Frontiers in Computing and Intelligent Systems
ISSN: 2832-6024 | Vol. 9, No. 1, 2024

Reliability Evaluation of Nonlinear Degraded Equipment based on Wiener Process

Dong Liu, Peng Xin, Xiaoqiang Wen

School of Information and Control Engineering, Jilin Institute of Chemical Technology, Jilin, Jilin 132022, China

Abstract: In unstable environments, many devices exhibit non-linear degradation characteristics, and their degradation rate will also change accordingly. In order to consider the degradation differences between individuals, We introduce a method for evaluating equipment reliability in the context of nonlinear degradation, leveraging the Wiener process. This technique utilizes the Wiener process to depict equipment degradation and employs a time scale model to linearize nonlinear data. At the same time, the drift coefficients of the Wiener process were randomized to propose a reliability model that considers individual differences. The unknown parameters in the model were determined by using two-stage maximum likelihood estimation, and the correctness and superiority of this method were verified through examples.

Keywords: Reliability Assessment; Wiener Process; Nonlinear Data; Individual Variation.

1. Introduction

With the popularization and large-scale application of modern industrial equipment, the precision of equipment is getting higher and higher. These devices integrate complexity and intelligence, and may fail during equipment operation, leading to serious accidents. In 2005, a serious explosion accident occurred in the biphenyl unit of Jilin Petrochemical Company, which was caused by the blockage of the nitration unit P-102 tower in the aniline unit, resulting in huge economic losses [1]. A large number of practical engineering practices have shown that reliability assessment technology can provide theoretical basis for health management activities such as product maintenance decisions and optimization of spare parts inventory. Therefore, it is necessary to carry out early maintenance management of equipment [2]. Therefore, the reliability evaluation of equipment has become crucial. However, in practical work, many devices operate in unstable environments, and their degradation rates also exhibit non-linear characteristics. Therefore, traditional reliability assessment methods often find it difficult to accurately describe the degradation process of equipment and individual differences [3]. To address this issue, this paper proposes a new reliability evaluation method for nonlinear degraded equipment based on the Wiener process.

The Wiener process is widely used to describe stochastic processes, characterized by continuity, Markov nature, and independent incrementality, which can effectively describe the degradation process of equipment [4]. At the same time, we combine a time scale model to linearize nonlinear data, in order to more accurately describe the degradation characteristics of equipment. In addition, we also studied the degradation differences between individuals and proposed a reliability model that considers individual differences by randomizing the drift coefficients of the Wiener process [5]. We used a two-step maximum likelihood estimation to determine the unknown parameters in the model, verifying the accuracy and superiority of this method [6].This article aims to propose a new nonlinear degradation equipment reliability evaluation method to more accurately describe the degradation process of equipment and individual differences, providing theoretical support and guidance for practical engineering applications.

2. Wiener Process

The Wiener process is a type of diffusion motion with a linear drift term, driven by the standard Brownian motion, which is a Gaussian process with a mean of 0 and variance dependent on time. Originally used to describe the random walk of small particles. Given this, Wiener processes are generally used to model non monotonic degradation processes with linear trends.

Generally, the Wiener process{X(t), t≥0} can be expressed as

$$X(t) = \lambda t + \sigma B(t) \tag{1}$$

In the formula: λ is the drift coefficient; $\sigma > 0$ for diffusion coefficient; {B(t), t≥0} for standard Brownian motion. At the current detection time t_k ,The remaining lifespan of the equipment L_k can be defined as{X(t),t≥0}The first arrival time to reach the failure threshold ω :

$$L_k = \inf\{l_k : X(t_k + l_k) \ge \omega | X(t_k) < \omega\} \tag{2}$$

From this, the probability density function can be obtained

$$f_{L_k}(l_k) = \frac{\omega - x_k}{\sqrt{2\pi l_k^3 \sigma^2}} \exp\left\{-\frac{(\omega - x_k - \lambda t)^2}{2 l_k \sigma^2}\right\} \tag{3}$$

3. Reliability Modeling of Nonlinear Degraded Data

3.1. Remaining Life Function

In engineering practice, the degradation process of many products often exhibits nonlinear and stochastic characteristics. In order to better describe these data, we usually perform linear transformations on nonlinear degraded data to enable the use of Wiener processes for description [7]. Subsequently, by randomizing the drift coefficients of the Wiener process, we can more comprehensively depict the stochastic characteristics of product degradation process.

