Optimization Model of Heliostat Field Layout based on Genetic Algorithm

Zhenhao Bu †, Chengshuo Wu †, Huining Li †
School of Information Science and Technology, Taishan University, Taian, China
† These authors also contributed equally to this work

Abstract: In this paper, starting from the relevant data of the heliostat field, we firstly built an accurate computational model and established a 3D coordinate system with the tower as the origin, and visualized the data through Matplotlib to present the results clearly. Finally, we constructed a reliable mathematical geometric model and explored how to optimize the heliostat field by a genetic algorithm: maximizing the output power per unit area with the annual power already determined.

Keywords: Genetic Algorithm; Heuristic Algorithm; Fixed Sun Mirror Field Optimization Problem; EB Layout.

1. Introduction

The construction of a new power system mainly based on new energy is an important initiative for China to realize the goals of "carbon peak" and "carbon neutrality". The newly launched tower solar thermal power generation is a new type of low-carbon and environmentally friendly clean energy technology. The tower solar power plant uses a large number of heliostats to form a heliostat field, which is the key component of the tower solar thermal power plant and is used to collect solar energy. The heliostat mirrors are planar rectangles, consisting of a longitudinal rotating axis and a horizontal rotating axis. The planar mirrors are mounted on the horizontal axis. The longitudinal axis is perpendicular to the ground and can be used to control the azimuth of the mirror. The horizontal axis is parallel to the ground and can be used to control the pitch angle of the reflector. The mounting height of the heliostat is the height of the intersection of the two axes (also the center of the heliostat) above the ground. The heliostat field allows the sunlight to be reflected onto the collector placed on top of the heat-absorbing tower. The heat-conducting medium in the collector is heated, storing the solar energy in the form of heat and converting it into electrical energy through heat exchange. In operation, the heliostat is adjusted to the normal direction of the heliostat according to the position of the sun through a real-time control system, so that the light emitted from the center of the sun is reflected from the center of the heliostat and finally directed to the center of the collector at the top of the heat-absorbing tower.

This paper combines the local solar altitude angle, the height and size of the heliostat mirror, and uses the model to calculate the annual average optical efficiency of the heliostat field, the annual average output thermal power, and the average output thermal power per unit mirror area. Through the research reference of related professional literature and considering the latitude and longitude where the heliostat field is located, we initially think that the main factor affecting the efficiency of the heliostat field is the shading of the heliostat mirror.

Secondly, with the known output power related data and assuming that all the sidereal mirrors have the same size and mounting height, the optimal solution is obtained by heuristic algorithm to design the parameters of the sidereal mirror field, which include the position coordinates of the absorption tower, the sidereal mirror size, the mounting height, the number of sidereal mirrors, and the position of sidereal mirrors. The final realization is to make the annual average thermal power output per unit mirror area as large as possible under the condition of satisfying the rated power.

2. Modeling and Solving

2.1. Matplotlib-based Visual Geometric Model Construction

We begin by calculating the fundamental quantities, all of which directly or indirectly affect the final result. One of these is the solar declination angle $\delta$, which is presented as equation (1):

$$ \sin \delta = \sin \frac{2\pi D}{365} \sin \left( \frac{2\pi}{365} 23.45 \right) $$

(1)

In this formula, there is only one variable, D, which is the number of days counted from the equinox as day 0. Then we can roughly extrapolate the number of days from March 21 for each time based on the times on the table, and here we chose to create a list to store the results of the number of days from March for the months of January-December. Similarly, we can store the time period of the days in each month in the list ST. ST allows us to calculate the solar time angle, which is determined for each fixed time period for each of the 12 months. We bring the constructed list into Eq, as presented in equation (2):

$$ \omega = \frac{\pi}{12} (ST - 12) $$

(2)

According to the above formula, we can get the sin value of the angle of declination, and store the data of the sine value of the angle of declination into the corresponding new list. At this time, the value of $\omega, \sin \alpha_S$ has been obtained, which is presented as equation (3), according to the solar altitude angle $\alpha_S$ formula [1].

