Project Investment Decision Under Uncertain Information

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Abstract: With the change of environment and market, the current investment project decision is facing a more complex environment, the competition of enterprises intensifies, the decision complexity increases, and the demand for scientific decision is increasing. The advantage of multi-attribute decision making in investment project decision is increasingly prominent. This paper studies the application of multi-attribute decision making under triangular fuzzy uncertainty information in investment decision making. Considering the decision makers' risk aversion and preference attitude in multi-stage decision making process, a dynamic reference point setting method based on stage development characteristics is proposed. Considering that the attribute value and attribute value will produce compensation in the decision-making process, the attribute value difference formula is constructed to reduce the decision-making bias, and on this basis, an improved TODIM method is proposed. The triangular fuzzy number uncertainty information is introduced to evaluate the attributes of each scheme in each decision stage, and the information is aggregated by the aggregation operator. Combining the dynamic multi-attribute decision model, the practical problem is solved. The practicability and feasibility of this method are demonstrated by the example analysis and method comparison.

Keywords: Dynamic reference point, TODIM, Triangular fuzzy number, Project investment decision.

1. Introduction

In today's constantly developing global economic environment, we are faced with complex and changeable market conditions and uncertainties. Project investment decision is a crucial part of enterprise strategic management, but because of information asymmetry and uncertainty, project investment decision is faced with great challenges. In this context, uncertain information and prospect theory has become one of the important theories to explain people's psychological behavior when facing uncertain decisions. In 1965, Zadeh [1] introduced fuzzy sets into the field of decision making, combining fuzziness with mathematics and promoting the development of decision theory. In the actual decision-making process, the complexity of the decision-making environment, the uncertainty of information itself and people's subjective cognition lead to the uncertainty of the decision maker in the decision-making process, and the generation of fuzzy set can solve the problem of fuzzy uncertainty.

In 2009, Eddie et al. [2] introduced the fuzzy logic system into the evaluation of the real estate market, providing a systematic theoretical basis for the investment decision of real estate projects, and then helping the investment decision makers to better solve the difficulties in the investment decision of real estate projects. In 2011, William C. Waheato et al. [3] proposed the time series method to effectively quantify the risks in the real estate market, which is widely forward-looking. In 2013, based on selection experiments, Marmolejo et al. [4] combined commercial factors and social and political factors in the real estate market, and adopted the traditional polynomial regression model to comprehensively evaluate each decision attribute of the real estate project, and then selected the best real estate project. Based on the fuzziness of traditional real option values, Liu Pengyang [5] built a fuzzy real option pricing model for large-scale real estate projects with the idea of fuzzy mathematics, and verified that the real option method is a relatively scientific and reasonable real estate investment method through examples. Wang Junwu [6] et al. built a hierarchical structure model of real estate project investment decision by conducting in-depth analysis of many influencing factors that affect real estate project investment decision, classifying them and using AHP method. Jia Yuanhui [7] believes that there are many factors affecting wind power decision making. He proposes to make investment decision on wind power projects from the perspective of risk, and establishes a multi-attribute decision model based on the four major risks of economy, technology, policy and environment, aiming to realize project decision making through risk control. Liu Min [8] pointed out that wind power projects are an effective way to control CO2 emissions, calculated the impact of CO2 trading mechanism on wind power project decision-making, and considered the uncertainty of CDM trading price and the flexibility of delaying investment in the investment decision model. Through the above literature, it is found that with the complexity of the investment environment, domestic and foreign scholars no longer only study from a single economic perspective, but from the perspective of national policies, social preferences and green mechanisms, research on wind power project investment is gradually increasing, and many scholars also consider uncertain factors in the research of investment decision.

2. Theoretical Knowledge

2.1. Triangular fuzzy number

Definition 1. [9] If $\tilde{a} = [a^l, a^m, a^u]$, where $0 \leq a^l \leq a^m \leq a^u \leq 1$, Angular fuzzy number $\tilde{a}$, whose characteristic function (membership function) can be expressed as
\[
\mu_{\tilde{a}} = \begin{cases} 
\frac{x-a_l}{a_{M}-a_l}, & a_l \leq x \leq a_M, \\
\frac{x-a_l}{a_{M}-\sigma^2}, & a_M \leq x \leq a_u, \\
0, & \text{otherwise}.
\end{cases}
\] (1)

Where \(a_l\) and \(a_u\) are the lower and upper bounds of \(\tilde{a}\), respectively, and \(a^M\) is the median of \(\tilde{a}\).

