Carbon Futures Volatility Forecasting: The Role of Spillover Effect from Crude Oil Futures Market

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Abstract: Accurate prediction of the volatility of EU carbon emission quota futures is an important foundation for pricing derivative products, portfolio allocation, and risk management. On the basis of the GARCH-MIDAS model involving mixed frequency data sampling, this article considers the spillover effect of Brent crude oil futures market on carbon quota futures market, and incorporates the spillover effect into the prediction of carbon quota futures volatility to construct the VS-GARCH-MIDAS model. Using the monthly realized volatility of Brent crude oil futures as a proxy variable to reflect spillover effects, empirical results indicate the existence of a one-way spillover effect of Brent crude oil futures market on carbon quota futures market. Moreover, the GARCH-MIDAS model considering spillover effects has better in sample fitting performance and stronger out of sample predictive ability than the original GARCH-MIDAS model. To ensure the robustness of this conclusion, this paper also investigates the impact of spillover effects generated by WTI crude oil futures on the volatility of EUA futures, and concludes that the GARCH-MIDAS model considering spillover effects can improve the prediction accuracy of EUA futures volatility.

Keywords: Brent crude oil futures, EUA futures, VS-GARCH-MIDAS model, volatility prediction.

1. Introduction

In order to promote the development of China's low-carbon economy and curb the growth of energy consumption, the government has proposed the vision of achieving the "dual carbon" goal. In this context, after ten years of regional pilot projects for carbon emission quota trading, China has established a unified carbon emission quota trading market. At the beginning of the establishment of China's national carbon trading market, the current trading products are limited to spot carbon emission quotas in the power industry, and the limited variety of trading products has led to a lack of liquidity in the market. The driving force of spot price changes is insufficient, making it difficult to fully utilize resource allocation functions, and the efficiency of carbon trading market operation is low. Developing carbon financial derivative products target to improve the liquidity of carbon trading market and provide risk avoidance functions for carbon emission reduction enterprises can help to improve the diversified development of carbon trading market. The European Union Carbon Trading System (EU ETS) is currently the most well-developed carbon trading system, with Carbon Emission Quota Spot (EUA) and Carbon Emission Quota Futures (EUA Futures) being the main trading products under this system. Carbon futures can help companies lock in spot prices in advance and are a powerful tool for carbon reduction companies to avoid risks. The main exchange of EUA futures, the Intercontinental Exchange of London (ICE), has developed option products based on EUA futures. On the one hand, accurate prediction of the volatility of EUA futures can help market managers price EUA option products; On the other hand, it can provide reference for investment portfolio allocation and risk management for participants in the carbon trading system, except for carbon reduction enterprises.

2. Literature Review

Regarding the modeling and prediction of volatility in the stock market, due to the characteristics of sharp peaks, thick tails, conditional heteroscedasticity, and time-varying and clustering characteristics of volatility, the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model and Stochastic Volatility (SV) model are commonly used to describe the dynamic changes in financial time series data. GARCH models, due to their concise form and easy implementation of parameter estimation, have become more widely used models for modeling and predicting stock market volatility. Byun and Cho (2013) conducted an empirical study using the daily data of EUA futures in the second stage of EU ETS. The results showed that compared with the implied volatility of carbon option prices and k-nearest neighbor models, GARCH models had better predictive ability for EUA futures volatility. However, traditional GARCH models, such as GARCH models, EGARCH models that depict stock market leverage effects, and GJR GARCH models, typically only model volatility based on daily data of stock returns, making it difficult to characterize the impact of other low-frequency variables on stock volatility. Ignoring the role of low-frequency variables, GARCH models have certain shortcomings in predicting volatility.

On this basis, Ghysels (2004) first proposed the Mixed Frequency Data Sampling (MIDAS) regression model, which involves time series data sampled at different frequencies. Due to the potential loss of effective information caused by reducing high-frequency data to low-frequency variables, empirical studies have shown that parameter estimation in MIDAS regression considering mixed frequency data is more effective than low-frequency data regression, thus enabling more accurate prediction of volatility. Engle (2013) combined mixed frequency data sampling with GARCH models that decompose volatility into long and short-term components to characterize the impact of low-frequency macroeconomic or financial variables on asset volatility, known as the GARCH-MIDAS model. This model decomposes volatility into short-term components and long-term components measured by monthly realized volatility. Through variance ratio, it is concluded that the long-term component contributes about 30%
to volatility prediction. It can be seen that the long-term component driven by low-frequency variables plays an important role in accurate volatility prediction. Zheng Tingguo (2014) introduced macroeconomic variables into the single factor GARCH-MIDAS model to obtain a multi factor GARCH-MIDAS model. Empirical results showed that the multi factor GARCH-MIDAS model had better predictive performance than the single factor GARCH-MIDAS model.

