

# Quantum fuzzy neural network based on fuzzy number

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**Abstract:** Neural network is one of the AI algorithms commonly used to process data, and has an extremely important position in scenarios such as image recognition, classification, and machine translation. With the increase of data volume explosion, the required computing power of neural networks is also significantly increased. The emergence of quantum neural networks improves the computational power of neural networks, but the accuracy of neural networks and quantum neural networks is not high in the face of the complexity and uncertainty of big data. In order to improve the efficiency and accuracy, the cross-fusion of "fuzzy number theory + quantum neural network" is proposed to study the quantum fuzzy neural network (FQNN) based on fuzzy number. The Gaussian fuzzy function is used to generate the corresponding fuzzy affiliation matrix to describe the uncertain information in the data. The fuzzy independent variables are trained through the FQNN model, and the model is output after changing the parameters of the quantum forward propagation layer. Simulation experiments show that the quantum fuzzy neural network model based on fuzzy number is more efficient and accurate in this study compared with the quantum neural network model.

**Keywords:** Fuzzy set theory; Fuzzy book; Quantum computing; Artificial intelligence; Quantum fuzzy neural network.

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## 1. Introduction

With the rapid development and extensive use of information technology, the explosive growth of business data in the information field has announced the arrival of the era of big data. Big data is not only a significant increase in the volume of data, but also implies a lot of valuable information. By mining and analyzing big data, valuable information can be obtained to support people's decision making. Neural network [1] with powerful computation and learning ability is one of the most effective tools to analyze big data. In recent years, although neural networks have made rapid development in both theoretical research and practical applications. However, we also see that there are some problems in the face of the growing, complex, and uncertain big data [2]: first, as the amount of data increases, neural networks need to consume a large amount of computational resources, which leads to an increase in computation time; second, neural networks cannot effectively deal with the uncertainty problem that big data has, and the accuracy of computation will decrease.

Quantum machine learning is an approach that combines quantum computers with machine learning to achieve more efficient computation in many machine learning tasks. Recent research results include the use of quantum machine learning for tasks such as image classification and target recognition. Quantum neural networks are one of the very important branches. In quantum neural network research, quantum variational circuits are a very important part of quantum neural networks. Francesco [3] et al. proposed quantum variational circuits based on variational protocols, which have better adaptability in quantum neural networks. arthur [4] et al. proposed a hybrid quantum classical neural network structure in which each neuron is a variational quantum circuit, compared to a single variational quantum circuit for significantly better accuracy in classification. There are also classical learning methods combined in quantum neural networks to improve efficiency and performance. For example, Kawase [4] et al. proposed to use the parameter t-

SNE of quantum neural network to reflect the properties of high-dimensional quantum data on low-dimensional data to improve the efficiency of neural network in processing data. osakabe [5] et al. proposed a Hebb rule-based learning method for quantum neural network, in which Hebb and inverse Hebb rules improve the learning performance of neural network.

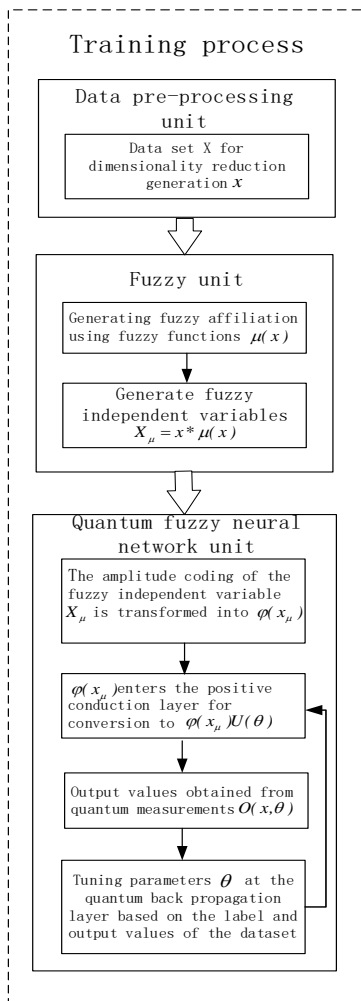
Fuzzy theory [6-8] is a mathematical theory for dealing with uncertainty and vagueness, which was proposed by the American mathematician Lotfi Zadeh. Fuzzy theory can be used to describe concepts such as fuzzy, fuzzy boundaries and fuzzy rules, and can be applied in modeling and analysis of many practical problems. Meanwhile, with the development of technologies such as neural networks, fuzzy theory has been widely used. For example, in neural networks, researchers can use fuzzy theory to fuzzify the output of neural networks to improve the performance and robustness of neural networks. Gu [9] et al. We extracted knowledge from deep neural networks (DNNs) into a Takagi-Sugeno-Kang (TSK)-based fuzzy inference system, which is able to express fuzzy rules based on knowledge obtained from DNNs, which makes it easier to explain specific decisions and has better generalization capabilities.

