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Experimental Research on Damage Recognition of T-shaped Beam on an Areospacecraft

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Abstract: The paper pre-set some damages in different level in the T-shaped beam test specimen by wire cutting, and obtained modal data of the damaged T-shaped beam by experiment. It adopted two methods, i.e. element modal strain energy change rate and element modal strain energy basis, to recognize damage locationally, and it adopted two methods, i.e. sensitivity matrix and second order generalized flexibility matrix, to recognize damage quantificationally. The result showed that the element modal strain energy basis had better locational recognition result while the second order generalized flexibility matrix had better quantitative recognition result. Therefore, using a combination of the two methods will get good recognition result, i.e., firstly using the element modal strain energy basis to recognize damage locationally, and then using the second order generalized flexibility matrix to recognize damage quantificationally.

Keywords: T-shaped beam; damage recognition; experiment; location; quantification.

1. Introduction

T-shaped beam is one of the main load-bearing components in the space vehicle, whose function is to strengthen the structure. Once the T-shaped beam is damaged and its bearing capacity decreases, the structure may be damaged in flight, resulting in flight failure and heavy losses. Therefore, it is very important to detect and identify the damage of T-shaped beam and obtain timely and accurate information so as to get timely and appropriate disposal.

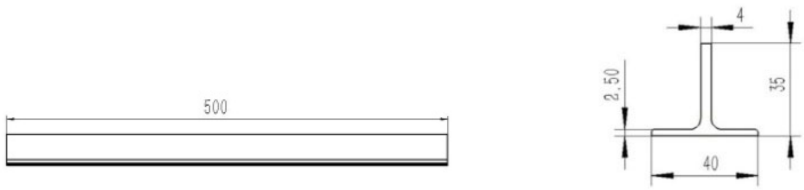
Many damage recognition methods have been proposed by domestic and foreign scholars, among which the method based on modal strain energy is a kind of important damage recognition method, which has attracted the attention of many scholars [1] [2]. For example, in 1995, Stubbs and Kim [3] first proposed the damage recognition index based on modal strain energy; From 1998 to 2002, Shi Zhiyu et al. [4] systematically studied the damage recognition index of element modal strain energy change rate by means of experiment and simulation; In 2008, Yan Wangji [5] proposed the damage recognition index of element modal strain energy sensitivity; in 2015, Ma Liyuan et al. [6] put forward the damage recognition index of element modal strain energy curvature difference, and so on. Some damage recognition indexes are sensitive to the location of damage, while others are sensitive to the degree of damage, each of which has its advantages and disadvantages.

In this paper, the excitation test of T-shaped beam with preset damage is carried out in laboratory, and the modal data are obtained by sensors [7]. Several damage recognition methods are used to locate and quantitatively identify the damage at the same time, so as to determine the better damage recognition method.

2. Test Methods

2.1. Test specimen

The test specimen is T-shaped beam, the material is aluminum alloy, the material parameters are as follows: elastic modulus E = 71GPa, Poisson's ratio $\mu = 0.33$, density $\rho = 2770 kg/m^3$, and its size is shown in Fig.1. Beam elements are used to divide the grid of test specimens, each element is 50mm long, and there are 20 elements and 21 nodes in total. Set the boundary condition to be free at both ends. As shown in Fig. 2.



(a) Main view of T-shaped beam (b) Left view of T-shaped beam

Figure 1. Dimension of Test specimen

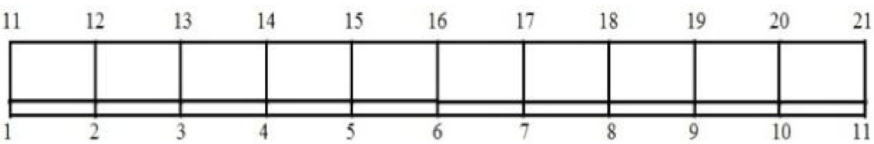
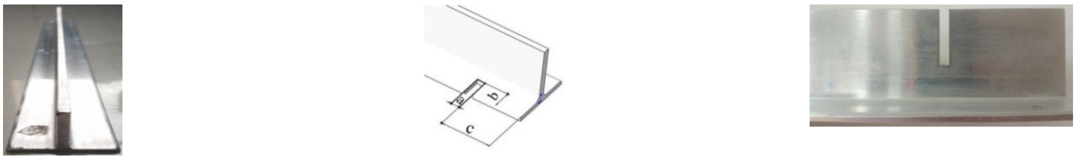


