ratio

Table 2. The results from different EGR ratios

		Outlet 1	Outlet 2	Outlet 3	Outlet 4
10% EGR	Mass flow rate	0.040981	0.268839	0.338252	0.371939
	CO ₂ Mass flow rate	0.000253	0.000964	0.007937	0.009213
	CO ₂ Mass fraction	0.006162	0.003587	0.023465	0.024771
	CO ₂ Uniformity	0.975758	0.953852	0.972725	0.971069
20% EGR	Mass flow rate	0.002926	0.246129	0.359945	0.411162
	CO ₂ Mass flow rate	0.000073	0.009997	0.014609	0.012038
	CO ₂ Mass fraction	0.024898	0.040619	0.040587	0.029278
	CO ₂ Uniformity	0.544470	0.988547	0.955379	0.966828
30% EGR	Mass flow rate	0.026029	0.281353	0.331964	0.392578
	CO ₂ Mass flow rate	0.002503	0.013138	0.019119	0.020317
	CO ₂ Mass fraction	0.096146	0.046696	0.057594	0.051753
	CO ₂ Uniformity	0.459624	0.755868	0.922558	0.975256

As shown in Table 2, the simulation results indicate a balance relationship determined by the exhaust gas recirculation ratio. On the one hand, a higher EGR ratio promotes a more balanced distribution of CO₂ concentration across cylinders. This can be demonstrated by changes in the relative standard deviation (RSD) of the CO₂ mass fraction at the four outlets, which varied from 77.07% at 10% EGR to 23.65% at 20% EGR to 35.71% at 30% EGR. This indicates that increasing the EGR gas proportion results in a more uniform CO₂ concentration in the cylinder, a positive sign for emission control in EGR applications. On the other hand, however, the mixing quality within each runner deteriorates as the EGR rate increases. This can be demonstrated by the decrease in the CO₂ uniformity index at the outlet of each runner at the 30% EGR case compared to the 20% and 10% cases. The reduction in local uniformity will lead to incomplete combustion and increased particulate matter emissions.

At a 20% EGR ratio, the relative standard deviation (RSD) is 23.65%, and it increases to 35.71% at a 30% EGR ratio. The emergence of significant flow instability explains this. Specifically, in the 20% EGR case, a considerable backflow occurred at outlet 1. Simulation results showed that this reverse flow effectively circulates and uniformly mixes the mixture, thereby improving its uniformity across different cylinders. However, this backflow phenomenon is not supposed to occur at that location. Its emergence is a manifestation of the interaction between increased flow resistance and higher EGR introduction volume. This finding indicates that, as the EGR ratio increases, a more complex mixing environment in the intake manifold is required to achieve good mixture uniformity. A more complex intake manifold design is needed to accommodate the high EGR ratios under working conditions. Furthermore, it also points out an important future research direction. Develop multiple strategies and optimize the intake manifold design to make the mixture efficiency and the EGR ratio independent of each other, allowing the distribution advantage of a high EGR ratio to be maintained without negative impacts from poor local mixing.

3.3. Effect of EGR Introduction Location

Table 3. The results from different introduction locations

		Outlet 1	Outlet 2	Outlet 3	Outlet 4
Near-end	Mass flow rate	0.044112	0.245853	0.353297	0.376736
	CO ₂ Mass flow rate	0.000503	0.001542	0.007158	0.009159
	CO ₂ Mass fraction	0.011393	0.006272	0.020260	0.024311
	CO ₂ Uniformity	0.979807	0.943194	0.900169	0.973607
Middle	Mass flow rate	0.040981	0.268839	0.338252	0.371939
	CO ₂ Mass flow rate	0.000253	0.000964	0.007937	0.009213
	CO ₂ Mass fraction	0.006162	0.003587	0.023465	0.024771
	CO ₂ Uniformity	0.975758	0.953851	0.972725	0.971068
Far-end	Mass flow rate	0.057010	0.271360	0.327971	0.363932
	CO ₂ Mass flow rate	0.000449	0.001375	0.007401	0.009144
	CO ₂ Mass fraction	0.007868	0.005065	0.022566	0.025125
	CO ₂ Uniformity	0.980516	0.948035	0.955149	0.976159

As shown in Table 3, the simulation results for the EGR introduction position provide crucial insights into how the mixing path length affects mixing. When EGR is introduced near the plenum, the CO₂ mass fractions at the four outlets appear to reach a balanced state. However, this apparent uniformity does not indicate a good mixing of EGR and fresh air. Instead, this is the result of the combined effect of an inappropriate mixing process and specific geometric structures. When entering the plenum, a portion of the EGR airflow flows almost directly into the nearest outlet (Outlet 1). This is because the injected EGR airflow has significant momentum, and there is a short, unobstructed path between the EGR inlet and Outlet 1, which causes an astonishingly high CO₂ mass fraction in Outlet 1. And when it averaged with the lower mass fractions from the other outlets, the data were skewed and showed uniformity. Therefore, unless the intake manifold is specifically designed to accommodate this intake point, the EGR injection point should be placed farther from the plenum to create a longer, more tortuous mixing path and achieve uniformity.

Although abnormal data were collected when EGR was introduced near the plenum, the trend in the CO₂ uniformity index still provides a clear standard. The results demonstrate that as the introduction point is moved farther from the plenum, the CO₂ uniformity index is increased. This trend strongly validates a fundamental principle of fluid mechanics. That is, longer residence times and longer flow paths can promote mixing by providing more time for molecular diffusion. This also provides a clear design suggestion for the EGR introduction position in the intake manifold. When space and other conditions allow, the EGR inlet should be positioned far enough from the plenum to ensure thorough mixing of EGR with fresh air, achieving good uniformity.