The above-mentioned quantum neural networks [10-12] are the product of the fusion of quantum computing and artificial neural networks, aiming to improve the efficiency of neural networks in processing big data by introducing quantum computing into traditional neural networks. There are many quantum neural network models currently available, and they are still under continuous development and improvement. However, quantum neural networks do not deal well with the problem of uncertainty in big data, resulting in low accuracy. The fuzzy neural network formed by introducing fuzzy theory into neural network learning has a factual description of the uncertainty of big data, and the above research results have higher accuracy compared with traditional neural networks, but due to the bottleneck of classical computer's computing power, fuzzy neural networks are inevitably limited in the face of big data.

How to combine fuzzy theory and quantum neural network, and take advantage of the properties of quantum parallel computing and exponential storage capacity and the natural advantages of fuzzy theory in dealing with the uncertainty of big data is one of the current researches focuses of quantum neural network.

## 2. The framework of quantum fuzzy neural network model based on fuzzy number

The quantum fuzzy neural network model based on fuzzy numbers is an improved model combining fuzzy theory and quantum neural network, which combines fuzzy theory to better describe uncertainty in real data and the advantages of quantum computing with high parallelism and exponential storage capacity. The quantum fuzzy neural network model based on fuzzy numbers is mainly divided into data preprocessing unit, fuzzy unit and quantum fuzzy neural network unit. The model needs to be trained before the output model can enter the use process, which can be divided into training process and use process according to different processes. The quantum fuzzy neural network model based on fuzzy number in the training process is shown in Figure 1.



**Figure 1.** Framework of quantum fuzzy neural network model based on fuzzy number in the training process

In the training process, the data pre-processing unit extracts features from the huge data, retains the valuable parameters, reduces the model computation and improves the computing efficiency. In the fuzzy unit, the Gaussian fuzzy number is

used to calculate the fuzzy affiliation of the data and combine with the data to generate fuzzy independent variables. In the quantum fuzzy neural network unit, it is further divided into quantum amplitude encoding layer, quantum forward propagation layer, quantum measurement layer and quantum back propagation layer. The quantum amplitude encoding layer encodes the quantum states of the input fuzzy independent variables and transfers the information to the quantum states; the quantum forward propagation layer puts the quantum states into linear and nonlinear you matrix changes simulating the propagation process of traditional neural networks; the quantum measurement layer measures the quantum states and uses them as output values; in the quantum back propagation layer the gradient is calculated based on the labeled and output values to make the parameters of the forward propagation layer adjustment. In use, there is no quantum backpropagation layer.

## 3. Quantum fuzzy neural network model based on fuzzy number

### 3.1. Data pre-processing unit

Due to the increasing size of the input data and the limitations of the quantum neural network circuits, preprocessing of the data is a very critical step. Preprocessing of the data cleans up errors, missing and outliers in the data, which can improve the accuracy and stability of the model. It also reduces the dimensionality of the dataset by retaining key information and reducing redundant information, which can improve the speed and efficiency of the model.