$$ \sin \alpha_S = \cos \delta \cos \varphi \cos \alpha + \sin \delta \sin \varphi $$

(3)

Calculations yielded 12 sine and cosine values for each of the sun's declination angles, and 5 cosine values for the sun's hour angle, which can be used to calculate 60 sine values for the sun's altitude angle, resulting in the following scatter plot Figure 1 of the sun's altitude angle versus month.
And save the results in a new excel sheet using the file operation. We currently have the sun's altitude angle, with a known latitude of 39.4°, brought into the formula (4):

$$\cos \gamma_{S(D,ST)} = \frac{\sin \delta - \sin \alpha_g \sin \varphi}{\cos \alpha_g \cos \varphi}$$

$$D \in \{60, 30, 0, 30, 60, 90, 120, 150, 180, 210, 240, 270\},$$

$$ST \in \{9, 10, 5, 12, 13.5, 15\};$$

We can find 60 solar azimuths $$\gamma_S$$ [2]. In the calculation process, we partition the list into groups of five, for a total of 12 cycles.

There is only one variable $$d_{HR}$$, in the atmospheric transmittance, i.e., the distance from the center of the mirror to the center of the collector, and we establish a threedimensional coordinate system based on the coordinates given in the annex, with the geographic location of the heat-absorbing tower as the z-axis as the origin, as Figure 2 presented:

It is easy to find $$d_{HR}$$ by the distance between two points in space, and then the atmospheric transmittance (mathematical notation), we find that the atmospheric transmittance is the same for each circle of mirrors, and the values of atmospheric transmittance are very similar, and the atmospheric transmittance can be regarded as a constant value in the same area for a certain period of time, which is represented here by the average value.

Among the specular efficiency losses, the cosine efficiency loss is the most serious one. In order to calculate the cosine efficiency, in this paper, we set the angle between the sun's incident light and the normal direction of the mirror surface of the heliostat as $$\theta$$, and the angle of deviation from the vertical direction of the ground as the zenith angle $$\beta$$, i.e., the angle of the vertical line (zenith line) measured from the ground surface.

Where cosine loss is an important concept that often comes up in the solar energy field, especially when evaluating solar panel performance and designing solar energy systems. Its main function is to describe the energy loss due to the angle between the direct sun rays and the solar panel.

According to the schematic diagram of the overall scenario of a fixed-sun mirror concentrating light, a visual representation of the cosine efficiency loss of the mirror is shown. The geometric relationship yields that the cosine efficiency of a single heliostat mirror can be expressed by the following equation (5):

$$\eta_{\cos} = \cos \theta = \left| \frac{1 + \cos(2\theta)}{2} \right|$$

(5)

From the nature of the mirror field power generation, it is known that the cosine efficiency value is equal to the ratio of the actual light-gathering area of the fixed-sun mirror to the area of the mirror, so set the main incoming ray of the sunlight cone to be $$\bar{r}$$, and the main outgoing ray to be $$\bar{r}$$, and then according to the $$d_{HR}$$ obtained from 5.1.5, and then the following relationship can be obtained as equation (6).

$$i = (-\cos \alpha_g \sin \gamma_S, -\cos \alpha_g \cos \gamma_S, -\sin \alpha_g)$$

$$s.t. \quad d_{HR} = \sqrt{x^2 + y^2 + (H-z)^2}$$

$$\bar{r} = (-x, -y, H-z) / d_{HR}$$

(6)

So, the smaller the angle $$\theta$$, the higher the cosine efficiency and vice versa. In the ideal case, when the direct sunlight hits the panel vertically, i.e., the angle between the sunlight rays and the panel is 0 degree, the cosine loss is minimized and the energy absorption is maximized.

In order to obtain the cosine efficiency more easily, this paper chooses to find the cosine efficiency through the zenith angle $$\beta$$. However, due to the rotation of the earth and the solar displacement caused by the rotation, the zenith angle $$\beta$$ will constantly change, and therefore, the effective energy received by the panel will also change accordingly. The zenith angle $$\beta$$ is defined as the angle of deviation from the upright direction of the ground, i.e., the angle of the vertical line (zenith line) measured from the surface.