Subsequently, many scholars have expanded the GARCH-MIDAS model to conduct a series of studies on the volatility of financial assets. Lei Likun (2018) introduced the Economic Policy Uncertainty Index (EPU) into the mixed frequency GARCH model to predict the volatility of the Shanghai Composite Index. Zhang Yifeng (2020) constructed a new measurement index of herd effect within the framework of the mixed frequency GARCH model to predict the volatility of the Shanghai Composite Index. Liang Chao (2021) introduced the Google Trend Index into the long-term component of the GARCH-MIDAS model to predict the volatility of China's gold price. The empirical results indicate that the GARCH-MIDAS model considering the influence of three exogenous indices has higher prediction accuracy than the original GARCH-MIDAS model. Liu (2021) examined the impact of the Global Economic Policy Uncertainty Index (GEPU) and the EU Economic Policy Uncertainty Index on the volatility of EUA futures, and pointed out that the GARCH-MIDAS model has better predictive ability than the GARCH model, and the prediction accuracy of the EU EPU index is higher than that of GEPU. From this, it can be seen that introducing exogenous low-frequency variables in the long-term component, except for the low-frequency realized volatility of financial assets themselves, helps to improve the prediction accuracy of the mixed frequency GARCH model.

Introducing low-frequency macroeconomic variables into long-term components, such as sampling mixed frequency data, can help improve the predictive accuracy of the model. The potential spillover effects between other energy markets on the EUA futures market can also have an impact on its volatility. Liu (2020) constructed a time-varying volatility spillover index based on the TVP-VAR-SV model to study the time-varying volatility spillover between four major crude oil markets, pointing out that the volatility spillover between crude oil markets increases slowly and exhibits cyclical changes. Atukeren (2021) examined the spillover effects between WTI crude oil spot market and Brent crude oil spot market, and found that the transmission direction of volatility spillover between the two markets was different at different periods. Gong (2021) used a stochastic volatility model with time-varying parameters to study the spillover effects between the EUA futures market and the oil futures market, and the empirical results showed that there were significant spillover effects between the markets.

From this, it can be seen that there is usually a volatility spillover effect between energy markets, that is, spillover effects from other energy markets may have an impact on price fluctuations in the EUA futures market. Due to the fact that Brent crude oil futures and EUA futures belong to the ICE exchange, this article selects the monthly realized volatility of Brent crude oil futures prices as a proxy variable to test whether the crude oil futures market has spillover effects on the EUA futures market. A mixed frequency GARCH (VS-GARCH-MIDAS) model considering spillover effects is constructed, using the single factor GARCH-MIDAS model as the benchmark. The model used in this article evaluates the effectiveness of spillover effects in improving the volatility prediction of EUA futures while examining the spillover effects between markets.

3. The VS-GARCH-MIDAS Model

3.1. The GARCH-MIDAS model

In order to compare the effectiveness of introducing spillover effects in the Brent crude oil futures market for predicting EUA futures volatility, this paper selects the single factor GARCH-MIDAS model as the benchmark model. The original GARCH-MIDAS model decomposes volatility into short-term and long-term components. Among them, the short-term component usually follows the GARCH (1,1) process, while the long-term component is obtained by weighting the monthly realized volatility of the financial asset itself. The single factor GARCH-MIDAS model is as follows.

\[ r_{ij} = \mu + \sqrt{h_{ij}} e_{ij} \mid F_{t-1} \sim N(0,1) \]  \hspace{1cm} (1)

\[ h_{ij} = g_{ij} \times \tau_t \]  \hspace{1cm} (2)

\[ g_{ij} = 1 - \alpha - \beta + \alpha \left( \frac{r_{ij,t-1} - \mu}{\tau_t} \right)^2 + \beta g_{ij,t-1} \]  \hspace{1cm} (3)

\[ \ln \tau_t = m + \theta \sum_{k=1}^{K} \phi_k (\omega_k) \ln(RV_{t-k}) \]  \hspace{1cm} (4)

