

A Method for Calculating the Band Change of Gauss Projection Coordinates

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Abstract: In geodesy, many surveying projects are carried out on the plane, which requires the transformation of geodetic coordinates, the element on the ellipsoid, into plane coordinates, and each independent coordinate system has new contradictions due to zoning. Based on the basic geometric parameters and the mathematical description of the earth ellipsoid, this paper introduces the principle of Gauss projection method. Since Gauss Krueger projection is currently used in China, this paper introduces the principle and method of Gauss projection, and deduces the correlation between the Gauss plane coordinates (x, y) and the geodetic coordinates (L, (B)), that is, the forward and reverse calculation formulas of Gauss projection. With the help of Gauss projection as a communication "bridge", the coordinate transformation calculation is studied from geodetic coordinate transformation, and its advantages and disadvantages are compared with other methods. Based on the 1954 Beijing coordinate system and 1980 Xi'an coordinate system commonly used in China, a geodetic coordinate conversion system is compiled using C # language, and the forward and inverse calculation of Gaussian projection is realized.

Keywords: Gaussian projection, Geodetic coordinates, Belt change calculation, Common coordinate system.

1. Introduction

In order to make the production practice in social life and the construction of surveying engineering more convenient, it is sometimes necessary to calculate the coordinates on the ellipsoid to the plane, which constitutes the contradiction between the ellipsoid and the plane. [1, 2] Gaussian projection transformation has solved this problem, but the length deformation is more serious because it ensures that there is no angular deformation. In order to limit the length distortion of Gaussian projection, the partition projection method is used to limit the boundary of the projection range, so that its deformation degree does not exceed a certain limit. However, this method of zoning makes the original unified coordinate system become multiple independent coordinate systems, which creates a new problem between independent coordinate systems of each zone.

Therefore, it is necessary to study the conversion method of adjacent bands of Gauss projection coordinates to solve the problems in the first line of engineering construction and production.

Gaussian projection uses orthomorphic projection to ensure that the projection angle does not deform, but its length deformation is more serious. The projection of the central meridian on the Gaussian projection plane is a straight line with constant length. The rest of the meridians are concave to the central meridian. The farther away from the central meridian, the greater the deformation of the length. In order to limit the deformation of Gauss projection length, the ellipsoid is divided into different projection zones through the meridian of a certain longitude; Or to compensate for the length distortion, select the meridian of a certain longitude as the central meridian of the measurement area. Due to the different longitude of the central meridian, the unified geodetic coordinate system on the ellipsoid becomes an independent plane rectangular coordinate system. It is necessary to convert the plane rectangular coordinates of one projection zone into the plane rectangular coordinates of another projection zone, which is called coordinate

transformation zone.

Surveyors often encounter large deformation in the length of the projection area or different coordinate systems used in the process of measurement when carrying out engineering survey in real life, which leads to difficulties in the process of office calculation. At this time, we can solve the problem of excessive length deformation of the projection area through the zonation of GAUSS projection, and when conducting coordinate conversion, we can also use the method of Gaussian projection to obtain a unified coordinate system and reduce the error in office calculation. [3,4] Calculation of Gauss projection coordinate transformation The independent coordinate systems of different projection zones are connected through the transformation between geodetic coordinates (B, L) and plane coordinates (x, y). It has solved many problems in surveying engineering construction, thus greatly facilitating the geodetic work.

2. Application of Coordinate Belt Change

(1) As shown in Figure 1, A, B, 1, 2, 3, 4, C and D are control networks located at the edge of two adjacent zones and spanning two projection zones (east and west zones). If the starting coordinates of starting points A, B, C and D are brought out by two lines respectively, in order to measure and calculate in the same zone, it is necessary to convert the starting coordinates of points A and B in the west zone to the coordinates of the east zone, or convert the starting coordinates of points C and D in the east zone to the coordinates of the west zone.

(2) When mapping the area near the boundary meridian, it is often necessary to use the triangle points in another area as the control, and the coordinates of these points must be converted into the same area; In order to realize the splicing and use of the topographic maps of two adjacent zones, the triangle points located at 45 '(or 37.5') overlapping need to have the coordinate values of the adjacent zones, as shown in Figure 2.