Figure 2. Grid Division of Test Specimen

In order to simulate the damage case, the damage condition is preset on the T-shaped beam. Use wire cutting machine to cut the test specimen at the preset damage position, and measure the damage degree by cutting the volume of the test specimen, as shown in Fig. 3. The test damage cases are shown in Table 1.



(a) Presetting of T-shaped beams before damage (b) Presetting of Damage Dimensions © Presetting of T-shaped beams after damage

Figure 3. Damage Case Presetting of Test Specimen

Table 1. Damage Case Design of T-shaped Beam

Damage Case	Damage presetting
Case 1	Unit 1, 3% (a=5mm,b=12mm,c=30mm)
Case 2	Unit 19, 6% (a=5mm,b=20mm,c=440mm)

2.2. Test Equipment

The PULSE system of Danish B & K Company is used in the test, which is divided into two parts: hardware and software. Hardware includes host computer, 3050 data acquisition front end, acceleration sensor, LC-01A force hammer, JZ-2A vibration exciter and B & K2692 charge amplifier, etc., as shown in Fig. 4. The software part is 7700 platform software and its application software. Labshop software is used to collect data in this experiment. In addition, Virtual.lab software developed by Siemens is used to simulate the excitation position and sensor layout of the test specimen.

(a) 3050 Data Acquisition Front End

(b) LC-01A Force Hammer

(c) JZ-2A Exciter

(d) B&K2692 Charge Amplifier

Figure 4. Hardwar for Experiment

2.3. Test Process

(1) Using Virtual.lab software developed by Siemens Company, the excitation position and sensor arrangement of the test specimen are simulated and optimized. According to the optimization results, acceleration sensors are arranged on the test specimen.

(2) Put the test piece and test instrument into the anechoic room, as shown in Fig. 5.

(a) Anechoic chamber

(b) T-shaped beam test

Figure 5. T-shaped Beam Being Testing in Anechoic Chamber

(3) The modal test of non-damaged T-shaped beam is carried out before the damage is preset. Respectively force hammer and vibration exciter in the best excitation position to excite, adjust the charge amplifier, collect modal data. Three sets of data were collected from each test specimen, and the first three modal parameters were taken.

(4) The damage of T-shaped beam is preset by wire cutting machine, and then the modal test of T-shaped beam with damage is carried out, and the method is the same as above.

3. Damage Recognition Method

Damage recognition mainly includes damage location recognition and damage quantitative recognition.

3.1. Damage location and recognition method

Modal strain energy is a function of structural stiffness and mode shape. When damage occurs in a certain part of the structure, it will cause the change of local material mechanical properties, which is manifested in the decrease of local stiffness of the structure, thus changing the modal strain energy at the damaged position. Therefore, modal strain energy can be used to identify structural damage. There are many damage recognition methods based on modal strain energy. Here we mainly analyze two methods: the rate of change of element modal strain energy and the basis of element modal strain energy.

3.1.1. Rate of change of element modal strain energy

The modal strain energy of Euler_Bernoulli beam j element is defined as:

$$MSE_j = \frac{1}{2}\int_{l_j}^{l_{j+1}} D_j \left(\frac{d^2\Phi_j(\mathrm{x})}{dx^2}\right)^2 dx = \frac{1}{2}\{\phi\}^T [k]_j \{\phi\} \tag{1}$$

Where D_j is the stiffness of the beam, $\varphi_j(x)$ is the modal displacement of element j, $[k]_j$ is the stiffness of element j, $\{\phi\}$ is the modal vector of the beam, and $\{\phi\}^T$ is the transposition of $\{\phi\}$.