Consider general nonlinear stochastic processes $\{X(t), t \ge 0\}$,to describe the degradation process of nonlinear stochastic degraded equipment. Specifically, $X(t)$ represents the degradation amount at time t, and there is

$$X(t) = X(0) + \int_0^t \mu(\tau, \theta) d\tau + \sigma B(T) \tag{4}$$

The drift coefficient of the $\mu(\tau;\theta)$ -degradation process in the equation is a nonlinear function of time t, used to characterize the nonlinear characteristics of the model. θ --Parameter vector

Furthermore, different forms of $\mu(t;\theta)$ functions can describe different forms of nonlinear characteristics. When the drift coefficient $\mu(t,\theta) = abt^{b-1}$ is about a time power function and nonlinearity=1, it is a linear standard Wiener process.

Using the concept of first arrival time, the lifespan T can be defined as:

$$T = \inf\{t; X(t) > | X(0) < W\} \tag{5}$$

We can obtain the probability density function

$$f(t) \cong \frac{w - \mu_\theta t^b (1-b)}{\sigma\sqrt{2\pi t^3}} \exp\left[\frac{-(w - \mu_\theta t^b)^2}{2\sigma^2 t}\right] \tag{6}$$

The probability density function of equations (6) does not consider random effects, but due to the interaction between many degraded devices of the same batch number, it can affect the prediction performance. Therefore, it is obtained that (7).

$$f_{T_1}(t) \approx \frac{1}{\sqrt{2\pi t^3(\sigma_\theta^2 t^{2b-1} + \sigma_\theta^2)}} (w - (t^b - bt^b) \frac{w\sigma_\theta^2 t^{b-1} + \mu_\theta \sigma^2}{\sigma_\theta^2 t^{2b-1} + \sigma^2}) \times \exp\left\{-\frac{(w - \mu_\theta t^b)^2}{2t(\sigma_\theta^2 t^{2b-1} + \sigma^2)}\right\} \tag{7}$$

In the formula, b - is about the time nonlinear index

3.2. Lifetime Reliability Function

In the field of reliability engineering, the life reliability function is a probability distribution function that evaluates the normal operation of a system or component within a specific lifespan. The study of life reliability function is crucial for evaluating the reliability and stability of systems. By analyzing the reliability function of the system's lifespan, it is possible to predict the operating status of the system in the future, guide the maintenance and improvement work of the system, and thereby improve the reliability and performance of the system [8]. By integrating the probability density function of the lifespan obtained in 2.1, the reliability function of the lifespan can be obtained

$$R_{T_1}(t) = \phi(\frac{\omega - \mu_\theta t}{\sqrt{\sigma_\theta^2 t + \sigma^2 t^2}}) \exp\left[\frac{2\omega}{\sigma_\theta^2}(\omega + \frac{\sigma^2}{\sigma_\theta^2})\right] \cdot \phi\left[-\frac{2\sigma^2 \omega t + \sigma_\theta^2(\omega + \mu_\theta t)}{\sigma_\theta^2 \sqrt{\sigma_\theta^2 + \sigma^2 t^2}}\right] \tag{8}$$