**Definition 2.** [10] Triangle fuzzy number \(a = (a^l, a^m, a^u)\) and \(b = (b^l, b^m, b^u)\) the distance between the \((a, b)\) can be defined as follows:

\[
d(a, b) = \sqrt{\frac{1}{3} \left( (a_l - b_l)^2 + (a^m - b^m)^2 + (a_u - b_u)^2 \right)}
\] (2)

Where \(0 \leq \lambda \leq 1\), the risk situation pursued by the decision maker determines the choice of \(\lambda\) value. When \(\lambda > 0.5\), the decision maker is taking a risk. The decision maker does not have a great position when \(\lambda < 0.5\) indicates that the decision maker is cautious. In general \(\lambda = 0.5\).

**2.2. Multi-attribute decision method TODIM**

TODIM (Multi-objective decision making) is a multi-objective decision making method proposed by Brazilian scholars J. Figueira, S. Greco and M. Ehrgott in 2005. TODIM method is a combination of judgment matrix and ranking technology, aiming to solve the multi-objective decision-making problem, and by introducing cognitive preferences and descriptive information into the decision-making process, the decision is more in line with the preferences and descriptive information into the decision-making process. TODIM method is based on pairwise comparison of schemes and calculates the dominance of each scheme under another scheme under each attribute to sort.

The introduction of dynamic reference points makes the model more close to the psychological and risk attitude of actual decision makers. The concept of prospect theory is considered to have good applicability in project investment because it can capture the decision maker's dynamic assessment of benefits and risks at different points in time. However, multi-attribute decision making still faces a series of challenges in project investment. TODIM method effectively overcomes these challenges and improves the accuracy and reliability of investment decisions.

In this section, after improving TODIM, a TODIM multi-attribute group decision-making model based on the dynamic reference point of prospect theory will be established and applied to project investment decision-making. The specific steps are as follows:

1. Assume a dynamic multi-stage \(T_\sigma = \{1, 2, ..., \zeta\}\) under multi-attribute decision problem, where \(A = \{A_1, A_2, A_m\}\) is the set of \(m\) alternatives, \(C = \{C_1, ..., C_n\}\) is the set of \(n\) attributes, \(E = \{E_1, ..., E_{\zeta}\}\) is a group of decision experts. At stage \(T_\sigma\), decision expert \(E_k(k = 1, 2, ..., t)\) for scheme \(A_i(i = 1, 2, ..., m)\) in the attribute \(C_j(j = 1, 2, ..., n)\).

2. The evaluation information under \(n\) can be represented by the decision matrix \(X_{ij}^\omega = \left(x_{ij}^\omega\right)_{m \times n}\) as follows:

\[
x_{ij}^\omega = \left(\begin{matrix}
x_{11}^\omega & \cdots & x_{1n}^\omega \\
\vdots & \ddots & \vdots \\
x_{m1}^\omega & \cdots & x_{mn}^\omega
\end{matrix}\right)
\]

3. Step 1 Under stage \(T_\sigma\), the expert's weight vector is \(\omega_i^\sigma = (\omega_i^{\sigma 1}, \omega_i^{\sigma 2}, ..., \omega_i^{\sigma n})^T\), with the number of uncertain fuzzy aggregation operator to aggregate each expert decision matrix, aggregate after each stage of matrix \(X_{ij}^\omega\) as follows:

\[
\begin{align*}
x_{ij}^{\sigma 1} &= \frac{\sum_{r=1}^{n} w_{ir} x_{ij}^{\sigma r}}{\sum_{r=1}^{n} w_{ir}}, \\
x_{ij}^{\sigma 2} &= \frac{\sum_{r=1}^{n} w_{ir} x_{ij}^{\sigma r}}{\sum_{r=1}^{n} w_{ir}}, \quad \text{and so on.}
\end{align*}
\]

4. Step 2 calculate attribute \(C_{ij}\) and relative attributes \(C_{ij}\) standardized weights \(w_{ij}^\sigma = \frac{w_{ij}^\sigma}{\sum_{r=1}^{n} w_{ij}^{\sigma r}}\) among them, the reference weight, \(w_{ij}^\sigma = \max\{w_{ij}\}_{j, r = 1, 2, ..., n}\).