Then the i-th modal strain energy of the beam (total n elements) is as follows:

$$MSE^{(i)} = \sum_{j=1}^{n} MSE_j = \frac{1}{2}\Phi^{(i)T}[K]\Phi^{(i)} \tag{2}$$

Where $\Phi^{(i)}$ is the i-th mode shape of the beam, $\Phi^{(i)T}$ is the transposition of $\Phi^{(i)}$, and $[K]$ is the overall structural stiffness matrix of the beam.

When the structure is damaged, the stiffness matrix of the system is often reduced. Therefore, from equation (2), it can be obtained that the i-th modal strain energy of the damaged structure is:

$$MSE^{(i)d} = \frac{1}{2}(\Phi^{(i)T} + \Delta\Phi^{(i)T})([K]+[\Delta K])(\Phi^{(i)} + \Delta\Phi^{(i)}) \tag{3}$$

Where $MSE^{(i)d}$ is the i-th modal strain energy of the damaged structure, ΔK is the change of stiffness before and after damage, and $\Delta\Phi^{(i)}$ is the change of i-th modal mode shape before and after damage.

By expanding the above equation and ignoring the higher-order term, the i-th modal strain energy of the damaged beam is obtained as follows:

$$\begin{aligned} MSE^{(i)d} \approx & \frac{1}{2}\Phi^{(i)\mathrm{T}}[K]\Phi^{(i)} + \Phi^{(i)\mathrm{T}}[K]\Delta\Phi^{(i)} \\ & + \frac{1}{2}\Phi^{(i)\mathrm{T}}[\Delta K]\Phi^{(i)} + \Phi^{(i)\mathrm{T}}[\Delta K]\Delta\Phi^{(i)} \end{aligned} \tag{4}$$

The modal strain energy of j element of damaged beam is as follows:

$$\begin{aligned} MSE_j^{(i)d} \approx & \frac{1}{2}\Phi^{(i)\mathrm{T}}[K]_j\Phi^{(i)} + \Phi^{(i)\mathrm{T}}[K]_j\Delta\Phi^{(i)} \\ & + \frac{1}{2}\Phi^{(i)\mathrm{T}}[\Delta K]_j\Phi^{(i)} + \Phi^{(i)\mathrm{T}}[\Delta K]_j\Delta\Phi^{(i)} \end{aligned} \tag{5}$$

Then the modal strain energy change rate of the i-th order element of the j element of beam structure can be expressed as:

$$MSECR_j = \left| \frac{MSE_j^{(i)d} - MSE_j^{(i)}}{MSE_j^{(i)}} \right| \tag{6}$$

Because it is impossible to measure all the modes of the structure in the actual test, the mean value of strain energy change rate of the first m modes is taken as the damage recognition index β_j:

$$\beta_j = \frac{1}{m}\sum_{i=1}^{m} MSECR_j^{(i)} \tag{7}$$

3.1.2. Damage recognition index based on element modal strain energy

In 2012, Seyedpoor et al. [8] constructed a damage recognition index based on modal strain energy basis. The modal strain energy of each element is normalized relative to the total modal strain energy of the structure:

$$nMSE_j^{(i)} = \frac{MSE_j^{(i)}}{MSE^{(i)}} \tag{8}$$

The mean value of normalized modal strain energy in the first m modes can be used as an effective parameter for structural damage location:

$$mnMSE_j = \frac{\sum_{i=1}^{m} nMSE_j^{(i)}}{m} \tag{9}$$

Since the damage of a structural element can be considered to be a decrease in the stiffness matrix, the corresponding modal shapes are expected to increase, and so is the $mnMSE_j$. The change rate of element modal strain energy is calculated, and the value is a positive number, that is, element modal strain energy basis (*MSEBI*):

$$MSEBI_j = \max[0, \frac{(mnMSE_j)^d - (mnMSE_j)^h}{(mnMSE_j)^h}] \tag{10}$$

Where superscript d and h represent damaged structure and healthy structure respectively. In this metric, the value of *MSEBI* is assumed to be positive for potentially damaged cells and zero for undamaged cells. Because the location and degree of damage are unknown, the element stiffness matrix of healthy structure is used to calculate the modal strain energy of damaged structure.

