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A Survey of Formal Concept Analysis and Concept Lattices

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Abstract: Formal concept is the basic unit of knowledge representation. Concept lattice comes from formal concept analysis theory, which is the ordered hierarchy of all formal concepts. Concept lattice structure model is a powerful tool for data analysis. In recent years, due to the visualization of concept lattices and the clarity of knowledge discovery, FCA has developed into an effective tool for data analysis, and has been widely used in various fields such as data mining, information retrieval, software engineering, etc., and it has achieved good economic and social benefits. Therefore, it is necessary to study the theory of formal concept analysis. At the same time, in order to meet the different needs of life, different kinds of formal concept analysis also emerged. This paper mainly analyzes the current research status of several mainstream concept analysis theories, and puts forward the prospect of them.

Keywords: Formal Concept; Concept Lattice; Data Analysis.

1. Introduction

Concept lattice is a powerful tool for data analysis. It comes from formal concept analysis theory and describes the relationship between objects and attributes. A concept lattice is a poset with a concept as an element that can be visualized through a Hasse graph, where each node is a concept. Except classical concept lattices, the relatively mature concept lattices mainly include three-way concept lattices, fuzzy concept lattices, approximate concept lattices, rough concept lattices, etc., and the study of three-valued concept lattices is also in hot progress. So far, the research and application of concept lattices have been extended to other research fields, such as the combination of fuzzy sets, rough sets, and three-way decision ideas, and applied to different scenarios

2. Main Research Content and Progress

2.1. Classical Formal Concept Analysis and Concept Lattice

Formal concept analysis was proposed by German mathematician Wille in 1982 [1]. It is a powerful data analysis technique in modern life. Formal concept is the basic unit of knowledge representation. Formal concept is produced on the basis of formal context. It is a triplet, which includes object set, attribute set and the binary relation between them. In philosophy, concept is composed of denotion and connotation [2]. In a formal context, binary pairs of object sets and attribute sets and their corresponding derived operators formalize concepts in philosophy from a mathematical perspective [3]. A concept represents the semantics of "co-ownership" between a subset of objects and a subset of attributes. In a formal context, different maximum states can occur between multiple objects and multiple properties, that is, there is usually more than one formal concept in a formal context.

Concept lattice is an ordered hierarchical structure composed of all formal concepts [4], which is built according to the binary relationship between objects and attributes in the formal context. Such relationship can be superimposed, that is, the relationship between multiple objects and multiple attributes can also be discussed. Once multiple objects and multiple attributes reach a maximum state, formal concepts will be formed. The generalization relation and specialization relation between concepts are clearly expressed [5]. The mathematical basis of concept lattice is lattice theory, and the visualization tool is Hasse graph. Due to the visualization of concept lattices and the clarity of knowledge expression, FCA has developed into an effective tool for data analysis, and has been widely used in various fields such as data mining, information retrieval and software engineering, and has achieved effective resource utilization.

2.2. Three-way Formal Concept Analysis and Concept Lattice

The traditional theory of formal concept analysis studies the properties (objects) that objects (attributes) have in common, or the properties (objects) that objects (attributes) do not have in common. Reflected in the practical application, is either "accept" or "reject". However, in many cases there is a third possibility, which is neither explicitly accepted nor explicitly rejected. Therefore, Yao Yiyu [6] proposed a three-way decision theory. The concept of three-way decision is a generalization of two-way decision. In addition to acceptance and rejection, a third choice is added, namely delayed decision method. There is a great deal of theoretical, model and applied research in various fields such as scientific research and engineering. Yao [7] discussed the realization process of the three-way decision, and expounded the superiority of this theory from different perspectives. In 2014, Qi et al. [8] applied the three-way decision theory to the formal concept analysis theory and proposed the three-way concept analysis theory, which not only has the basic form and specific research content of the formal concept analysis theory, but also reflects the idea of "three division and rule" of the three-way decision theory.

Qi et al. [9] proposed the negative operator in combination with the three-way decision theory, defined the meaning of "common does not have", and extended the formal concept

analysis to the three-way concept analysis (3WCA). Its core idea is the consistency of information, that is to say, the object of study can be divided into three disjoint parts by means of unanimous ownership and unanimous non-ownership, in which the opposite parts of unanimous ownership and unanimous non-ownership are regarded as boundary fields. In addition, the three-way concept lattices and classical concept lattices can be transformed into each other, which means that the three-way concept lattices can promote the further study of classical concept lattices. In the analysis of three-way concept lattices, the classical operators proposed by Wille are called corresponding positive operators. There are two kinds of three-way concept lattices in 3WCA, namely object-induced three-way concept and attribute-induced three-way concept. From the point of view of object, the concept of tripartite attribute set can be defined as OE concept derived from object and three-way concept lattice derived from object. Dually, from the point of view of attributes, the object set can be defined from the point of view of tripartite, attribute-induced tripartite concept (AE concept) and attribute-derived tripartite concept lattice. The three concepts are essentially the same as the formal concepts, which are composed of denotation and connotation. The tripartite concept and the tripartite concept case summarize the formal concept and the conceptual case. The extension (or connotation) of the tripartite concept contains two parts: the positive part and the negative part, which express the meaning of "common ownership" and "common non-ownership" respectively in the formal concept.