Similarity coefficient is a statistic used to measure the similarity between two variables. It is a form of Pearson's correlation coefficient and is usually used to calculate the linear correlation between two numerical variables. It takes values ranging from -1 to 1, where 1 indicates a perfectly positive correlation, -1 indicates a perfectly negative correlation, and 0 indicates no linear correlation. The similarity coefficient is calculated in the same way as the Pearson correlation coefficient, by calculating the covariance and standard deviation between two variables. The specific calculation formula is as follows:

$$Corr(X, Y) = Cov(X, Y) / (std(X) * std(Y)) \quad (1)$$

where  $Cov(X, Y)$  is the covariance of  $X$  and  $Y$ , and  $std(X)$  and  $std(Y)$  are the standard deviations of  $X$  and  $Y$ , respectively. The closer the value of the similarity coefficient is to 1 or -1, the stronger the linear correlation between the two variables; while the closer the similarity coefficient is to 0, the weaker the linear correlation between the two variables. It is a good choice to use in data preprocessing to select useful data based on the similarity coefficients of the input data parameters. Data normalization is the scaling of data to the same range to ensure that the model does not fail due to too large or too small a range of values for some features. This is especially true in feature selection. In feature selection, a similarity coefficient can be used to determine if there is a linear relationship between two variables in order to determine if they both need to be included in the model.

Feature selection is performed using the dataset to generate a heat map about the data feature values, and one or more features are selected from those with high similarity coefficients. The number of these features is also combined with quantum lines to make a comprehensive decision.

### 3.2. Fuzzy unit

In fuzzy theory, a fuzzy number is usually described by an affiliation function that takes values in the range  $[0,1]$ . The affiliation function describes the relationship between an element and a fuzzy concept, and its value indicates the degree to which the element belongs to the fuzzy concept. The common fuzzy numbers are triangular fuzzy number, trapezoidal fuzzy number, square fuzzy number, Gaussian fuzzy number and exponential fuzzy number. This model uses Gaussian fuzzy number for fuzzing, which is used to measure the correlation between the input values and the members of a particular set.

The mathematical expression of Gaussian affiliation function is

$$\mu(x, m, \sigma) = e^{-\frac{(x-m)^2}{2\sigma^2}} \quad (2)$$

where  $m$  represents the membership value of the set and  $\sigma$  represents the standard deviation, i.e., the value of the affiliation function decreases sharply when the deviation of the variable  $x$  with respect to  $m$  exceeds  $\sigma$ . The Gaussian affiliation function can be used to describe different kinds of input values and can even be used for fuzzy sets in multivariate systems. It is important in expressing fuzzy sets and defining fuzzy rules because it can perfectly express and measure the interconnection between fuzzy variables.

From the data preprocessing dataset  $(x, y)$  containing  $p$  data points and  $n$  features, the autocovariance matrix  $X$  is generated as shown in Eq. (3).

$$X = \begin{bmatrix} x_0^1 & x_1^1 & \dots & x_{n-1}^1 \\ x_0^2 & x_1^2 & \dots & x_{n-1}^2 \\ \dots & \dots & \dots & \dots \\ x_0^p & x_1^p & \dots & x_{n-1}^p \end{bmatrix} \quad (3)$$

The mean and standard deviation of each feature are calculated, and the Gaussian fuzzy degree matrix  $\mu_x$  is generated by using the Gaussian fuzzy number function to calculate the Gaussian affiliation, as shown in Eq. (4).

$$\mu_x = \begin{bmatrix} \mu_{x_0^1} & \mu_{x_1^1} & \dots & \mu_{x_{n-1}^1} \\ \mu_{x_0^2} & \mu_{x_1^2} & \dots & \mu_{x_{n-1}^2} \\ \dots & \dots & \dots & \dots \\ \mu_{x_0^p} & \mu_{x_1^p} & \dots & \mu_{x_{n-1}^p} \end{bmatrix} \quad (4)$$

The matrix  $X_\mu$  of fuzzy independent variables is generated by multiplying the variable matrix  $X$  with the corresponding elements of the corresponding Gaussian fuzziness matrix  $\mu_x$  as shown in Eq. (5).

$$X_\mu = X * \mu_x = \begin{bmatrix} \mu_{x_0^1} x_0^1 & \mu_{x_1^1} x_1^1 & \dots & \mu_{x_{n-1}^1} x_{n-1}^1 \\ \mu_{x_0^2} x_0^2 & \mu_{x_1^2} x_1^2 & \dots & \mu_{x_{n-1}^2} x_{n-1}^2 \\ \dots & \dots & \dots & \dots \\ \mu_{x_0^p} x_0^p & \mu_{x_1^p} x_1^p & \dots & \mu_{x_{n-1}^p} x_{n-1}^p \end{bmatrix} \quad (5)$$

### 3.3. Quantum fuzzy neural network unit

#### 3.3.1. Quantum forward propagation layer

Each row of the fuzzy autocovariance matrix represents one data point information, as shown in the figure will be quantum amplitude coding of each row of the fuzzy autocovariance matrix data in turn into the quantum forward propagation layer, after the quantum forward propagation layer amplitude coding, linear and nonlinear changes, the output value is obtained, and its quantum circuit diagram is shown in Figure 2.

