Frontiers in Computing and Intelligent Systems
ISSN: 2832-6024 | Vol. 15, No. 3, 2026

Effect of Node Weight Distributions on Critical Percolation Behavior in Weighted Networks

Hui Wang

School of Economics and Management, Chang'an University, Xi'an, Shaanxi, 710064, China

Abstract: Percolation theory provides a fundamental framework for understanding connectivity transitions in complex networks. While previous studies have primarily focused on topological structures, the role of node weight distributions in shaping percolation behavior remains insufficiently explored. In this study, we systematically investigate how different node weight distributions affect critical percolation properties in weighted networks. Four representative distributions—uniform, normal, exponential, and power-law—are considered. Numerical simulations show that weight heterogeneity significantly influences both the percolation threshold and the size of the largest connected component at criticality. In particular, highly heterogeneous distributions lead to earlier phase transitions but may result in reduced connectivity at critical states. These findings highlight the importance of weight distribution characteristics in determining network robustness and provide insights for the design and analysis of real-world infrastructure systems.

Keywords: Percolation Theory; Complex Networks; Weight Distribution; Phase Transition; Network Robustness.

1. Introduction

Percolation theory has been widely used to characterize connectivity transitions in complex systems[1,2], including transportation, communication, and infrastructure networks. As a network approaches critical conditions, small perturbations can trigger abrupt fragmentation, leading to large-scale connectivity loss[3]. Understanding the factors that influence such phase transitions is therefore essential for improving system resilience.

Most existing studies have focused on the role of network topology, such as degree distribution, clustering, and structural heterogeneity. These studies have shown that different network structures exhibit distinct percolation thresholds and transition behaviors. However, real-world networks are not only shaped by topology but also by heterogeneous node attributes[4,5], such as traffic load, capacity, or demand intensity.

Despite this, the impact of node weight distributions on percolation behavior has received relatively limited attention. In many cases, node weights are assumed to follow simple or uniform distributions, which may not adequately reflect real-world conditions. In practice, node weights often exhibit strong heterogeneity, such as heavy-tailed or skewed distributions[6,7].

To address this gap, this study investigates how different node weight distributions influence percolation dynamics in weighted networks. By systematically comparing multiple representative distributions, we aim to reveal the relationship between weight heterogeneity and critical percolation properties, including the percolation threshold and the size of the largest connected component at criticality.

2. Model and Method

2.1. Network Model

We consider an undirected network $G=(V,E)$, where V denotes the set of nodes and E denotes the set of edges. The total number of nodes is $N=|V|$. Each node $vi\in V$ is associated with a normalized weight $wi\in[0,1]$, representing its load or activity level.

To focus on the effect of weight distributions, we perform simulations on representative synthetic networks, such as Erdős–Rényi (ER) and Watts–Strogatz (WS) networks, while keeping the network size and average degree fixed.

2.2. Weight Distributions

Four typical node weight distributions are considered:

Uniform distribution:$wi\sim U(0,1)$, Normal distribution (truncated to [0,1]):$wi\sim N(\mu,\sigma 2)$, Exponential distribution: $wi\sim Exp(\lambda)$, Power-law distribution: $P(w)\sim w-\gamma$

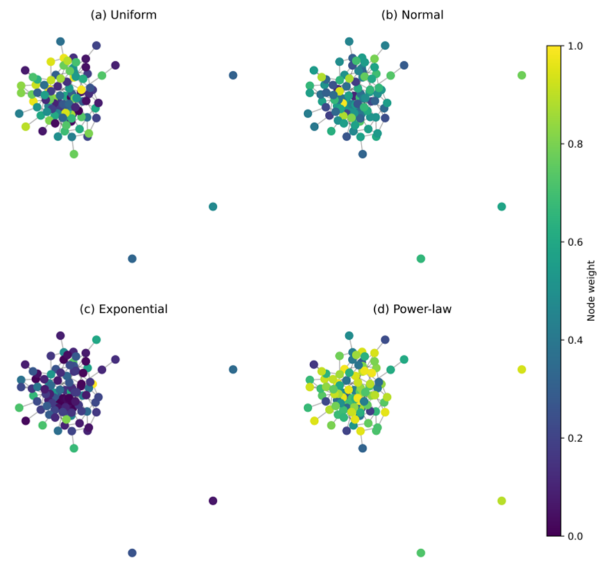

Figure 1. Network topology with node weights under different distributions.

(a) Uniform distribution $U(0,1)$, (b) truncated normal distribution $N(0.5, 0.2^2)$ with values clipped to [0,1], (c) exponential distribution $Exp(\lambda= 0.5)$ normalized to [0,1], and (d) power-law distribution generated using a power distribution with exponent parameter $\gamma = 2.5$.