Qi et al. [9] put forward the basic theory of three-way concept analysis, including three-way concept, three-way concept lattice and formal concept, the relationship between concept lattice, and the construction method of three-way concept lattice. Nowadays, there are many interesting results in the research on the three-strand concept analysis. Qi et al. [10] proposed the three-way concept analysis, and analyzed the relationship between the object derived three-way concept lattices and the attribute derived three-way concept lattices and the classical concept lattices from the perspectives of elements, sets and orders, respectively, and gave the sufficient and necessary conditions and corresponding algorithms for constructing the three-way concept on the basis of the classical concept. Qian et al. [11] first made isomorphic transformation of formal context, and then constructed three conceptual lattices by means of juxtaposition or superposition, and studied the construction of classical concept lattices, three concept lattices, and three object-oriented (attribute) concept lattices, and discussed the relationship between them.

2.3. Fuzzy Formal Concept Analysis and Concept Lattice

Fuzziness refers to the uncertainty contained in the phenomenon that an object cannot be simply judged by "belonging" and "not belonging", and the concept with fuzziness becomes fuzzy concept. Fuzzy concept has no clear boundary, so membership degree is needed to describe the degree of an object's belonging to this concept. The membership function of fuzzy set generalizes the value of characteristic function of classical set from {0,1} to [0,1]. Fuzzy Formal Concept Analysis by Burusco et al. [12] combined the classical formal concept analysis with the fuzzy set theory to put forward the fuzzy formal background and fuzzy concept lattice for the uncertainty relation of concept connotation and extension. The extension from classical concept to fuzzy concept is not the only way, and scholars have studied it from a variety of angles.

Burusco et al. firstly applied fuzzy theory to concept lattices to form fuzzy concept lattices [12]. Duntsch and Gediga introduced the upper and lower approximation of rough sets into formal concept analysis, and proposed attribute-oriented concept lattices [13]. Similarly, Yao [14] gives the definition of object-oriented concept lattices. In order to improve the speed of concept lattice generation algorithm, Belohlavek et al. generated L-fuzzy concept [15] with the help of NextClosure algorithm and L-fuzzy concept lattice [16] with Linding analogy. Singh [17] studied the description of three fuzzy concept lattices from the perspective of middle wisdom set and discussed many of their properties. He [18] et al. defined L-fuzzy tripartite operators and their inverse operators under the background of fuzzy form based on the analysis idea of three-way formal concepts, gave L-fuzzy tripartite concepts, and constructed corresponding concept lattices.

2.4. Three-valued Formal Concept Analysis and Concept Lattice

Although the methods and purposes of formal concept analysis (three-way concept analysis), shadow set theory, and conflict analysis are different, the basic data sheets are similar. The similarity of this kinds of data description inspires us to study the three-valued context, which can be used as the unity of the three basic data. Among these three data descriptions, what they have in common is that the existence of a third attitude complicates meaning. That is, the two extreme attitudes, + and -(or 1 and 0), cannot describe another opposite meaning. In order to better describe the common information, this kind of similar data table is converted into three-valued formal context. Qi et al. [19] put forward the three-valued concept analysis (3VCA). They define and study three-valued formal contexts that can be converted by imperfect formal contexts and conflict situations as well as by Valued-set models. They define a three-valued operator by using the three-valued concept analysis. After the properties of operators are given, the three-valued concept and the three-valued concept lattices are established. Besides, the relationship between the three-valued concept lattices and the existing approximate concept lattices is discussed. In fact, three-valued formal contexts often appear in the real world. Therefore, it is necessary to discuss the concept of three-valued formal context.

3. Conclusion

Formal concept analysis can intuitively display effective information in formal context through concept lattice structure. Due to its effectiveness, more and more studies have introduced the thinking of formal concept analysis. At the same time, scholars have developed many more simple and effective concept analysis methods and concept lattice models, such as three-way concept analysis and three-way concept lattice, fuzzy concept analysis and fuzzy concept lattice, three-valued concept analysis and three-valued concept lattice, etc. However, the different types of concept analysis and the concept lattice model are still worth further study. For example, when the amount of data is very large, the concept lattice model is still time-consuming, so it is very necessary to study how to establish a more efficient concept lattice generation algorithm. In addition, in practical

applications, the amount of missing data is often very large, so how to deal with these data more reasonably is also a problem worth further investigation. Moreover, the relationship between concept lattices in the three-valued formal context is also worth discussing because of its wide application.

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