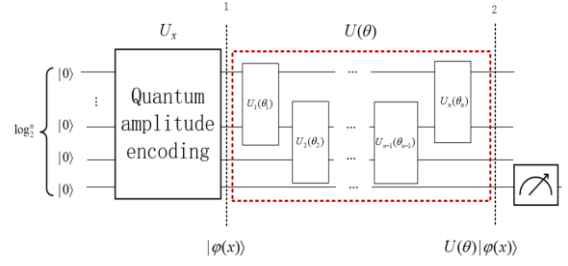


Figure 2. Quantum forward propagation layer diagram

Figure 2 mainly consists of two parts  $U_x$  and  $U(\theta)$ : after the first part of the quantum amplitude encoding quantum state as  $|\varphi_x\rangle$ , where the data of the  $i$ -th row of the fuzzy autocovariance matrix into the quantum forward propagation matrix as in Eq.(6) for the corresponding quantum superposition state to transfer the data information to the quantum amplitude.

$$|\varphi_x\rangle = U_x |0\rangle^{\otimes \log_2^n} = \sum_{j=0}^{n-1} \mu_{x_j} x_j^i |j\rangle \quad (6)$$

The second part is the quantum forward propagation part, where  $U(\theta)$  consists of  $N U_i(\theta_i)$  and the expression is shown in Eq. (7).

$$U(\theta) = \prod_{i=1}^N U_i(\theta_i) \quad (7)$$

The quantum line diagram of  $U_i(\theta_i)$  in Eq. (7) consists of the You matrix  $G$  gate (as shown in Eq. (8)) and the control non-gate, which realize the linear and nonlinear transformations of the quantum state, respectively, and each  $U_i(\theta_i)$  is located in a quantum line position different from that of  $U_{i-1}(\theta_{i-1})$  where  $\theta_i$  is a parameter that can be tuned by the quantum neural network and is also tuned later in the quantum backpropagation.

$$\begin{bmatrix} \cos(\theta_i) & \sin(\theta_i) \\ -\sin(\theta_i) & \cos(\theta_i) \end{bmatrix} \quad (8)$$

The  $|\varphi(x)\rangle$  quantum state is  $U(\theta)|\varphi(x)\rangle$  after passing through the quantum forward propagation layer state. The probability of measuring at the last quantum bit to be state  $|1\rangle$  is added with a classical bias  $b$  term as the output value  $O(x, \theta)$ , whose expression is shown in Eq. (9).

$$O(x, \theta) = p(R=|1\rangle) = \frac{1}{2}(1 + \langle \varphi(x) | U(\theta)^\dagger Z_n U(\theta) | \varphi(x) \rangle) + b \quad (9)$$

### 3.4. Quantum back propagation layer

The quantum back propagation layer implements the parameter learning update in the quantum neural network, and the parameter to be adjusted is  $\theta_1, \theta_2, \dots, \theta_n$  in  $U(\theta)$  in the quantum neural network layer. This model uses the mean square error as the loss function, as shown in Eq. (10).

$$E(x, \theta, y(x)) = \frac{1}{2}(O(x, \theta) - y(x))^2 \quad (10)$$

$y(x)$  in Eq. (10) is the label of the training sample data, and the optimal parameters need to be found during the training process to make the loss function smaller. This model is optimized using the gradient descent algorithm, and the formula for calculating the gradient  $\theta_i$  is as follows:

$$\frac{\partial_\theta E(x, \theta, y(x))}{\partial \theta_i} = \frac{1}{2} (O(x, \theta) - y(x)) \left( \langle \varphi(x) | \frac{U(\theta)^\dagger}{\partial \theta_i} Z_n U(\theta) | \varphi(x) \rangle + \langle \varphi(x) | U(\theta)^\dagger Z_n \frac{U(\theta)}{\partial \theta_i} | \varphi(x) \rangle \right) \quad (11)$$