5. Step 3 to calculate the properties under \(C_{ij}\), plan \(A_i\) and \(A_k\) of relative superiority degree \(\Phi_j(A_i, A_k)\), calculated as follows:

\[
\Phi_j(A_i, A_k) = \begin{cases} 
\frac{w_{jr}(x_{ij} - x_{kj})}{\sum_{r=1}^{n} w_{lr}}, & x_{ij} \geq x_{kj} \\
\frac{1}{\theta} - \frac{w_{jr}(x_{ij} - x_{kj})}{\sum_{r=1}^{n} w_{lr}}, & x_{ij} < x_{kj}
\end{cases}
\]

Among them, the \(\Phi_j(A_i, A_k)\) properties relative to the scheme \(A_j\) of dominance, \(j = 1, 2, ..., n; i, k = 1, 2, ..., m; \) the weight of the reference attribute, \(l = 1, 2, ..., n; \) the loss decay coefficient, \(\theta > 0\);
Among them, the $\theta_j^p(A_l, A_k)$ under the stage $T_\sigma$ properties under C solution $A_l$ relative to the scheme $A_k$ of dominance, $j = 1, 2, ..., n; i, k = 1, 2, ..., m$; $w_\sigma^p$ is the weight of the reference attribute, $l = 1, 2, ..., n$; $\theta$ is the loss decay coefficient, $0 < 0$.

Step 4 Construct comprehensive dominance degree

$$
\delta_j^\sigma(A_l) = \sum_{k=1}^{m} \omega_j^p(A_l, A_k), \text{max} \theta_j^p(A_l, A_k) - \min \theta_j^p(A_l, A_k)) \text{; Combined with TOPSIS thought, to find the ideal solution to the and non-ideal solution of the decision-makers now evaluate the best investment plan. At the same time the evaluation results are shown in Table 4.1) in order to calculate the optimal expected level of each scheme } D_j^\sigma = (\delta_j^\sigma, \delta_j^\sigma); \text{Ideal solution to different schemes are calculated distance } D_j^\sigma = (\delta_j^\sigma, \delta_j^\sigma); \text{Step 5 to calculate comprehensive score of each alternative, } C_j^\sigma = \frac{D_j^\sigma}{D_j^\sigma + D_j^\sigma}; \text{Step 6 In the stage } T_\sigma, \text{the optimal expected level of each scheme under the same attribute can be obtained from } C_j^\sigma. \text{The dynamic expectation level programming model is expressed as:}

$$
C_j^\sigma = \frac{\sum_{l=1}^{n} C_l^\sigma}{n} \geq C_j^\sigma \geq C_j^\sigma, 0 \leq \omega_j^\sigma \leq 1, \sum_{j=1}^{n} (\omega_j^\sigma)^2 = 1,
$$

Step 7 Under stage $T_\sigma$, the value function of the solution is defined as:

$$
v(\Delta C_j^\sigma) = \begin{cases} 
(\Delta C_j^\sigma)^{\alpha}, & \Delta C_j^\sigma \geq 0 \\
-\theta(-\Delta C_j^\sigma)^{\beta}, & \Delta C_j^\sigma < 0
\end{cases}
$$

As $\Delta C_j^\sigma = C_j^\sigma - C_j^{\sigma+1}$ ($C_j^{\sigma+1}$ as a reference point), theta risk preference, $0 < \alpha, \beta < 1$.

Step 8 Sort scheme $v(\Delta C_j^\sigma)$. The larger the value, the better.