Here, we redefine the zenith angle $$\beta$$ in order to accurately describe its relationship with cosine loss. When $$\beta$$ is equal to 0 degrees, the sun is in a direct position, perpendicular to the ground, at which time the cosine loss is at its lowest and can be considered as 0%, whereas when $$\beta$$ increases, i.e., when the sun gradually deviates from the midheaven, the cosine loss will gradually increase. In order to calculate the cosine loss accurately, we can formalize this process as the following equation [6] (7):

$$\eta_{\cos} = \frac{I_D}{DNI} \times 100\%$$

(7)

where $$\theta$$ is the zenith angle (the angle between the incident light and the direction of the local zenith), the vertical irradiation of the sunlight at any moment of the day can be described by the atmospheric quality, which was fitted from the test data from all over the world to calculate the formula as equation (8):
Where 1.353kW/m² is the solar constant, 0.7 means that roughly 70% of the solar capacity is transmitted from space to the Earth’s surface, and 0.678 is the fitted empirical value. By Ding Qi [4] et al. by calculating the shading and occlusion efficiency of a fixed-sun mirror field on four typical days, and then obtain the average effective mirror area of the fixed-sun mirror field. It can be seen that the shading and shading efficiency is the highest at noon in a day; among the four typical days, the shading and shading efficiency is the lowest on the winter solstice, the shading and shading efficiency is the highest on the summer solstice, and the change of the shading and shading efficiency is insignificant in a day, and the shading and shading efficiency curves of the spring equinox and autumn equinox have a similar trend of change.

In order to calculate the shading and occlusion efficiency, this paper adopts a method of fixed-sun mirror contour projection proposed by the Institute of Electrical Engineering of the Chinese Academy of Sciences. With this method, we can calculate the shading and occlusion losses between fixed-sun mirrors, which is realized using a dense ray-tracing technique. This ray tracing method is flexible and provides accurate calculation results. An example of neighboring heliostat shadow masking is shown in Figure 3.

### Figure 3. Example of shadow masking by neighboring heliostats

The sidereal mirror outline projection method is a flexible and natural method that utilizes the projected outlines of the sidereal mirror field and the solar tower on the ground to determine if the sidereal mirror is shadowed or obscured by nearby sidereal mirrors. At a given moment in time or position of the sun in the sky, we can project the outer contour of the heliostat and the cylindrical outer contour of the solar tower on a uniform ground plane to calculate the shadow region of the mirror. Similarly, we can project the outer contour of the heliostat as well as the reverse projection of the reflected solar rays to calculate the shaded region of the mirror. This approach effectively reduces the computational complexity of shading and occlusion efficiency and provides accurate results.

In the production process of heliostat mirrors, there are certain tracking accuracy and face shape accuracy requirements. Inaccuracies in tracking accuracy may affect whether the reflected spot can be accurately projected onto the target point, while face shape errors may cause the reflected spot to lose regularity. All of these factors may result in the mirror field reflecting a spot that does not fully cover the receiving range of the collector, and thus a portion of the light energy overflows. The schematic diagram of the solar cone light scheme is shown in Figure 4.

### Figure 4. Schematic diagram of the solar cone light scenario

Based on the dispersive nature of the conical light, we can conclude that if the heliostat is closer to the heat absorbing tower, the spot reflected to the surface of the collector is smaller; on the contrary, if it is farther away, the spot reflected to the surface of the collector is larger. Therefore, the energy distribution on the surface of the collector shows inhomogeneity.