3.2. Quantitative damage recognition

There are many quantitative damage recognition methods, and sensitivity matrix and second-order generalized flexibility matrix are analyzed here.

3.2.1. Sensitivity matrix

In 1999, Shi Zhiyu et al. used sensitivity matrix to evaluate the damage degree of two-story plane rigid frame structure. The results show that this method can evaluate the damage degree.

Assuming that the s-th element of the structure is damaged, the modal strain energy difference of the element is written as:

$$MSEC_j^{(i)} = \sum_{p=1}^{L}\{-2\alpha_p \Phi^{(i)T}[K]_j \sum_{s=1}^{n} \frac{\Phi^{(s)T}[K]_p \Phi^{(i)}}{\lambda^{(s)} - \lambda^{(i)}} \Phi^{(s)}\}(s \neq i) \tag{11}$$

Where λ is the natural frequency of the structure, L is the number of damaged elements, and α_p is the damage degree of element p.

If the modal strain energy changes of Y elements are selected in the calculation, and $Y \geq L$, then the above equation can be written as:

$$\begin{Bmatrix} MSEC_1^{(i)} \\ MSEC_2^{(i)} \\ \cdots \\ MSEC_Y^{(i)} \end{Bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1L} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2L} \\ \cdots & \cdots & \cdots & \cdots \\ \beta_{Y1} & \beta_{Y1} & \cdots & \beta_{YL} \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \cdots \\ \alpha_L \end{Bmatrix} \tag{12}$$

Where: $\beta_{xy} = -2\sum_{s=1}^{n^*} \Phi^{(i)T}[K]_x \frac{\Phi^{(s)T}[K]_y \Phi^{(i)}}{\lambda^{(s)} - \lambda^{(i)}} \Phi^{(s)} (s \neq i)$ and n*is the final calculated modal order.

The above equation is the sensitivity matrix. By solving the sensitivity matrix, the damage degree of each element determined by the i-th modal calculation can be obtained. Shi Zhiyu et al. [9] further studied and found that modal truncation has a great influence on the evaluation results of sensitivity matrix, so β_{xy} was improved to:

$$\begin{aligned} \beta_{xy} &= -2\Phi^{(i)}[K]_x[K^{-1}b_w(\Phi^{(i)T}[K]_y\Phi^{(i)}[M]\Phi^{(i)} - [K]_y\Phi^{(i)}) \\ &+ \sum_{s=1}^{n^*} -(\frac{\lambda^{(i)}}{\lambda^{(s)}})^w \frac{\Phi^{(s)T}[K]_y\Phi^{(i)}}{\lambda^{(s)} - \lambda^{(i)}}\Phi^{(s)}](s \neq i) \end{aligned} \tag{13}$$

Where: w is the weighting coefficient.

3.2.2. Generalized flexibility matrix

J Li et al. [10] define the generalized flexibility matrix (GFM) of order l as:

$$F^{g(l)} = \Phi\Lambda^{-1-l}\Phi^T \tag{14}$$

Where F is the flexibility matrix, Φ is the modal shape, and Λ^{-1} is the diagonal matrix composed of the square ω_i^2 of natural frequency.