3.4. Effect of EGR Injection Angle

As shown in Table 4, the simulation results indicate that the EGR injection angle is a key parameter influencing the mixture dynamics. Compared with upstream EGR, downstream EGR (aligned with the fresh air) significantly reduces the CO₂ uniformity index. This is because, in that case, the EGR

injection and the main flow are in the same direction, so they rarely interact or mix. This observation is supported by the substantially higher CO₂ mass flow rate at Outlet 4 compared to the other outlets, demonstrating that the EGR flows directly towards the rear of the plenum and is mainly drawn into the final runner.

In contrast, the mixture shows markedly greater uniformity when injected against the flow. The physical principle behind this enhancement is that at the interface between the reverse-flowing EGR inlet flow and the main flow, the instability of the shear layer and the generation of vortices significantly increase. This powerful interaction disperses the EGR flow into the main flow and enhances the turbulent kinetic energy, thereby facilitating a strong momentum exchange. This superior mixing efficiency can be confirmed by comparing the relative standard deviation (RSD) of the CO₂ mass fraction across the four outlets. This is 45.66% for the against-flow case, and 77.05% for the vertical case. It can be seen that when EGR is introduced against the main flow, the mixture shows higher uniformity.

It should be noted that the results on the impact of changes in EGR introduction angle on performance in this study will be influenced by the specific asymmetric geometric structure used. Therefore, although the against-flow introduction case demonstrates a straightforward principle, this study cannot provide a definite and universal conclusion on the optimal EGR introduction angle. However, this finding effectively highlights a promising research direction. Future research should focus on integrating the EGR injection angle with the overall manifold design to leverage these flow interactions and fully improve mixing.

Table 4. The results from different introduction angles

		Outlet 1	Outlet 2	Outlet 3	Outlet 4
With-Flow	Mass flow rate	0.106469	0.255313	0.327054	0.331586
	CO ₂ Mass flow rate	0.001288	0.001787	0.003733	0.011571
	CO ₂ Mass fraction	0.012095	0.006999	0.011415	0.034897
	CO ₂ Uniformity	0.954605	0.899925	0.787783	0.978631
Vertical	Mass flow rate	0.040981	0.268839	0.338252	0.371938
	CO ₂ Mass flow rate	0.000252	0.000964	0.007937	0.009213
	CO ₂ Mass fraction	0.006162	0.003587	0.023465	0.024771
	CO ₂ Uniformity	0.975759	0.953852	0.972725	0.971069
Against-Flow	Mass flow rate	0.019799	0.205168	0.389863	0.405375
	CO ₂ Mass flow rate	0.000171	0.001974	0.008412	0.007816
	CO ₂ Mass fraction	0.008628	0.009623	0.021577	0.019281
	CO ₂ Uniformity	0.993751	0.971324	0.924233	0.971660

3.5. Synthesis of Findings

This systematic investigation into EGR introduction strategies has revealed the key trade-off factors and design principles for achieving high mixture uniformity in multi-cylinder intake manifolds. A higher EGR ratio (e.g., 20-30%) typically promotes a more balanced distribution of CO₂ among the cylinders, as indicated by the reduction of the relative standard deviation of the CO₂ mass fraction. However, this is achieved at the expense of reduced mixing quality, as is evidenced by a decrease in

the CO₂ uniformity index in the simulation results, accompanied by local excessive dilution. For the introduction position, it is advisable to avoid setting the EGR inlet close to the plenum. Because EGR requires sufficient distance for momentum exchange and mass diffusion with fresh air, if the introduction point is too close to the plenum, the EGR gas, which carries higher kinetic energy, will not flow along its expected path. For the EGR injection angle, a with-flow (co-flow) introduction should be avoided, as it will reduce overall mixing uniformity. In contrast, an against-flow (counter-flow) introduction will intensify shear-layer instability and vortex phenomena, significantly enhance turbulent mixing, and provide greater uniformity. Ultimately, these findings indicate that an effective EGR system design requires an integrated approach that combines the injection strategy with the manifold geometry to balance flow distribution and internal mixing quality among cylinders.

4. Conclusion

This study systematically investigated the impact of EGR introduction strategies on mixture uniformity in a four-cylinder engine intake manifold using CFD simulations. The results clarify several key trade-off factors and design principles. Firstly, although a higher EGR rate can improve cylinder-to-cylinder uniformity, it simultaneously reduces local mixing quality, increasing the likelihood of incomplete combustion and particulate emissions. Secondly, the EGR inlet should be placed as far from the plenum as possible to create a longer, more tortuous mixing path, thereby enhancing both turbulent and molecular diffusion. This enables a more thorough EGR and air mixing. Furthermore, the against-flow injection angle has been proven to be significantly superior to co-flow or vertical injections. It will enhance shear-layer instability and vortex phenomena, considerably increasing turbulent mixing and improving mixing uniformity.

By employing an asymmetric geometric structure to represent the actual engine layout, this research not only reproduced the common uneven flow distribution found in practical applications but also established strict benchmarks for evaluating the uniformity of the mixture. Based on the resulting analysis and conclusions, quantitative support and direct, applicable guidance are provided for the design of EGR systems in multi-cylinder engines. However, it should be noted that there will still be a difference between the actual situation and the simulation results from the simplified CFD model using a steady-state simulation method. Additionally, the benefits of the against-flow injection angle identified in this study may be specific to the manifold geometry used. The application of this finding to other intake manifold layouts requires further research.

Future work should focus on several aspects. Experiments combining transient CFD simulations with experimental verification should be conducted to validate these findings under dynamic conditions. Additionally, future studies should optimize the EGR injection strategy in combination with different intake manifold models to develop a design that maintains mixture efficiency while achieving a high EGR ratio. All these efforts are crucial for advancing high-EGR and high-efficiency engine technologies.

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