(3) When large scale (above 1:1000) mapping, especially in engineering survey, requires the use of 3° zone, 1.5° zone or any zone, while the national control point usually only has 6° zone coordinates, then the problem of mutual coordinate exchange between 6° zone and 3° zone (or 1.5° zone or any zone) arises [5].

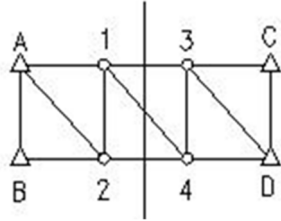


Figure 1. Central meridian

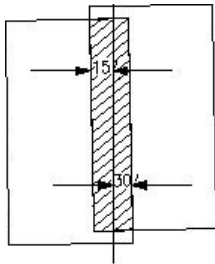


Figure 2. 3 degrees with central meridian

2.1. Calculation of indirect belt change

The plane coordinates of the first belt (east belt or west belt) can be converted to the plane coordinates of the second belt (west belt or east belt). The central idea of the conversion is based on the known Gauss plane coordinates of the first belt (x_1, y_1) And the longitude of the central meridian L_0^1 . Geodetic coordinates (B, L) can be obtained through the inverse operation formula of Gauss projection coordinates, and then according to (B, L) and the central meridian of the second zone L_0^2 , get longitude difference $l = L - L_0^2$, and then convert (B, l) to the Gauss plane coordinates in the second band using the Gauss projection coordinate forward calculation formula (x_2, y_2). Because the geodetic coordinate on the ellipsoid is used as the transition coordinate in the calculation of zone change, it is called the indirect zone change method [6].

For example, the plane coordinates of a point A in the 6° zone of the 54 coordinate system are $x_1=3589644.287$, $y_1=20679136.439$

Find the plane rectangular coordinates (x_2, y_2) of point A in the 3° zone

First, determine the longitude of the central meridian of the projection zone where point A is located. According to the specified value of abscissa, point A is located in the 20th zone of the 6° zone, and the longitude of the central meridian $L_0^1 = 6N - 3 = 117^\circ$, the natural value of the abscissa is $y_1=679136.493-500000=+179136.439\text{m}$, which is known to be on the east side of its central meridian; This coordinate can be determined as the plane coordinate within the range of the east of the central meridian of the 39th zone and the west of the central meridian of the 40th zone of the 3° zone.

Secondly, the known 6° zone coordinates are inversely calculated as geodetic coordinates. For this reason, the coordinate inverse calculation formula derived in Chapter 3 can be directly applied for calculation, and the result is

$$B=32^\circ 24' 57.6522''$$

$$L=118^\circ 54' 15.2206''$$

Judging from the geodetic longitude L , $n = \frac{L}{3}$ it is concluded that point A is located in the 40th zone of 3°, and the central meridian is $L = 120^\circ$.

Finally, according to the forward calculation formula of Gauss projection coordinates, calculate the plane rectangular coordinates of point A in the 40th zone of the 3° zone from the known latitude B and longitude difference l $x_2=3588576.591$, $y_2=40396922.874$.

2.2. Direct belt change calculation

The direct strip exchange calculation is calculated by using the geometric relationship between the two points, and a relationship bridge is constructed with the help of "symmetry point" P2 and "auxiliary point" M [7].

Point P2 is a point on the ellipsoid. P2 and P1 are symmetrical about the zonal meridian, that is, they are on the same parallel circle, and the absolute value of the meridional difference relative to the zonal meridian is equal, but the sign is opposite. M is the "auxiliary point" of P2's coordinates in the west zone and P1's coordinates. Because P2 and P1 are symmetrical about the zonal meridian, the geodetic length $MP_1=MP_2=S$, and the sum of the geodetic azimuth angles of MP1 and MP2 is 360° , that is, $A_1+A_2=360^\circ$.

Through the symmetry point and auxiliary point, the coordinate relationship is established, and the coordinates of P2 point in the west zone are calculated (x'_2, y'_2). To get the coordinates of P2 (x_2, y_2).

2.3. Calculation of complex variable function band change

Complex functions are often used to solve isometric problems because of their advantages in solving isometric projection problems. Because Gaussian projection is isometric projection, complex variable function plays an important role in solving Gaussian problems.