Then the second order GFM (that is, l=2) is:

$$F_m^{g(2)} = \Phi_m \Lambda^{-3} \Phi_m^T \tag{15}$$

Two vectors (δ_1 and δ_2) are defined by taking the elements on the diagonals and diagonals of the second-order GFM:

$$\begin{aligned} \delta_1 &= \{F_m^{g(2)}(1,1), F_m^{g(2)}(2,2) \ldots F_m^{g(2)}(ndf, ndf)\} \\ \delta_2 &= \{F_m^{g(2)}(1,ndf), F_m^{g(2)}(2, ndf-1) \ldots F_m^{g(2)}(ndf, 1)\} \end{aligned} \tag{16}$$

The difference of GFM between damaged structure and healthy structure is:

$$\Delta F = F_m^{g(2)\mathrm{d}} - F_m^{g(2)\mathrm{h}} \tag{17}$$

Two vectors (δ_3 and δ_4) are defined by taking the elements on the diagonals and diagonals of ΔF:

$$\begin{aligned} \delta_3 &= \{\Delta F(1,1), \Delta F(2,2) \ldots \Delta F(ndf, ndf)\} \\ \delta_4 &= \{\Delta F(1, ndf), \Delta F(2, ndf-1) \ldots \Delta F(ndf, 1)\} \end{aligned} \tag{18}$$

Since damage can be seen as a reduction in structural stiffness, the change in element stiffness can be expressed as:

$$k_i^d = (1 - d_i)k_i^h \tag{19}$$

In the equation, k_i^d and k_i^h represent the stiffness matrix of the i-th element under damaged and undamaged conditions, and d_i represents the damage degree of the i-th element.

In the process of reverse optimization, $F_m^{g(2)d}$ is used to replace $F_m^{g(2)}$, and δ_1^d, δ_2^d, δ_3^d and δ_4^d are calculated. Using these 8 vectors, MAC confidence index is constructed:

$$MAC_i = \frac{\left|(\delta_i)^T \cdot (\delta_i^d)\right|^2}{[(\delta_i)^T \cdot (\delta_i)][(\delta_i^d)^T \cdot (\delta_i^d)]}, i = 1,2,3,4 \tag{20}$$

The objective function is constructed as:

$$f(d_1, d_2, ..., d_n) = \sqrt{(\frac{1-e_1}{e_1})^2 + (\frac{1-e_2}{e_2})^2}, 0 \le d_i \le 1 \tag{21}$$

Among them:

$$\begin{aligned} e_1 &= MAC_1 \times MAC_3 \\ e_2 &= MAC_2 \times MAC_4 \end{aligned} \tag{22}$$

In equation (21), d_i is the damage degree value. By solving the objective function value, the damage degree of damage location can be obtained.

Using the generalized flexibility matrix for damage recognition requires a relatively large amount of calculation. Usually, other damage recognition indicators are used to identify the damage location first, and then the generalized flexibility matrix is used to evaluate the damage degree of the identified damaged units. The higher the level of generalized flexibility matrices, the better the damage recognition effect, but the computational cost is also greater, so the order should not be too high; second-order generalized flexibility matrices are more appropriate.

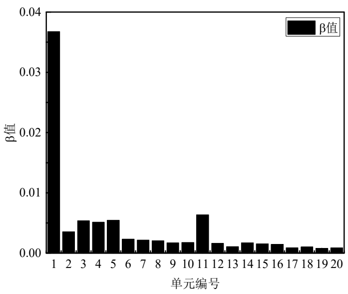
As can be seen from the above, both quantitative damage recognition methods are based on known damage units, so when applied, other methods must first be used for damage location recognition.

4. Damage Recognition Results

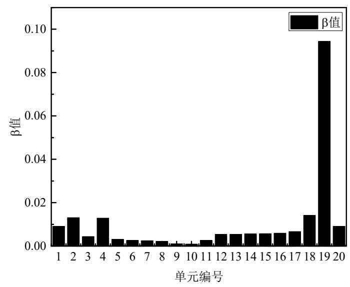
Due to the limitations of experimental conditions, the number of modal vibration mode coordinates obtained in the experiment is relatively small. The modal shape data obtained in the test was expanded according to the modal expansion method proposed by Wang Xiaohong and others [11]. According to the damage recognition index in Chapter 3, use MATLAB programming to import the expanded modal shape data, first calculate the modal strain energy of each unit, and then perform the damage recognition calculation.