4. Parameter Estimation

Assuming parameters $\Theta = (\mu_\theta, \sigma_\theta^2, \sigma^2, b)$,Assuming there are N independent devices, each with a degradation count of K, the testing time point for the i-th degraded device is $t_1, t_2 \cdots t_n$,The corresponding degradation measurement data represents the degradation data detected by each degradation device $Y_i = (y_{i,1}, y_{i,2} \cdots y_{i,n})$.The independent incremental property of standard BM motion can be obtained that Y_i follows a multidimensional normal distribution, with the mean and covariance being:

$$\widetilde{u}_\theta = \mu_\theta T_n, \sum\nolimits_n = \Omega_n + \sigma_\theta^2 T_n T_n'.$$

$$Q_i = \begin{bmatrix} t_{i,1} & t_{i,1} & \cdots & t_{i,1} \\ t_{i,1} & t_{i,2} & \cdots & t_{i,2} \\ \vdots & \vdots & \vdots & \vdots \\ t_{i,1} & t_{i,2} & \cdots & t_{i,k_i} \end{bmatrix} \quad \Omega_n = \sigma_n^2 T_n T_n'$$

Since different individuals in the same batch number are independent of each other, the logarithmic likelihood function of data Y with respect to $\vartheta = (\mu_\theta, \sigma_\theta^2, \sigma^2, b)^\tau$ an be expressed as follows (9)

$$\ell(\Theta | X) = -\ln(2\pi)\sum\nolimits_{n=1}^N k_n - \frac{1}{2}\sum\nolimits_{n=1}^N \ln|\sum\nolimits_n| - \frac{1}{2}\sum\nolimits_{n=1}^N (Y_n - \mu_\theta T_n)' \sum\nolimits_n^{-1} (Y_n - \mu_\theta T_n)$$

$$|\sum\nolimits_n| = |\Omega_n|(1 + \sigma_\theta^2 T_n' \Omega_n^{-1} T_n) \tag{9}$$

$$\sum\nolimits_n^{-1} = \Omega_n^{-1} - \frac{\sigma_\theta^2}{1 + \sigma_\theta^2 T_n' \Omega_n^{-1} T_n} \Omega_n^{-1} T_n T_n' \Omega_n^{-1}$$

$$\frac{\partial \ell(\eta | Y)}{\partial \mu_\theta} = \frac{\sum_{n=1}^N T'\Omega^{-1} Y_n - N\mu_\theta T'\Omega^{-1} T}{1 + \sigma_\theta^2 T \Omega^{-1} T} \tag{10}$$

Further derive the first-order partial derivatives of the likelihood function above regarding μ_θ and σ_θ

$$\frac{\partial \ell(\eta | Y)}{\partial \sigma_\theta} = -\frac{N\sigma_\theta T'\Omega^{-1} T}{1 + \sigma_\theta^2 T'\Omega^{-1} T} + \frac{\sigma_\theta \sum_{n=1}^N (Y_n - \mu_\theta T)'\Omega^{-1} TT'\Omega^{-1}(Y_n - \mu_\theta T)}{(1 + \sigma_\theta^2 T'\Omega^{-1} T)^2} \tag{11}$$

According to equations (11) and (12), it can be concluded that

$$\mu_\theta = \frac{\sum_{n=1}^{N} T'\Omega^{-1}Y_n}{NT'\Omega^{-1}T} \tag{12}$$

$$\sigma_\theta = \left\{ \frac{1}{N(T'\Omega^{-1}T)^2} \sum_{n=1}^{N} (Y_n - \mu_\theta T)'\Omega^{-1}TT'\Omega^{-1}(Y_n - \mu_\theta T) - \frac{1}{T'\Omega^{-1}T} \right\}^{1/2} \tag{13}$$

Based on this, the maximum likelihood profile likelihood functions of σ and bwith respect to μ_θ and σ_θ^2 can be expressed as

$$\ell(\sigma, b \mid Y, \mu_\theta, \sigma_\theta) = -Nk\ln(2\pi) - \frac{N}{2} - \frac{N}{2}\ln|\Omega| - \frac{1}{2}\left\{ \sum_{n-1}^{N} Y_n'\Omega^{-1}Y_n - \frac{\sum_{n=1}^{N}(T'\Omega^{-1}Y_n)^2}{T'\Omega^{-1}T} \right\} \tag{14}$$

$$-\frac{N}{2}\ln\left\{ \frac{\sum_{n=1}^{N}(Y_n'\Omega^{-1}Y_n)^2}{NT'\Omega T} - \frac{\sum_{n=1}^{N} T'\Omega^{-1}Y_n}{N^2T'\Omega^{-1}T} \right\}$$