The value of Eq. (12) is calculated by quantum calculation. Setting Eq. (3-12) to  $D(\theta_i, x)$ , we obtain the gradient of  $\theta_i$  as  $\frac{1}{2} (O(x, \theta) - y(x)) D(\theta_i, x)$ .

$$\langle \varphi(x) | \frac{U(\theta)^\dagger}{\partial \theta_i} Z_n U(\theta) | \varphi(x) \rangle + \langle \varphi(x) | U(\theta)^\dagger Z_n \frac{U(\theta)}{\partial \theta_i} | \varphi(x) \rangle \quad (12)$$

Update the parameter  $\theta_i$ , calculated as follows:

$$\theta_{new} = \theta_{old} + \frac{1}{2} \alpha (\pi(x, \theta) - y(x)) D(\theta_i, x) \quad (13)$$

Next, the quantum gradient calculation algorithm is used to find  $D(\theta_i, x)$ .

$U_i(\theta_i)$  is composed of a Your matrix G gate and a control non-gate. The control non-gate has no parameters and can be viewed as a constant, so the derivative of  $U_i(\theta_i)$  is equivalent to the derivative of the G gate, and the derivative of the G gate is shown in Eq. (14):

$$\frac{G(\theta_i)}{\partial \theta_i} = \begin{bmatrix} -\sin(\theta_i) & \cos(\theta_i) \\ -\cos(\theta_i) & -\sin(\theta_i) \end{bmatrix} = G(\theta + \frac{\pi}{2}) \quad (14)$$

The partial derivative of  $\theta_i$  for  $U(\theta)$  is given by

$$\frac{U(\theta)}{\partial \theta_i} = U_n(\theta_n) U_{n-1}(\theta_{n-1}) \dots U(\theta_i + \frac{\pi}{2}) \dots U_1(\theta_1) \quad (15)$$

### 3.5. Quantum gradient calculation algorithm

The line of the quantum gradient calculation algorithm is shown in Figure 3, where the first initial value is the auxiliary quantum bit  $|0\rangle_c$  and the remaining quantum bits have the initial value  $|0\rangle$ .

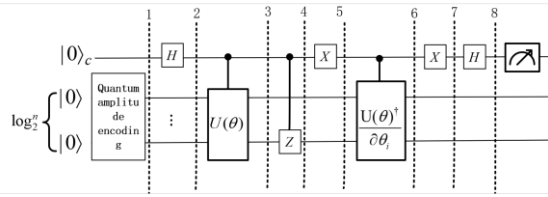


Figure 3. Quantum gradient calculation algorithm diagram

After a series of changes, the auxiliary quantum bits are measured and the probability result of measuring as  $|1\rangle$  state is

$$p(R=|1\rangle) = \frac{1}{2} - \frac{1}{4} \left( \langle \varphi(x) | \frac{U(\theta)^\dagger}{\partial \theta_i} Z_n U(\theta) | \varphi(x) \rangle + \langle \varphi(x) | U(\theta)^\dagger Z_n \frac{U(\theta)}{\partial \theta_i} | \varphi(x) \rangle \right) \quad (16)$$

## 4. Simulation experiments and analysis

This subsection is based on IBM quantum platform for simulation experiments and analysis. Firstly, the cancer dataset and evaluation metrics are introduced, then, finally, the fuzzy quantum sub neural network model experiments and analysis are performed.

### 4.1. Breast cancer dataset

Breast cancer is one of the most common types of cancer

and is an abnormal growth of malignant cells in the breast tissue. Breast cancer can occur in both women and men, but women are much more likely to develop it. The Breast cancer dataset is a commonly used medical dataset for the diagnosis and prediction of breast cancer. The dataset contains digitized images sampled from breast tissue and its associated clinical data. Originally collected by the University of Wisconsin Medical Physics Laboratory, the dataset has become one of the standard datasets in the UCI Machine Learning Warehouse.

The dataset contains 569 samples, each with 30 features, including characteristics such as size, shape, and texture of breast tissue nuclei, as well as clinical information such as the patient's age, gender, and tumor size. Each sample was labeled as benign or malignant, with 357 benign samples and 212 malignant samples.

The breast cancer dataset is a classical dataset for classification and regression problems and is commonly used for machine learning and deep learning exercises and studies. Some of the data are shown in Table 1.