4.1. Damage location and recognition

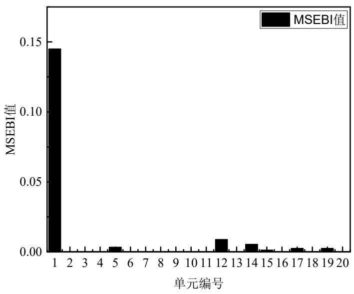
The modal data obtained by the test were calculated by using the two damage recognition indexes of unit modal strain energy change rate and unit modal strain energy basis, and the damage location results are shown in Fig. 6.



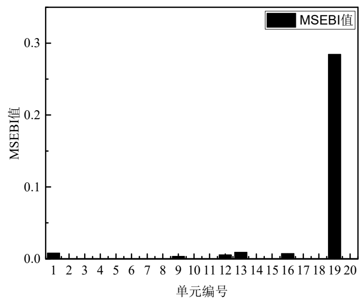
(a) Rate of change of strain energy in Case 1



(b) Rate of change of strain energy in Case 2



(c) Strain energy basis of element mode in Case 1



(d) Strain Energy Basis of Element Modal in Case 2

Figure 6. Damage Recognition Result of T-shaped Beam

It can be seen from the figure that both damage recognition indicators have identified the preset damage unit (unit 1 for case 1 and unit 19 for case 2), and there is no positioning error or missed recognition. As can be seen from the recognition results of the unit modal strain energy change rate, the indicators for units 3, 4, 5, and 11 in case 1 and units 2, 4, and 18 in case 2 are also high, which can easily cause misrecognition. It can be seen from the recognition results of the unit modal strain energy basis that the indexes of the undamaged units are not high, and there is no misrecognition. Therefore, overall, among the two recognition methods, the damage location recognition effect of the unit-modal strain energy base is the best.

At the same time, it can also be seen from the recognition results that there is no precise numerical relationship between the indicators obtained by the two damage recognition methods and the preset degree of damage, so it cannot be used for quantitative damage recognition.

4.2. Quantitative recognition results

Two quantitative damage recognition algorithms, sensitivity matrix and second-order generalized flexibility matrix, are used for calculation, and the quantitative damage results are shown in Table 2.

Table 2. Damage Quantification Result of T-shaped Beam

Damage condition		Damage degree presetting	Damage degree recognition	
			Sensitivity matrix	Second-order GFM
Case 1	Damage degree of unit 1/%	3	1.9	2.5
	Error/%		36.67	16.67
Case 2	Damage degree of unit 19/%	6	4.68	5.28
	Error/%		22	12

As can be seen from the table, both quantitative damage recognition methods give quantitative damage values, but the quantitative errors are large. This is mainly due to some errors in the test. For example, installing a sensor on the T-shaped beam will inevitably change the modal parameters of the T-shaped beam. Also, if the test only obtains the first 3 modes, ignoring the higher-order modality will inevitably cause some errors, and furthermore, the noise present in the data will also cause some errors.

For the two quantitative damage recognition methods, the second-order generalized flexibility matrix has the smallest error, so it is the most promising quantitative damage recognition method.

5. Conclusion

Using two damage location recognition indicators and two damage quantitative recognition indicators, the pre-set damaged T-shaped beam test mode data were calculated separately, and the position and quantitative value of the damage unit were obtained. Among the two damage location recognition methods, the recognition effect of the damage recognition index based on unit modal strain energy is better than the damage recognition index based on unit modal strain energy change rate. Among the two quantitative damage recognition methods, the recognition effect of the second-order generalized flexibility matrix is better than that of the sensitivity matrix. Therefore, using a combination of the two methods, the unit modal strain energy base is first used for damage location recognition, and then the damage unit position is obtained, and then a second-order generalized flexibility matrix is used for quantitative damage recognition, thereby achieving the goal of quantitative recognition of damage positioning, and a good recognition effect will be obtained.

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