Among them, \( r_{ij} \) is the yield sequence of financial assets, representing the asset yield on the i-th trading day of month t, \( \mu \) is the unconditional mean of the return sequence, \( h_{ij} \) is the conditional variance, and \( e_{ij} \) is the independent return information that follows the conditional standard normal distribution. \( F_{t-1} \) is the information set for the i-th trading day of month t. \( g_{ij} \) is the short-term component of volatility, \( \alpha \) is a coefficient reflecting the persistence of short-term component volatility; For long-term components, \( \beta \) is the weighted value of monthly realized volatility under the Beta weight function, as shown in equation (5). K is the lag order of the selected monthly realized volatility. This article introduces the monthly realized volatility \( RV_t \) with a lag of one year as a long-term component into the model, that is, \( K=12 \). \( RV_t \) is the sum of squares of daily returns, where is the number of trading days in month t.

\[ \phi_k (\omega_k) = \frac{(1-k/K)^{\omega_k-1}}{\sum_{j=1}^{K} (1-k/K)^{\omega_j-1}} \]  \hspace{1cm} (5)

\[ RV_t = \sum_{i=1}^{N} r_{ij}^2 \]  \hspace{1cm} (6)

The above equations (1) - (6) together constitute the GARCH-MIDAS model based on monthly realized volatility of return information.
3.2. The VS-GARCH-MIDAS model

This paper introduces the potential spillover effects of the Brent crude oil futures market on the EUA futures market into the GARCH-MIDAS model, and uses the monthly realized volatility of the Brent crude oil futures market as a proxy variable to reflect the spillover effects on the EUA futures market. The long-term components of the VS-GARCH-MIDAS model are shown below.

\[
\ln r_t = m + \theta \sum_{k=1}^{K} \phi_k (\omega_k) \ln(RV_{t-k}) + \theta \ln(RV_t')
\]

\[
RV_t' = \sum_{i=1}^{N_t} (r_{it})^2
\]

Among them, \(RV_t'\) is the monthly realized volatility of Brent crude oil futures. Unlike the long-term components driven by the monthly realized volatility of EUA futures in their own lagged period, spillover effects can usually be quickly and timely transmitted between related markets. Therefore, this article selects the monthly realized volatility of Brent crude oil futures for the current period and introduces the GARCH-MIDAS model to examine the impact of immediate spillover effects on the volatility of EUA futures. \(r_{it}\) is the daily yield of Brent crude oil futures, \(RV_t'\) is measured by the sum of the squares of the daily returns, \(N_t\) represents the number of trading days in month \(t\). \(\theta\) reflecting the spillover effect of Brent crude oil futures market on EUA futures market, it significantly indicates the existence of one-way spillover effect, while the opposite does not exist.

4. Empirical Analysis

4.1. Data

This article selects Brent crude oil futures price data and EUA futures price data under the ICE exchange from January 1, 2008 to May 11, 2022, sourced from the Wande database. Among them, let the logarithm settlement price of the i-th trading day in the t-th month of EUA futures be \(p_{it}\), the daily yield is denoted as \(r_{it} = p_{it} - p_{i-1, t}\), and \(p_{it}'\) is the settlement price of Brent crude oil futures on the i-th trading day of the t-th month. The daily yield is denoted as \(r_{it}' = p_{it}' - p_{i-1, t}'\), \(r_{it}\) and \(r_{it}'\) are shown in Figure 1.

From Figures 1 and 2, it can be seen that there is a significant temporal variability and volatility clustering between the EUA futures yield series and the Brent crude oil futures yield series. Overall, the volatility of Brent crude oil futures is regular, with stronger volatility clustering. During periods of high volatility in Brent crude oil futures, the yield series of EUA futures also show significant fluctuations, and there may be spillover effects between the two markets. The descriptive statistics of the two are shown in Table 1.
According to Table 1, the skewness of the yield series of EUA futures and Brent crude oil futures is less than 0, the kurtosis is greater than 3, and there is a clear peak and thick tail feature. The mean and standard deviation of the daily yield of EUA futures are higher than those of Brent crude oil futures, and the volatility of their yield series is greater. The kurtosis of the daily yield distribution of EUA futures is higher, and the probability of extreme values or abnormal fluctuations is higher. And the values of the J-B statistic are both greater than 0, indicating that both sample populations do not follow a normal distribution.