So the maximum likelihood estimates of σ and bcan be obtained by maximizing the profile likelihood function using a two-dimensional search method. Then, the maximum likelihood estimates of σ and b obtained from the search are taken into equations (13) (14), To obtain the corresponding maximum likelihood estimates of μ_θ and σ_θ^2.

5. Instance Verification

The inertial navigation system, as an important component of the strategic missile control system, can navigate on its own without the need for external signals, and plays a crucial role in the accuracy of guidance and system controllability [9]. The gyroscope, as the core equipment of the inertial navigation system, determines the position of the carrier by calculating its angular velocity. The quality of the gyroscope directly affects the guidance accuracy. In practical applications, gyroscopes are easily affected by the combined effects of external environment and internal wear, and the drift value of gyroscopes will tend to increase [10]. When the drift value of the gyroscope increases to the pre-set failure threshold, it is considered that the performance of the gyroscope has failed. The failure of the gyroscope's performance will have an impact on the effectiveness of the inertial navigation system. In practical engineering, modern detection technology can be used to detect the degradation data of gyroscopes and historical data of the same batch number, thereby estimating the remaining service life of gyroscopes. Si et al. [11] provided 5 sets of degradation data for the same batch of gyroscopes. According to expert experience, the failure threshold for the preset drift value was 0.6 (°)/h, and the monitoring starting point was taken as the zero moment.

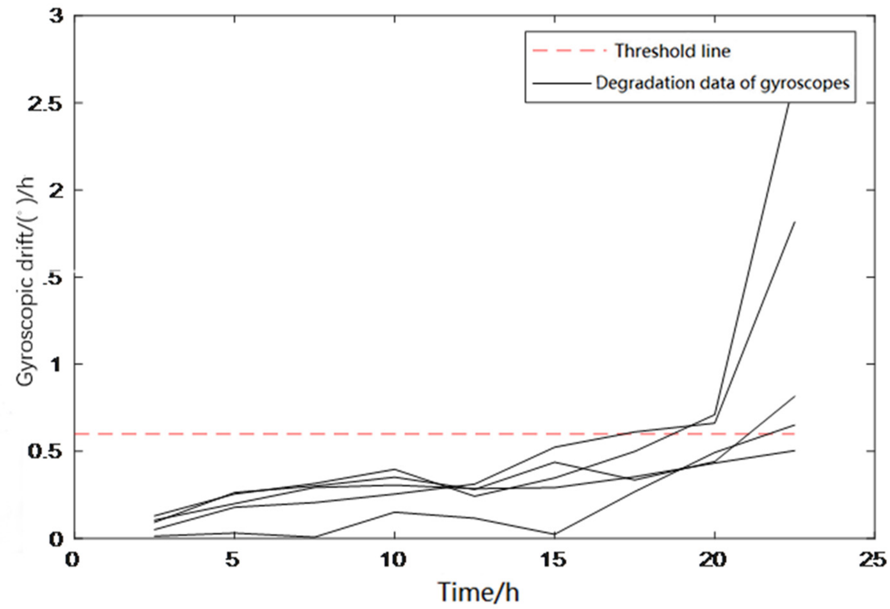

Figure 1. Gyroscope Drift Degradation Trajectory of Inertial Navigation System

According to Figure 1, it can be seen that the drift degradation of the gyroscope is comprehensively affected by factors such as actual operating environment and internal wear, exhibiting degradation characteristics slightly different from linear features. It is evident from the graph that the degradation of the gyroscope exhibits significant nonlinear characteristics, therefore a nonlinear modeling method is needed.