In this method [8,9], the main idea of forward solution of Gaussian projection is shown in Figure 4. In the figure: φ is the isometric latitude, q is the isometric latitude, and X is the meridian arc length, $\omega = q + i\Delta l$ is the plural equivalent latitude; φ_{CV} is complex isometric latitude, and $z=z+iy$ is Gaussian complex coordinates. Conformal latitude φ is a function of equal latitude q , and the meridian arc length X can be expressed as equal latitude φ expression, namely:

$$\varphi = \sin^{-1} \tan h q$$

$$X = a (\alpha_0 \varphi + \alpha_2 \sin 2\varphi + \alpha_4 \sin 4\varphi + \alpha_6 \sin 6\varphi$$

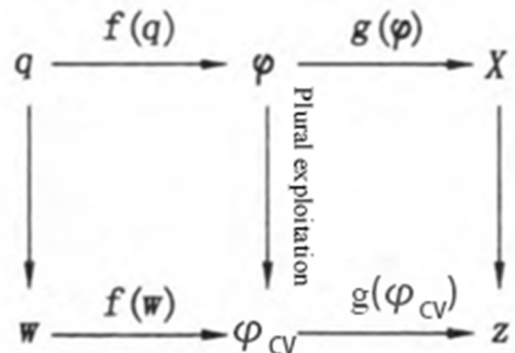


Figure 4. Plural exploitation

With the help of the theory of complex functions, expand (4-1) to the complex field, and use the complex equivalent latitude ω substitute for isometric latitude q , complex isometric latitude $\varphi_{\text{复}}$ replace isometric latitude q the Gaussian complex coordinate z replaces the meridian arc length X , and the complex function of the positive solution of the Gaussian projection is obtained:

$$\varphi_{cv} = \sin^{-1} \tan h\omega$$

$$X = a (\alpha_0 \varphi_{cv} + \alpha_2 \sin 2\varphi_{cv} + \alpha_4 \sin 4\varphi_{cv} + \alpha_6 \sin 6\varphi_{\text{复}})$$

3. Comparison of Calculation Methods for Belt Replacement

Gauss projection is widely used in the control survey and the compilation and mapping of large-scale topographic maps in social life. The calculation of band change is a practical problem often encountered in the survey and mapping work. Scholars at home and abroad have put forward a variety of calculation methods for strip exchange, such as direct exchange algorithm, indirect exchange algorithm, complex variable function method, numerical analysis method, and the use of triangular method [10,11,12].

Indirect band exchange method: it is rigorous in theory, easy to understand and universal. It is applicable to coordinate band exchange between 6° band and 6° band, 3° band and 3° band, and 6° band and 3° band, but it should be within the range of $1 < 3.5^\circ$. This method requires a large amount of calculation, and the process is complex. If the manpower is calculated, it will waste a lot of time and reduce the work efficiency. However, this function can be realized by computer programming, so it has become the most basic method in coordinate transformation.

Direct transformation algorithm: in its calculation process, Gauss-Clueger coordinate transformation table is needed to calculate its coordinates in the adjacent zone, and these transformation tables are only applicable to conventional calculation, so its scope of application will be limited.

Complex variable function method: because of its advantages in solving isometric mapping problems, it is often used to solve isometric problems. Therefore, complex variable function has the effect of getting twice the result with half the effort in solving Gaussian problems, and is not limited to Gaussian projection with 6° or 3° bandwidth. The "transformation formula between Gaussian projections of different central meridians" applicable to a wider range of longitude can be derived.

4. Purpose and Significance of The Program

Because the use of indirect band exchange method has become the most basic method in coordinate band exchange, but it also has some difficulties, that is, its calculation process is relatively complex and cumbersome, and the amount of calculation is very large, which requires the use of software

programs to achieve the calculation of the positive and negative coordinates, convenient office processing in surveying and mapping engineering, reducing the waste of time, manpower and material resources, greatly improving the efficiency of surveying work, and saving costs.