Node colors represent normalized weights, where darker tones indicate higher values. The network structure and layout are identical across all panels to ensure comparability. It can be observed that the uniform distribution produces evenly distributed node weights, while the normal and exponential distributions exhibit moderate heterogeneity. In contrast, the power-law distribution results in strong heterogeneity, characterized by a small number of high-weight nodes and a large number of low-weight nodes.

These distributions represent different levels of heterogeneity, ranging from homogeneous (uniform) to highly heterogeneous (power-law).

2.3. Percolation Process

We adopt a threshold-based percolation model. Given a global control parameter $p\in[0,1]$, the state of node vi is defined [8]as:

$$\phi_i(p)=\begin{cases}1, & w_i \le p\\ 0, & \text{otherwise}\end{cases} \tag{1}$$

The set of active nodes is $V(p)=\{vi\in V|\phi i(p)=1\}$, and the induced subgraph is denoted as $G(p)$.

Let $S(p)$ denote the fraction of nodes in the largest connected component (LCC) of $G(p)$.

The percolation threshold pc is defined as the value of p at which the size of the second-largest connected component reaches its maximum.

3. Results

3.1. Percolation Curves under Different Distributions

Figure 2 illustrates the evolution of the largest connected component $S(p)$ under different node weight distributions. Significant differences in percolation behavior can be observed.

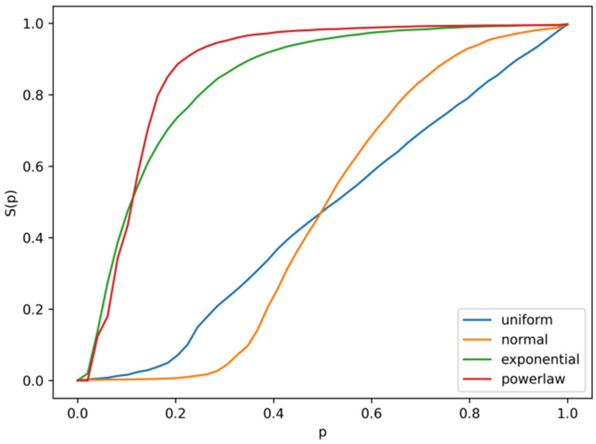

Figure 2. Percolation curves under different node weight distributions.

This figure illustrates the evolution of the normalized size of the largest connected component S(p) as a function of the percolation control parameter p for four representative node weight distributions: uniform, normal, exponential, and power-law. Each curve corresponds to a different distribution, highlighting how weight heterogeneity affects the phase transition process. The results show that the power-law distribution leads to an earlier emergence of the giant component (lower percolation threshold), while the uniform distribution exhibits a smoother and more gradual transition. Intermediate behaviors are observed for the normal and exponential distributions.

For the uniform distribution, the percolation transition occurs relatively smoothly, with a gradual increase in connectivity. In contrast, the power-law distribution exhibits a much earlier transition, indicating a lower percolation threshold. However, the growth of the largest connected component is less stable near the critical point.

The normal and exponential distributions show intermediate behavior, with transition characteristics lying between the uniform and power-law cases.

3.2. Comparison of Percolation Thresholds

The corresponding percolation thresholds under different weight distributions are summarized in Figure 3.

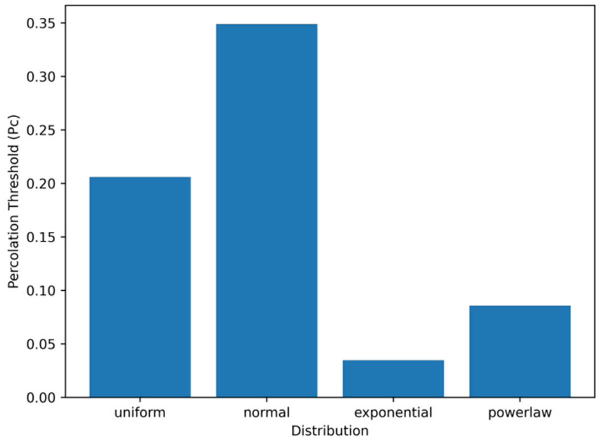

Figure 3. Comparison of percolation thresholds under different node weight distributions

This figure presents the percolation threshold pc for different node weight distributions, where pc is defined as the value of the control parameter p at which the second-largest connected component reaches its maximum size. The results indicate that the power-law distribution yields the lowest percolation threshold, reflecting accelerated phase transition due to strong heterogeneity. In contrast, the uniform distribution shows the highest threshold, indicating more stable activation dynamics. The normal and exponential distributions exhibit intermediate threshold values.

The results indicate that the power-law distribution yields the smallest percolation threshold, suggesting that strong heterogeneity accelerates the onset of percolation. In contrast, the uniform distribution has the largest threshold, reflecting more stable and homogeneous activation dynamics. The normal and exponential distributions produce intermediate values.