Step 9 In phase $T_{\sigma+1}$, $C_j^{\sigma+1}$ can obtain the optimal expected level of each scheme with the same attribute. The dynamic expectation level programming model is expressed as:

$$
C_j^{\sigma+1} = \max C_j^{\sigma+1}
$$

### Table 4.1 Triangular fuzzy number evaluation information in each stage

<table>
<thead>
<tr>
<th>$T_\sigma$</th>
<th>$A_1$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^1$</td>
<td>$A_1$</td>
<td>[0.60,0.70,0.80]</td>
<td>[0.80,0.85,0.90]</td>
<td>[0.35,0.40,0.50]</td>
</tr>
<tr>
<td></td>
<td>$A_2$</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.70,0.75,0.80]</td>
<td>[0.40,0.45,0.50]</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>[0.75,0.80,0.90]</td>
<td>[0.80,0.85,0.95]</td>
<td>[0.30,0.40,0.45]</td>
</tr>
<tr>
<td></td>
<td>$A_4$</td>
<td>[0.80,0.85,0.90]</td>
<td>[0.70,0.75,0.85]</td>
<td>[0.45,0.50,0.65]</td>
</tr>
<tr>
<td></td>
<td>$A_5$</td>
<td>[0.80,0.85,0.90]</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.30,0.35,0.40]</td>
</tr>
<tr>
<td>$T^2$</td>
<td>$A_2$</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.75,0.80,0.85]</td>
<td>[0.35,0.40,0.45]</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>[0.75,0.80,0.85]</td>
<td>[0.65,0.70,0.80]</td>
<td>[0.30,0.40,0.45]</td>
</tr>
<tr>
<td></td>
<td>$A_4$</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.80,0.90,0.95]</td>
<td>[0.55,0.60,0.65]</td>
</tr>
<tr>
<td></td>
<td>$A_5$</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.80,0.85,0.95]</td>
<td>[0.30,0.35,0.40]</td>
</tr>
<tr>
<td>$T^3$</td>
<td>$A_2$</td>
<td>[0.80,0.85,0.90]</td>
<td>[0.70,0.75,1.00]</td>
<td>[0.25,0.30,0.40]</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.80,0.85,0.85]</td>
<td>[0.40,0.45,0.50]</td>
</tr>
<tr>
<td></td>
<td>$A_4$</td>
<td>[0.80,0.85,0.95]</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.35,0.40,0.50]</td>
</tr>
</tbody>
</table>

The manuscript should include a conclusion. In this section, summarize what was described in your paper. Future directions may also be included in this section. Authors are strongly encouraged not to reference multiple figures or tables.
in the conclusion; these should be referenced in the body of
the paper.

4.2. Calculation process

The specific arithmetic steps of TODIM method based on
fuzzy information of triangular fuzzy numbers in this section
are as follows:

The evaluation information of the above three stages is
integrated and processed to obtain the decision matrix, as
shown in the following table:

Table 4.2 Decision matrix of each stage

<table>
<thead>
<tr>
<th>$T^a$</th>
<th>$A_i$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^1$</td>
<td>$A_1$</td>
<td>[0.60,0.70,0.80]</td>
<td>[0.80,0.85,0.90]</td>
<td>[0.35,0.40,0.50]</td>
</tr>
<tr>
<td></td>
<td>$A_2$</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.70,0.75,0.80]</td>
<td>[0.40,0.45,0.50]</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>[0.75,0.80,0.90]</td>
<td>[0.80,0.85,0.95]</td>
<td>[0.30,0.40,0.45]</td>
</tr>
<tr>
<td></td>
<td>$A_4$</td>
<td>[0.80,0.85,0.90]</td>
<td>[0.70,0.75,0.85]</td>
<td>[0.45,0.50,0.65]</td>
</tr>
<tr>
<td></td>
<td>$A_5$</td>
<td>[0.80,0.85,0.90]</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.30,0.35,0.40]</td>
</tr>
<tr>
<td>$T^2$</td>
<td>$A_2$</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.75,0.80,0.85]</td>
<td>[0.35,0.40,0.45]</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>[0.75,0.80,0.85]</td>
<td>[0.65,0.70,0.80]</td>
<td>[0.30,0.40,0.45]</td>
</tr>
<tr>
<td></td>
<td>$A_4$</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.80,0.90,0.95]</td>
<td>[0.55,0.60,0.65]</td>
</tr>
<tr>
<td></td>
<td>$A_5$</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.80,0.85,0.95]</td>
<td>[0.30,0.35,0.40]</td>
</tr>
<tr>
<td>$T^3$</td>
<td>$A_2$</td>
<td>[0.80,0.85,0.90]</td>
<td>[0.70,0.75,1.00]</td>
<td>[0.25,0.30,0.40]</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.80,0.85,0.85]</td>
<td>[0.40,0.45,0.50]</td>
</tr>
<tr>
<td></td>
<td>$A_4$</td>
<td>[0.80,0.85,0.95]</td>
<td>[0.85,0.90,0.95]</td>
<td>[0.35,0.40,0.50]</td>
</tr>
</tbody>
</table>