<table>
<thead>
<tr>
<th>Table 1. Descriptive Statistics</th>
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<tbody>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>$r_{it}$</td>
</tr>
<tr>
<td>$r'_{it}$</td>
</tr>
</tbody>
</table>

4.2. In-sample parameter estimation

This article uses the maximum likelihood estimation method based on EUA futures price data and Brent crude oil futures price data from January 1, 2008 to November 11, 2019, for a total of 3001 samples. That is, 3000 daily return rate data are used for intra sample parameter estimation. At the same time, the GARCH (1,1) model was used as the benchmark model for comparison to examine the within sample fitting performance of GARCH-MIDAS and VS-GARCH-MIDAS models with spillover effects. The GARCH (1,1) model is shown in equations (9) - (10), and the parameter estimation results are shown in Table 2.

$$
r_i = \mu + a_i = \mu + \sqrt{\alpha_{t-1}} \epsilon_i | F_{t-1} \sim N(0,1) \tag{9}
$$

$$
h_t = m + \alpha \epsilon_{t-1}^2 + \beta \epsilon_{t-1} \tag{10}
$$

Firstly, from Table 2, it can be seen that the GARCH model satisfies the stationarity conditions of the model: $\alpha + \beta < 1$. $\beta > 0.9$ indicates that the GARCH model captures the persistence characteristics of EUA futures volatility. Secondly, in the GARCH-MIDAS model's, $\beta > 0.8$ indicates that the short-term component well characterizes the clustering characteristics of EUA futures volatility. The coefficient of the monthly realized volatility weighted value of EUA futures is significantly positive and as high as 0.89, indicating that this low-frequency variable can significantly affect the estimation of volatility. The logarithmic likelihood value and AIC and BIC information accuracy of the GARCH MIDAS model are better than those of the GARCH model. Therefore, it can be concluded that the GARCH model considering mixed frequency data sampling has a better intra sample fitting effect than the GARCH model. On the other hand, $\theta_2$ of the VS-GARCH-MIDAS model is significantly positive, indicating a spillover effect between the Brent crude oil futures market and the EUA futures market, reflecting the spillover effect of the monthly realized volatility of Brent crude oil futures on the EUA futures market. Moreover, the logarithmic likelihood value of the VS-GARCH-MIDAS model is higher than that of GARCH-MIDAS, and the AIC and BIC values are smaller, indicating that the within sample fitting effect of the VS-GARCH-MIDAS model is better than that of the GARCH-MIDAS model.

The long-term components within the samples fitted by the two models are shown in Figure 3. It can be seen that the volatility range of EUA futures was relatively large between December 13, 2011 and November 22, 2015. During this period, when fitting the peaks and valleys of volatility, the long-term components of the VS-GARCH-MIDAS model were fitted more accurately, indicating that the introduction of spillover effects helps improve the within sample fitting performance of the model.

<table>
<thead>
<tr>
<th>Table 2. Parameter estimation results</th>
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<tbody>
<tr>
<td>GARCH</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>$m$</td>
</tr>
<tr>
<td>$\alpha$</td>
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<tr>
<td>$\beta$</td>
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<tr>
<td>$\omega_k$</td>
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<tr>
<td>$\theta_1$</td>
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<tr>
<td>$\theta_2$</td>
</tr>
<tr>
<td>$LLF$</td>
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<tr>
<td>$AIC$</td>
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<tr>
<td>$BIC$</td>
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</table>

Figure 3. Conditional variance and long-term component (first: from the GARCH-MIDAS model, second: from the VS-GARCH-MIDAS model)
4.3. Out-of-sample prediction evaluation

To further compare the advantages and disadvantages of the models, this article examines the out of sample predictive ability of different models. Due to the unmeasurable nature of real volatility, this article selects the squared series of EUA futures averaged returns as the proxy variable, and uses three different loss functions to evaluate the difference between the predicted volatility of the model and the real volatility. Considering that the intra sample fitting performance of the GARCH model is significantly lower than the other two models, this article only compares the out of sample predictive ability of the GARCH-MIDAS model and the VS-GARCH-MIDAS model. The prediction sample is the EUA futures price data from November 12, 2019 to May 9, 2022, with a total of 641 trading days. Based on the estimation results within the aforementioned samples, the rolling time window method is used to predict volatility. The selected evaluation indicators for the model's out of sample predictive ability are mean absolute error (MAE), mean square error (MSE), and gaussian quasi likelihood (QLIKE). The definitions of the three evaluation indicators are as follows.

\[
MAE = \frac{1}{N} \sum_{i=1}^{N} | h_i - \hat{h}_i | \\
MSE = \frac{1}{N} \sum_{i=1}^{N} (h_i - \hat{h}_i)^2 \\
QLIKE = \frac{1}{N} \sum_{i=1}^{N} \left( \ln(\hat{h}_i) + \frac{\hat{h}_i}{h_i} \right)
\]

Among them, N represents the number of out-of-sample predictions, is the proxy variable for the true volatility, and is the volatility fitted by the model. Generally speaking, the smaller the loss function value calculated by the model, the stronger its predictive ability. This article evaluates the short-term, medium-term, and long-term predictive abilities of two models by taking 1 step forward, 10 steps forward, and 30 steps forward. The loss functions calculated by the two models are shown in Table 3a-3b.