Thus, the following equation (9) and (10) can be obtained from the above analysis:

\[
\begin{aligned}
E_{\text{all}} &= \eta_{\text{trunc}} \quad &\text{s.t.} \\
E_{\text{op}} - E_{\text{sh}} &= \eta_{\text{ref}} \\
E_{\text{op}} &= DNI \times S \times \eta_{\text{ref}} \\
E_{\text{sh}} &= DNI \times S \times 100\% \\
\eta_{\text{trunc}} &= \frac{\eta_{\text{ref}}}{1 - \eta_{\text{sh}}} 
\end{aligned}
\]

### 2.2. Model Analysis based on EB Layout

In the optimized heliostat layout problem of solar thermal power generation system, azimuthal spacing is the horizontal distance between heliostats. A reasonable setting of azimuthal spacing can ensure that the solar rays can be fully utilized and avoid shadows, thus improving the system energy harvesting efficiency. In EB layout, azimuthal spacing factor (Asf) and reset limit factor (Arlim) are key parameters, but their values have not been studied in detail. Therefore, we decided to explore the optimal values of these two parameters by combining examples and optimization algorithms to obtain a better performing heliostat field. In this study, we explore two key parameters in the EB layout, the azimuthal spacing factor (Asf) and the reset limiting factor (Arlim), by combining examples and optimization algorithms. In order to
quantitatively represent the spatial relationship between the heliostat and the heat-absorbing tower, two parameters, radial spacing $\Delta R$ and azimuthal spacing $\Delta A$ are then introduced [5] as equation (11) presented:

$$\begin{align*}
\Delta R &= (1.1442 \cot \theta_i - 1.0935 + 3.0648 \theta_i - 1.1266) \cdot H_S \\
\Delta A &= (1.791 + 0.6396 \theta_i) + 0.08373 \\
\theta &= \tan^{-1}(1/|r|)
\end{align*}$$

Relationship between the azimuthal spacing of the heliostat and the radius of the mirror field: the azimuthal spacing of the heliostat between regions in the mirror field increases with the radius of the mirror field. In order to control the maximum azimuthal distance between regions, we introduce the azimuthal distance reset limit factor (Arlim). Specifically, when the azimuthal spacing between the outermost heliostat and the innermost heliostat in a region is greater than Arlim, we decide to start arranging heliostats in the next region. The conditions for this decision can be expressed as follows: define the EB layout in the XY plane by some basic parameters, as shown in Figure 5.

Figure 5. EB Layout Schematic

According to the genetic algorithm, we first randomly initialize a set of solutions as the initial population, and in each generation of the algorithm, we find the optimal solution by calculating the fitness value of each solution. Then, crossover, selection, specifically, the position of individual mirrors is changed and then observe the conditions of the optimal solution, and so on, through continuous iteration eventually produce the optimal solution, optimal solution constraints.

The conditions are as follows:

$$\begin{align*}
100 &\leq \sqrt{x_i^2 + y_i^2} \leq 350 \\
P &\geq 60MW \\
5 + b &\leq \sqrt{(x_n - x_{n-1})^2 + (y_n - y_{n-1})^2} \\
5 + b &\leq \sqrt{(x_n - x_{n-1})^2} \\
5 &\leq a \leq b \leq 8
\end{align*}$$

In genetic algorithm, without adding constraints to a,b, it is allowed to find the optimal group (a,b) within 300 populations, and repeating the process 2000 times, 1750 different sets of a,b values are automatically obtained, and it can be seen that all these 2000 times eventually converge to almost the same value. Through 2000 iterations, we get a set of optimal solutions: i.e., $a = 5.834m$, $b = 7.2452m$.

3. Model Evaluation

In the task of calculating the heliostat parameters, we use a genetic algorithm for the optimization search, which allows a direct global search. Since the fixed-sun mirror parameter space may make the optimization objective have multiple local optimal solutions, the genetic algorithm can avoid falling into local optima and increase the possibility of finding the global optimal solution. At the same time, by maintaining a population of parameter solutions, the genetic algorithm can explore multiple possible solutions in parallel, which is particularly useful in finding the optimal values of layout parameters, since there may be multiple valid values for each parameter.

However, genetic algorithms, due to their trial-and-error approach, may require more number of generations to converge to the optimal solution, which is resource intensive when computational resources are limited or for computational time, where a number of 1000 iterations takes half an hour. At the same time, there are numerous parameters to rely on (e.g., population size, crossover probability, and mutation probability). These parameters need to be set meticulously according to the specific problem, and the parameter sizes need to be constantly adjusted and the results observed during the iteration process, which is a tedious and complex operation when the problem is too large and the results are too many.

References

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