The results obtained by calculating each step are as follows:

Table 4.3 Decision-making results of each stage

<table>
<thead>
<tr>
<th>$T^a$</th>
<th>$A_i$</th>
<th>Expected value</th>
<th>reference point</th>
<th>value function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^1$</td>
<td>$A_1$</td>
<td>0.127</td>
<td>0.065</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>$A_2$</td>
<td>1.268</td>
<td>0.934</td>
<td>0.381</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>1.176</td>
<td>0.597</td>
<td>0.618</td>
</tr>
<tr>
<td></td>
<td>$A_4$</td>
<td>1.000</td>
<td>0.344</td>
<td>0.689</td>
</tr>
<tr>
<td></td>
<td>$A_5$</td>
<td>-0.504</td>
<td>0.876</td>
<td>-2.989</td>
</tr>
<tr>
<td>$T^2$</td>
<td>$A_2$</td>
<td>0</td>
<td>0.123</td>
<td>-0.356</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>0.140</td>
<td>0.099</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>$A_4$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$A_5$</td>
<td>0.127</td>
<td>1</td>
<td>-1.99</td>
</tr>
<tr>
<td>$T^3$</td>
<td>$A_2$</td>
<td>1.268</td>
<td>0</td>
<td>1.232</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>1.176</td>
<td>0.009</td>
<td>1.146</td>
</tr>
<tr>
<td></td>
<td>$A_4$</td>
<td>1.000</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

From the analysis of the final results, it can be seen that in
the $T^1$ stage, the scheme order is $A_4 > A_3 > A_2 > A_1$; in
the $T^2$ stage, the sequence is $A_3 > A_4 > A_2 > A_1$. In the $T^3$
phase, the scheme is ordered as $A_2 > A_3 > A_4 > A_1$. If an
optimal project investment needs to be selected in each stage,
the corresponding optimal project in these three stages is:
$A_4 \rightarrow A_3 \rightarrow A_2$.

5. Research Conclusions

In today's rapidly developing economic environment, project
investment decision is an important link in the
development of enterprises. Therefore, when making project
investment decisions, it is necessary to conduct adequate
market research and risk assessment to ensure the
effectiveness and sustainability of the investment. Multi-
attribute decision making is an important part of modern
decision science, system engineering and management
science, and its theory and method are widely used in many
fields such as economy, management, engineering, military
and social life. However, both theoretical research and
method application, especially in fuzzy multi-attribute
decision theory and method research, are still not perfect, still
facing new challenges, need further research. This paper
mainly focuses on fuzzy information multi-attribute (group)
decision-making problem. The main content of this paper
considers the risk aversion and preference attitude of decision
makers in the multi-stage decision-making process,
establishes a multi-stage decision-making analysis
framework based on prospect theory, and proposes a dynamic
reference point setting method based on stage development
characteristics. A multi-attribute decision making method
based on improved TODIM method is proposed. The
practicability and feasibility of the proposed method are
demonstrated by example analysis and method comparison.
To sum up, the investment decision model based on prospect
theory under uncertain information proposed in this paper
provides a new decision method for decision makers and has
important significance in solving practical problems.

Acknowledgment

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improvement and perfection of this paper.

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