4.1. Geodetic coordinate system commonly used in China

In the early 1950s, at the beginning of the establishment of the astronomical geodetic network in China, a national unified geodetic coordinate system was established according to the historical conditions at that time, and was once named the Beijing coordinate system of 1954. The plane coordinate positions on the topographic map of China were calculated based on this coordinate. The origin of the coordinate system is actually in Pulkovo, the former Soviet Union, rather than in Beijing. The ellipsoid used is Krasovsky ellipsoid. In April 1978, the National Astronomical and Geodetic Network Adjustment Conference was held in Xi'an and decided to reposition and establish a new coordinate system in China. For this reason, the 1980 National Geodetic Coordinate System was established. The geodetic origin of the coordinate system is located in Yongle Town, Jingyang County, Shaanxi Province, central China, so it is called the 1980 Xi'an coordinate system. The parameters of ellipsoid are recommended by IUG in 1975

4.2. Program design

4.2.1. Program design process

The preliminary work is to master the forward and inverse calculation formula of Gaussian projection and have a deep understanding of it, but the expression of the forward and inverse calculation formula in the ordinary sense is relatively complex, which requires the collection of simple computer formulas for coordinate transformation.

The program uses VS2013 version as the development platform, the development language is C#, and is designed as a WindowsForm program. Create a new project, design the program interface, and then add the required Button and Textbox components (such as the input geodetic coordinates, forward and backward calculation buttons, etc.). When applying the coordinate system, add the drop-down bar, and move it to the appropriate position to align its format, so as to keep the interface simple and beautiful. Then define the types of characters to be used (such as long and short half axes a , b , etc. of different coordinate systems). Edit the button, input the positive and negative calculation formulas of Gaussian coordinates respectively, and import the results into the corresponding text box. Finally, debug the program and check whether it can run normally and the accuracy of data.

4.2.2. Initial interface and application display

Initial interface display: input geodetic coordinates (B , L), input Gaussian coordinates (x , y), and obtain the central meridian of the point within the 6° or 3° zone from the known longitude. When outputting coordinate y , add 5000000 to ensure that its value is always positive.

Figure 5. Software interface display

Figure 6. Positive calculation test under 1954 Beijing coordinate system

Figure 7. Negative calculation test under 1954 Beijing coordinate system.

Figure 8. Positive calculation test under the 1980 Xi'an coordinate system.

Figure 9. Negative calculation test under the 1980 Xi'an coordinate system.

5. Conclusion

In the process of transforming Gaussian projection coordinates to plane coordinates, we need to rely on the nature of Gaussian projection, the conditions of orthomorphic projection and its essential characteristics that are different from other projections, and we need to use the calculation formula of the radius of curvature of the meridian and the prime unitary circle, the positive and negative calculation formula of the meridian arc length and the calculation formula of the latitude of the base point, through a series of mathematical operations (differential, integral, series expansion, etc.). Although the operation is complicated and complicated, it is not easy to understand the mathematical formula alone, but the mathematical meaning of each formula can be well understood with its geometric meaning. When studying the problem of coordinate transformation, we should do a good job in the early stage of knowledge, and form the three-dimensional geometric relationship of the projection between geodetic coordinates and plane coordinates in our mind, which will make the research ideas in the future clearer.

Because the use of indirect zone change method has become the most basic way of coordinate zone change, but its calculation process is complex and cumbersome, and the amount of calculation is very large, which requires the use of software programs to realize the calculation of the positive and negative coordinates, which is convenient for the office processing in surveying and mapping engineering, greatly improving the efficiency of surveying work and saving costs. The C# language is used for programming because it not only maintains the powerful functions of C and C++ languages, but also removes some of their complex features. The forward and inverse calculation formula of Gaussian projection is relatively complex, so the computer formula is used. The program designed in this project can simply carry out the forward and inverse calculation of Gaussian projection.

Adjacent band conversion of coordinates is widely used in real social production. Many scholars at home and abroad have also studied many different methods, but each method has its limitations [13]. At present, the most basic method is indirect conversion algorithm, that is, with the help of Gaussian projection coordinate forward and inverse calculation. However, with the explosive development of science and technology, the measurement technology will also develop rapidly, and the method of adjacent zone coordinate zone change will also burst out with the progress of science and technology, which will lead to a new method with a wider and more complete scope of application.

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