Table 1. Inertial Navigation Coefficients for Model 1 and Model 2

	M1	M2
μ_θ	0.055705	2.34*10^(-25)
σ_θ	0.0255	2.18*10^(-25)
b	-	18.1608
σ	0.2106	0.0659
Log-LF	-13.374	28.377
AIC	32.748	-48.754

Among them, Model 1 is a linear Wiener process without considering the value of b, Model 2 is a nonlinear Wiener process, and the AIC in the table represents the Akashi information criterion, which is used for performance measurement and comparison. The formula for the fitting degree of the model to the measurement data is shown in Equation (15):

$$AIC = -2(\max \ell) + 2M \tag{15}$$

In the formula, M - represents the number of independent parameters of the model;

$\max \ell$ ——The maximum logarithmic likelihood function of the model

The Akashi information criterion is mainly used in practice as a method for fitting models with good performance. It can achieve a balance between complex models and fitting accuracy, making the prediction results of the model better. The smaller the value, the better the fitting effect of the model. According to Table 1, it can be seen that the Akashi information criterion of Model 2 is the smallest, so the corresponding model has the best fitting degree.

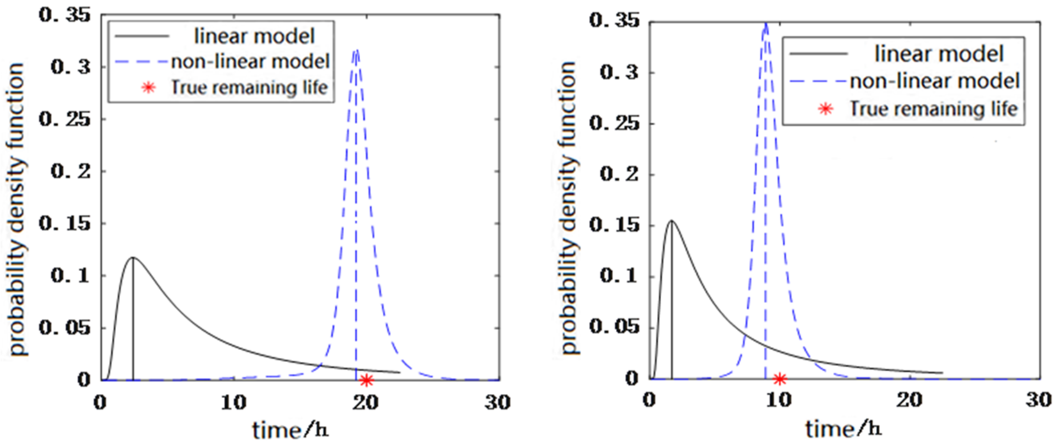

Figure 2. Probability density functions of two models at different time points

Figures 2 show the probability density function curves at different times, and it can be seen that the probability density function curves at different times show that the nonlinear model is closer to the actual values. In addition, the peak of the nonlinear model is higher and sharper, indicating lower uncertainty. This indicates that nonlinear models are more effective in describing gyroscope degradation data, and linear models are no longer applicable to nonlinear degradation equipment.

Based on the parameter estimation results of the model in Table 1, the parameter estimation results are inputted into the reliability function, and the reliability function curve is obtained as shown in Figure 3. Let the reliability model for nonlinear degradation be M2, and the reliability model for linear degradation be denoted as M1. The reliability model of linear degradation is obtained by integrating the probability density function of the first arrival time.