Table 1. Breast cancer data set

| Diagno<br>si | radius_mea<br>n | texture_mea<br>n | ... | concavity_me<br>an |
|--------------|-----------------|------------------|-----|--------------------|
| M            | 17.99           | 10.38            | ... | 0.4601             |
| M            | 20.57           | 17.77            | ... | 0.275              |
| M            | 19.69           | 21.25            | ... | 0.3613             |
| M            | 11.42           | 20.38            | ... | 0.6638             |
| M            | 20.29           | 14.34            | ... | 0.2364             |
| M            | 12.45           | 15.7             | ... | 0.3985             |
| M            | 18.25           | 19.98            | ... | 0.3063             |
| ...          | ...             | ...              | ... | ...                |
| M            | 19.81           | 22.15            | ... | 0.2768             |
| B            | 13.54           | 14.36            | ... | 0.2977             |
| B            | 13.08           | 15.71            | ... | 0.3184             |
| B            | 9.504           | 12.44            | ... | 0.245              |
| M            | 15.34           | 14.26            | ... | 0.4667             |
| M            | 21.16           | 23.04            | ... | 0.2822             |

### 4.2. Evaluation Indicators

Evaluation metrics are a quantitative indicator of model performance. One evaluation metric can only reflect part of the model performance, so different evaluation metrics should be selected for specific data and models. In this paper, we use the most commonly used evaluation metrics in classification problems to demonstrate the classification accuracy performance of the proposed algorithm model. Table 2 shows the evaluation metrics, and the higher the value of the metric, the better the classification accuracy of the model. For the evaluation of the classification efficiency performance of the algorithm model, the loss function value of the training and the accuracy of the test prediction are used.

Table 2. Evaluation indicators

| Metric    | Definition  |
|-----------|---|
| Recall    | $\frac{TP}{TP + FN}$  |
| Precision | $\frac{TP}{TP + FP}$  |
| F1-Score  | $2 \times \frac{Recall \times Precision}{Recall + Precision}$ |
| Accuracy  | $\frac{TP + TN}{TP + FP + TN + FN}$                           |

### 4.3. Experiment and analysis of quantum neural network model based on fuzzy number

#### 4.3.1. Data pre-processing

Use correlation coefficient corr to calculate the correlation coefficient of each feature and other features, and then make feature selection according to the correlation coefficient. For display convenience, the correlation coefficient of features is displayed using a heat map, and the diagonal of the heat map is the correlation coefficient of the univariate itself is 1. The lighter the color represents the greater the correlation. As shown in Figure 4.

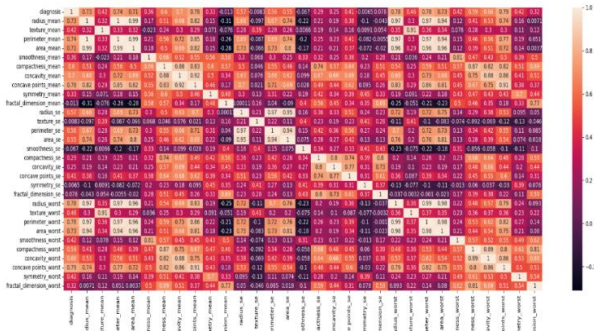


Figure 4. Heat map of Breast cancer dataset

The purpose of feature selection is to reduce the dimensionality and use a small number of features to represent the characteristics of the data, which can also enhance the generalization ability of the classifier and avoid overfitting the data. The quantum neural net can accept four parameters, according to which the mean is selected from mean, se, and worst, and then the radius mean, perimeter mean, compactness mean, and concavity mean are selected from the mean.

#### 4.3.2. Fuzzy processing

In this subsection, the mean and standard deviation of radius mean, perimeter mean, compactness mean and concavity mean are calculated respectively to support the calculation of Gaussian fuzzy affiliation. The results are shown in Tables 3.

The Gaussian fuzzy affiliation function is used to calculate each fuzzy affiliation degree, as shown in Table 4.

The values of the variable matrix and the fuzzy affiliation matrix are multiplied correspondingly to generate the fuzzy independent variable matrix.

Finally, the fuzzy independent variables are normalized to facilitate the next quantum neural network operation. The fuzzy autocovariance matrix was first randomly divided into 75% training set and 25% test set before the experiment.