These findings demonstrate that increasing weight heterogeneity tends to lower the percolation threshold.

3.3. Effect of Weight Heterogeneity

To further quantify the effect of heterogeneity, we introduce a dispersion measure:

$$H=\frac{\sigma_w}{\mu_w} \tag{2}$$

where σw and μwdenote the standard deviation and mean of node weights, respectively.

Figure 4 shows the relationship between H and pc. A clear negative correlation is observed, indicating that higher heterogeneity leads to smaller percolation thresholds.

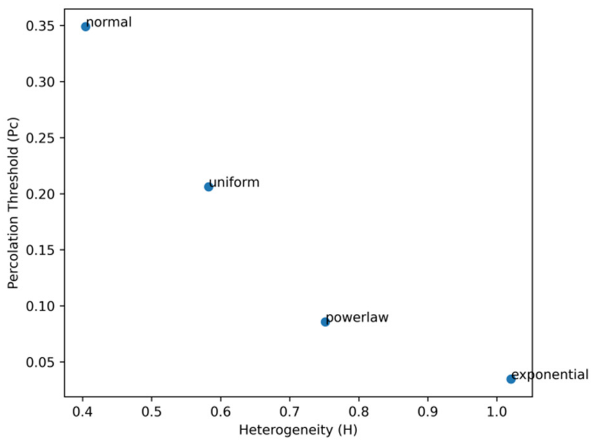

Figure 4. Relationship between weight heterogeneity and percolation threshold

This figure shows the relationship between the heterogeneity measure H (defined as the coefficient of variation of node weights) and the percolation threshold pc. Each point represents a different weight distribution. A clear negative correlation can be observed, indicating that higher weight heterogeneity leads to a lower percolation threshold. This suggests that heterogeneous networks tend to undergo phase transitions earlier, although such early transitions may be associated with reduced structural stability near criticality.

However, increased heterogeneity may also reduce the size of the largest connected component at criticality, suggesting a trade-off between early activation and structural robustness.

4. Discussion

The results highlight the critical role of node weight distributions in shaping percolation dynamics. Unlike topological factors, which determine the structural skeleton of the network, weight distributions influence the sequence and pattern of node activation.

Highly heterogeneous distributions, such as power-law, concentrate activation among a small subset of nodes, leading to earlier phase transitions. However, this also makes the network more sensitive to local fluctuations, potentially reducing stability near the critical point.

In contrast, homogeneous distributions promote more uniform activation, resulting in smoother transitions and more stable connectivity.

These observations suggest that both topology and weight distribution should be considered when analyzing and designing real-world networks, particularly in systems where load or demand is unevenly distributed.

5. Conclusion

This study investigated the impact of node weight distributions on percolation behavior in weighted networks. The results demonstrate that weight heterogeneity plays a crucial role in determining critical percolation properties.

In particular, highly heterogeneous distributions lead to lower percolation thresholds but may reduce connectivity stability at critical states. These findings provide new insights into the interplay between weight distribution and network robustness and may inform the design of resilient infrastructure systems.

References

[1] Stauffer, D. & Aharony, A. Introduction to Percolation Theory. Taylor & Francis (2018). https://doi.org/10. 1201/ 978131527 4386.

[2] Callaway, D. S. et al. Network robustness and fragility: Percolation on random graphs. Phys. Rev. Lett. 85, 5468–5471 (2000). https://doi.org/10.1103/PhysRevLett.85.5468.

[3] Albert, R., Jeong, H. & Barabási, A. L. Attack and error tolerance of complex networks. Nature 406, 378–382 (2000).

[4] Pastor-Satorras, R. & Vespignani, A. Epidemic dynamics and endemic states in complex networks. Phys. Rev. E 63, 066117 (2001). https://doi.org/10.1103/PhysRevE.63.066117.

[5] Moreno, Y., Pastor-Satorras, R. & Vespignani, A. Epidemic outbreaks in complex heterogeneous networks. Eur. Phys. J. B 26, 521–529 (2003). https://doi.org/10.1140/epjb/e20020122.

[6] Baxter, G. J. et al. Heterogeneous k-core versus bootstrap percolation on complex networks. Phys. Rev. E 83, 051134 (2011). https://doi.org/10.1103/PhysRevE.83.051134.

[7] Li, M., Liu, R. R., Lü, L. et al. Percolation on complex networks: Theory and application. Phys. Rep. 907, 1–68 (2021). https://doi.org/10.1016/j.physrep.2020.12.003.

[8] Li, D. et al. Percolation transition in dynamical traffic network with evolving critical bottlenecks. Proc. Natl. Acad. Sci. U.S.A. 112, 669–672 (2014).https://doi.org/10. 1073/pnas. 1419185 112.