<table>
<thead>
<tr>
<th>Table 3a. Loss values (GARCH-MIDAS)</th>
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<tbody>
<tr>
<td>horizon</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1-step</td>
</tr>
<tr>
<td>10-step</td>
</tr>
<tr>
<td>30-step</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>MAE</td>
</tr>
<tr>
<td>1.1031e-03</td>
</tr>
<tr>
<td>1.1390e-03</td>
</tr>
<tr>
<td>1.1553e-03</td>
</tr>
<tr>
<td>MSE</td>
</tr>
<tr>
<td>6.4662e-06</td>
</tr>
<tr>
<td>6.8106e-06</td>
</tr>
<tr>
<td>6.9588e-06</td>
</tr>
<tr>
<td>QLIKE</td>
</tr>
<tr>
<td>-6.0136</td>
</tr>
<tr>
<td>-5.9310</td>
</tr>
<tr>
<td>-5.9238</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3b. Loss values (VS-GARCH-MIDAS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizon</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1-step</td>
</tr>
<tr>
<td>10-step</td>
</tr>
<tr>
<td>30-step</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>MAE</td>
</tr>
<tr>
<td>1.0795e-03</td>
</tr>
<tr>
<td>1.1143e-03</td>
</tr>
<tr>
<td>1.1300e-03</td>
</tr>
<tr>
<td>MSE</td>
</tr>
<tr>
<td>6.4650e-06</td>
</tr>
<tr>
<td>6.8077e-06</td>
</tr>
<tr>
<td>6.9580e-06</td>
</tr>
<tr>
<td>QLIKE</td>
</tr>
<tr>
<td>-6.0179</td>
</tr>
<tr>
<td>-5.9300</td>
</tr>
<tr>
<td>-5.9233</td>
</tr>
</tbody>
</table>

From Table 3, it can be seen that the three loss function values of VS-GARCH-MIDAS are smaller than those of GARCH-MIDAS model in both short-term, medium-term, and long-term predictions, indicating that the out of sample predictive ability of VS-GARCH-MIDAS model is better than GARCH-MIDAS model in short-term, medium-term, and long-term predictions. From this, it can be concluded that the VS-GARCH-MIDAS model has better in sample fitting and out of sample prediction abilities than the GARCH-MIDAS model. Therefore, considering the Brent crude oil futures market spillover effect can significantly improve the prediction accuracy of EUA futures volatility.

4.4. The impact of WTI crude oil futures market

The above results indicate that the Brent crude oil futures market has a one-way spillover effect on the EUA futures market, and the introduction of this spillover effect helps to improve the prediction accuracy of EUA futures volatility. To examine the robustness of the conclusion that the introduction of spillover effects can improve the accuracy of EUA futures volatility prediction, this article examines the impact of the West Texas crude oil (WTI) futures market in the United States on the volatility of the EUA futures market. The monthly realized volatility of WTI crude oil futures, which is the sum of the squares of daily returns within the month, is introduced into the long-term component. The trend of the monthly realized volatility is shown in Figure 4.
accurately predict the volatility of EUA futures. The empirical
between markets, but also applies spillover effects to
market. This model not only considers spillover effects
MIDAS model based on the GARCH-MIDAS model by
enterprises. This article considers the spillover effects
allocation, risk management, and emission compliance
for the pricing of carbon finance derivatives, portfolio
component, the dual factor GARCH-MIDAS model that
realized volatility of EUA futures in the long-term
factor GARCH-MIDAS model that only considers the
realized volatility spillover effect of Brent crude oil futures is
significant, and there is a one-way volatility spillover effect
on the EUA futures market; (2) Compared with the single
factor GARCH-MIDAS model that only considers the
monthly realized volatility of EUA futures in the long-term
component, the dual factor GARCH-MIDAS model that introduces spillover effects has better intra sample fitting
effect and better predictive ability for short-term, medium-
term, and long-term volatility; (3) WTI crude oil futures have
spillover effects on EUA futures, and the mixed frequency
GARCH model considering this spillover effect improves the
prediction accuracy of EUA futures volatility. Therefore, the
conclusion that introducing spillover effects can improve the
predictive ability of the model for EUA futures volatility is
robust, as the transmission object of spillover effects does not change.