Table 3. Evaluation values and standard deviations of the data set

|                           | radius mean | texture mean | compactness mean | concavity mean |
|---------------------------|-------------|--------------|------------------|----------------|
| <b>Evaluation value</b>   | 14.1272917  | 19.28964851  | 0.09636028       | 0.10434098     |
| <b>standarddeviations</b> | 3.52095076  | 4.29725464   | 0.01405176       | 0.05276633     |

Table 4. Breast cancer fuzzy affiliation degree

|                    | radius mean | texture mean | compactness mean | concavity mean |
|--------------------|-------------|--------------|------------------|----------------|
| <b>0.300127054</b> | 0.013585964 | 0.08542755   | 2.078111231      |                |
| <b>0.035146036</b> | 0.88244759  | 0.504661744  | 0.788802888      |                |
| <b>0.082410168</b> | 0.812120513 | 0.411576441  | 0.33000344       |                |
| <b>0.553650592</b> | 0.93764864  | 2.07758E-05  | 9.3532E-06       |                |
| <b>0.046722086</b> | 0.265356495 | 0.924401884  | 0.747599312      |                |
| <b>0.79697432</b>  | 0.497686425 | 0.006697273  | 0.21259405       |                |
| <b>0.25384669</b>  | 0.974521948 | 0.984951842  | 0.992234261      |                |
| ...                | ...         | ...          | ...              |                |

#### 4.3.3. Quantum fuzzy neural network experiment and analysis

In this paper, a Gaussian fuzzy quantum neural network is designed based on IBM quantum platform for validation, and the fuzzy independent variable matrices are entered into the quantum neural network sequentially, and the quantum fuzzy neural network line is shown in Figure 5.

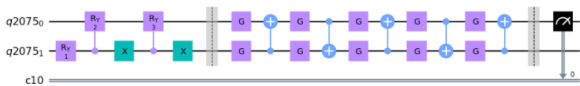


Figure 5. Quantum fuzzy neural network line diagram

Figure 5 includes the quantum line of two quantum bits and the classical data line of one. The first part is the quantum amplitude part, which realizes the assignment of the fuzzy autocovariance matrix to the quantum state; the second part is the quantum forward propagation part, which has ten G gates and five controlled non-gates to realize the linear and non-linear variation of the data; and finally, the measurement part, which performs the quantum measurement and saves the output value in the classical data line.

The output values are obtained and the loss function is calculated with the labeled values of the fuzzy autocovariance matrix. Using the quantum gradient calculation algorithm to quickly calculate the value of Eq. and bring it into the loss function, the gradient can be calculated quickly, thus improving the efficiency of the quantum neural network. The quantum gradient calculation algorithm is shown in Figure 6.

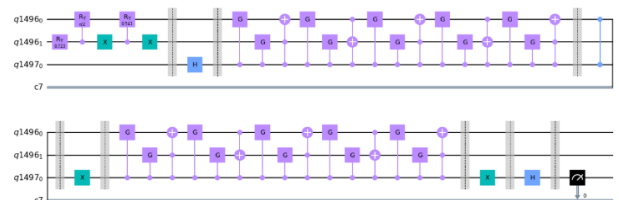


Figure 6. Line diagram of quantum gradient calculation algorithm

Figure 6 includes three quantum lines of quantum bits and one classical data line. The quantum line diagram is designed strictly according to Figure 3, and the partitioned lines are given for easy viewing, and the final measurements are made and the output values are stored in the classical data line.

After the training set is trained, the test set is tested in the

Gaussian fuzzy quantum neural network. We can get is the value of the loss function for each training in training and the accuracy of the predicted values in testing.

To illustrate the accuracy and efficiency of the quantum fuzzy neural network model, we will use the same Breast cancer dataset in the traditional quantum neural network model for training and testing, and also obtained the loss function values for each training and the accuracy of the predicted values in the test.

To visualize the classification efficiency of the quantum neural network QNN and the quantum fuzzy neural network FQNN model based on fuzzy number, the loss function values of the two models at different number of iterations are shown in Figure 7.