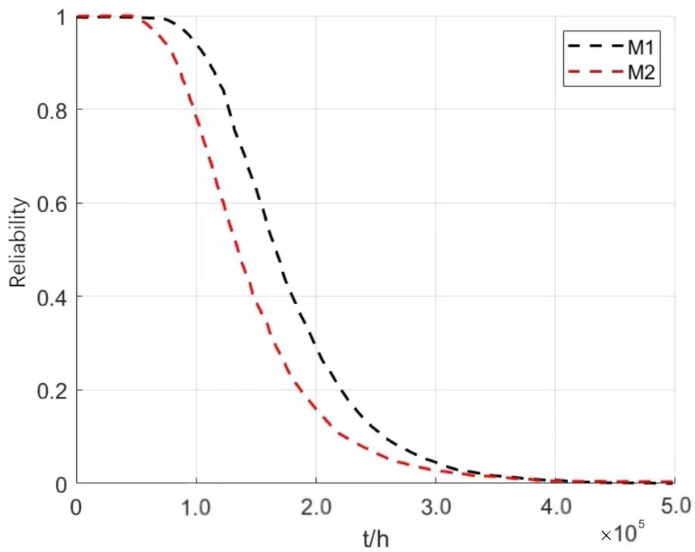

Figure 3. Reliability curves of two methods

Observing the M2 curve of the model, the reliability of the equipment is 1 before running for 9500 hours, indicating that the equipment is in a safe operating state. The reliability rapidly decreases from 9500 hours, indicating that the performance of the equipment gradually decreases with the increase of degradation. Staff should pay attention to the equipment and perform maintenance or upkeep at the appropriate time. When the reliability of 30000 hours of operation approaches 0, it indicates that the equipment is in a state of complete failure. Comparing the reliability curves of the two models, the reliability curve of Model M2 is steeper. Under the same operating time, the reliability obtained through model M2 is lower than that of model M1, and the reliability evaluation results are more "conservative".

6. Conclusion

This article studies the reliability evaluation method of nonlinear degraded equipment based on Wiener process by considering the degradation differences between individuals under constant stress accelerated degradation test conditions. By considering the degradation differences between individuals, the drift coefficients in the Wiener process are randomized and assumed to be normally distributed. This solution is superior to traditional methods that do not consider individual differences, as evidenced by the comparison of AIC values and reliability curves

References

[1] Wang Z , Fu H , Zhang . Analyzing Degradation by an Independent Increment Process[J]. Quality and Reliability Engineering International, 2014, 30(8): 1275-1283.

[2] Peng C Y, Tseng S T. Mis-Specification Analysis of Linear Degradation Models[J]. IEEE Transactions on Reliability, 2009, 58(3): 444-455.

[3] TANG S, GUO X, YU C, et al. Accelerated Degradation Tests Modeling Based on the Nonlinear Wiener Process with Random Effects [J]. Mathematical Problems in Engineering, 2014(2): 1-11.

[4] Si X S, Wang W, Hu C H , et al. Remaining Useful Life Estimation Based on a Nonlinear Diffusion Degradation Process[J]. IEEE Transactions on Reliability,2012,61(1):50-67.

[5] Ye Z.S, Chen N., Shen Y. A new class of Wiener process models for degradation analysis [J]. Reliability Engineering& System Safety, 2015, 139: 58-67.

[6] Chih-Chun, Tsai, and, et al. Mis-specification analyses of gamma and Wiener degradation processes[J]. Journal of Statistical Planning and Inference, 2011, 141(12): 3725: 3735.

[7] Li W , Liang Y Y, Pan G , et al. Step-down-stress degradation modeling based on Gamma process[C]. International Conference on Quality. IEEE, 2013: 312-315.

[8] GEBRAEEL, Nagi Z, LAWLEY, et al. A Neural Network Degradation Model for Computing and Updating Residual Life Distributions[J]. IEEE Transactions on Automation Science Engineering, 2008, 5(1)154-163.

[9] A, Khanh Le Son , et al. "Remaining useful life estimation based on stochastic deterioration models: A comparative study. Reliability Engineering & System Safety 112.4(2013):165-175.

[10] Liao H , Elsayed E A. Reliability inference for field conditions from accelerated degradation testing[J]. Naval Research Logistics (NRL), 2010, 53(6): 576-587.

[11] SI X S, WANG W B, HU C H, et al. Remaining useful life estimation based on a nonlinear diffusion degradation process [J]. IEEE Transaction on Reliability, 2012, 61 (1): 50-67.