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herd effect and its predictive effect on stock market volatility

futures with economic policy uncertainty using the


energy markets into EUA markets under EU ETS: a multi-

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volatility prediction based on mixed frequency conditional

measurement of EGARCH model based on skip and good/bad

\begin{table}[h]
\centering
\caption{Parameter estimation results}
\begin{tabular}{|c|c|c|c|}
\hline
 & GARCH & GARCH-MIDAS & VS-GARCH-MIDAS \\
\hline \(\mu\) & 0.0008 & 0.0011 & 0.0011 \\
\hline \(\sigma\) & -0.0000 & -3.1549 & -2.2734 \\
\hline \(\alpha\) & 0.0397 & 0.1104 & 0.1113 \\
\hline \(\beta\) & 0.9309 & 0.8595 & 0.8564 \\
\hline \(\omega\) & 0.0965 & 1.0077 & 1.0091 \\
\hline \(\theta\) & 0.0890 & 0.9507 & 0.9507 \\
\hline \(\phi\) & 0.1365 & 0.1365 & 0.1365 \\
\hline \hline \(LLF\) & 5807.6263 & 6149.5000 & 6151.5185 \\
\hline \(AIC\) & -11607.2527 & -12287.0000 & -12289.0370 \\
\hline \(BIC\) & -11583.2272 & -12250.9924 & -12246.9924 \\
\hline
\end{tabular}
\end{table}

From Table 5, it can be seen that the three loss function
values of VS-GARCH-MIDAS are smaller in the short,
medium, and long-term than those of the GARCH-MIDAS
model, indicating that the out of sample predictive ability of
the VS-GARCH-MIDAS model is better than that of the
GARCH-MIDAS model. The conclusion that considering the
spillover effects of WTI crude oil futures market can
significantly improve the prediction accuracy of EUA futures
volatility is robust. Therefore, considering the spillover
effects between energy markets can improve the prediction
accuracy of EUA futures volatility.

\begin{table}[h]
\centering
\caption{Loss values (GARCH-MIDAS)}
\begin{tabular}{|c|c|c|c|}
\hline
 & 1-step & 10-step & 30-step \\
\hline MAE & 1.2288e-03 & 1.2643e-03 & 1.2843e-03 \\
\hline MSE & 6.7806e-06 & 7.1365e-06 & 7.3214e-06 \\
\hline QLIKE & -5.9622 & -5.8974 & -5.8811 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Loss values (VS-GARCH-MIDAS)}
\begin{tabular}{|c|c|c|c|}
\hline
 & 1-step & 10-step & 30-step \\
\hline MAE & 1.1973e-03 & 1.2324e-03 & 1.2520e-03 \\
\hline MSE & 6.7621e-06 & 7.1174e-06 & 7.3031e-06 \\
\hline QLIKE & -5.9721 & -5.9030 & -5.8870 \\
\hline
\end{tabular}
\end{table}

5. Conclusion

The accurate prediction of EUA futures volatility is crucial
for the pricing of carbon finance derivatives, portfolio
allocation, risk management, and emission compliance
enterprises. This article considers the spillover effects
between energy markets and constructs the VS-GARCH-
MIDAS model based on the GARCH-MIDAS model by
introducing the spillover effects of Brent crude oil futures
market. This model not only considers spillover effects
between markets, but also applies spillover effects to
accurately predict the volatility of EUA futures. The empirical
results indicate that: (1) the coefficient reflecting the monthly
realized volatility spillover effect of Brent crude oil futures is
significant, and there is a one-way volatility spillover effect
on the EUA futures market; (2) Compared with the single
factor GARCH-MIDAS model that only considers the
monthly realized volatility of EUA futures in the long-term
component, the dual factor GARCH-MIDAS model that introduces spillover effects has better intra sample fitting
effect and better predictive ability for short-term, medium-
term, and long-term volatility; (3) WTI crude oil futures have
spillover effects on EUA futures, and the mixed frequency
GARCH model considering this spillover effect improves the
prediction accuracy of EUA futures volatility. Therefore, the
conclusion that introducing spillover effects can improve the
predictive ability of the model for EUA futures volatility is
robust, as the transmission object of spillover effects does not change.