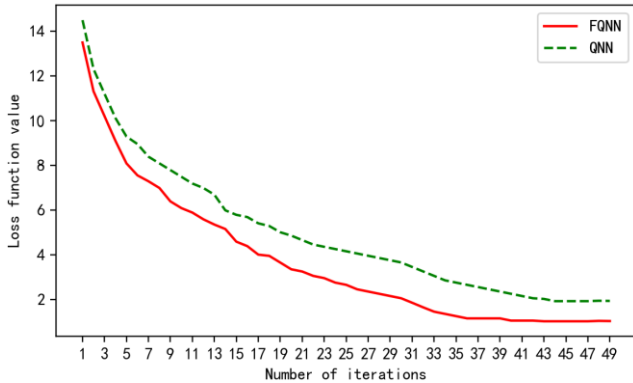


Figure 7. Plot of loss function values for QNN and FQNN with different number of iterations

Figure 7 shows the values of the loss functions of the QNN and FQNN models for different number of iterations in the Breast cancer dataset, where QNN and FQNN are shown by the green dashed line and the red solid line, respectively. By comparing QNN and FQNN, we can get that FQNN has a smaller value of the initial loss function compared with QNN, and the gradient decreases faster when the number of iterations increases, and the value of the loss function of QNN is around 1.9 after 50 iterations, while the value of the loss function of QFNN is around 1.0 after 50 iterations, and the final loss function of QFNN is also smaller. This indicates that QFNN has faster gradient descent speed and smaller loss function compared to QNN. It shows that QFNN has better classification efficiency in the same data set.

The accuracy of QNN and FQNN models at different number of iterations is shown in Figure 8.

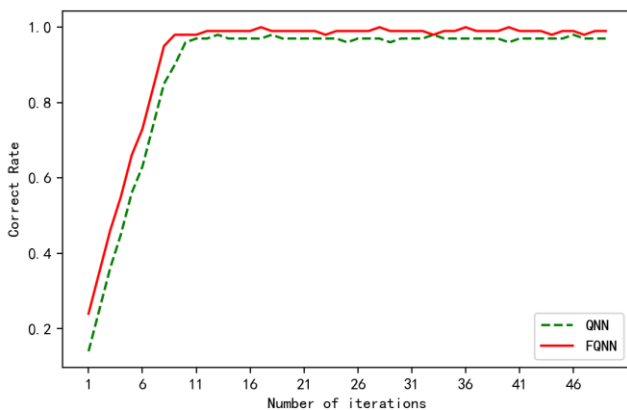


Figure 8. Shows the correctness of QNN and FQNN models for the test dataset with different number of iterations in the Breast cancer dataset, where QNN and FQNN are shown by green dashed line and red solid line respectively.

By comparing Figure 8, we can see that the QNN model stabilizes to 97% correct rate after about 11 iterations, while the FQNN only needs about 8 iterations to stabilize to 99% correct rate, and the initial correct rate of the FQNN model is also higher than that of the QNN model. It can be judged that FQNN requires fewer iterations to reach the final accuracy compared to QNN. For the same class of experiments, the QFNN has a higher correct rate than the QNN for the same number of iterations before reaching the final correct rate, indicating that fewer iterations are needed to achieve a higher correct rate and that the FQNN has a better classification efficiency in the same data set.

This is due to the fact that FQNN has a better description of the uncertainty in the real data set after the Gaussian fuzzy number operation, and the fuzzy operation of Gaussian fuzzy number on the data set also improves the model to deal with data uncertainty, which is more beneficial to improve the accuracy of the predicted values.

## 5. Conclusion

In this paper, we propose a quantum fuzzy neural network model (FQNN) based on fuzzy numbers, which can not only take advantage of the computational acceleration of quantum computing, but also introduce the concept of Gaussian fuzzy numbers into the learning of quantum neural networks, and fuse fuzzy numbers and quantum neural networks. In dealing with data uncertainty and fuzziness, Gaussian fuzzy numbers are used to calculate the affiliation degree on the input data set to describe the information of data uncertainty and fuzziness. FQNN model can effectively utilize the advantages of fuzzy numbers to deal with uncertainty and ambiguity problems and the advantages of parallel computing of neural networks to make up for the shortcomings of each. The simulation experiments show that the quantum fuzzy neural network model based on fuzzy numbers has higher efficiency and accuracy compared with the existing quantum neural network model (QNN).

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