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A Boolean Vector Method for Granule Calculation in Granular Computing

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Abstract: Granule calculation is one of the core issues in granular computing theory. This study proposes a method for the Boolean vector representation and calculation of granules. An isomorphism is defined on the universe of discourse, and a Boolean characteristic vector uniquely corresponding to a set is constructed, which transforms set operations including intersection, union, difference, and complement into vector addition, subtraction, and the Hadamard product. Based on this, the Boolean characteristic vector of a granule in granular computing is defined, and the computational methods for granule composition, decomposition, inheritance, similarity, and difference are reconstructed, and Boolean vector operations on granules provide a new approach to granular computation.

Keywords: Granular Computing; Isomorphic Mapping; Boolean Characteristic Vector; Hadamard Product.

1. Introduction

Granular Computing (GrC) is a novel computational paradigm in the field of intelligent information processing [1,2]. It is a method and means of solving complex problems from a multi-granularity, multi-perspective, and multi-level perspective using granules, aligning with the fundamental human approach to processing and solving complex problems, and has therefore attracted significant attention [3].

Granularization and granular computation are the two core issues of granular computing theory [4]. Granularization is the process of dividing the object under study into several granules according to a given granularization criterion to form a granular hierarchy. Given multiple granularization criteria, multiple granular layers are obtained, thereby forming a granular hierarchical structure. Granular computation takes granules as its basic units and characterizes the evolutionary processes between different granular layers by defining operations such as granularity, similarity, difference, intersection, decomposition, inheritance, combination, extension, and inference, thereby enabling free transformation between different granularities within the granular hierarchical structure. Common granular computation models aimed at multi-granularity computation—such as fuzzy sets, rough sets, quotient spaces, term computation, and cloud models—all have finite set-theoretic domains as their objects of study [5].Since granules are subsets of the universe of discourse, granular computation naturally satisfies the operational rules of sets. Algebraic systems are important mathematical tools for studying abstract operations on sets, as well as their properties and structures; algebraic granular computing models have become a key research direction in the development and application of granular computing theory.

Boolean algebra is a crucial component of algebraic systems. From the perspective of Boolean algebra, a set can be represented as a Boolean vector, and set operations can be implemented through logical operations on Boolean vectors; these Boolean logical operations can be directly realized within a computer via binary operations. Boolean granular computing models have become a major research direction in the development and application of granular computing theory. Liu [6,7] defined a binary representation of granules based on the correspondence between elements within a granule and their positions in the domain, and implemented the intersection operation of granules using the Boolean AND operation on binary numbers. Zheng et al [8] proposed a binary granular computing model, defining bitwise AND, OR, NOT, XOR, and other operations. All binary granules within a granular layer can be integrated into a Boolean granular matrix representing the structure of that layer. Chen et al[9–11] (2009, 2010, 2012, 2015) described the containment relationships between different granules through matrix multiplication. Zhong et al (2011) [12] and Wu et al(2017) [13] defined the AND operation of granular matrices by performing bitwise AND operations on all binary granules one by one, thereby characterizing the intersection of granules.

Based on the aforementioned research, this study first proves the isomorphic mapping between granules and Boolean characteristic vectors, then utilizes Boolean vector addition, subtraction, and the Hadamard product to reconstruct computational methods for granule composition, decomposition, inheritance, similarity, and difference in granular computing, thereby providing a new approach to granular computation.

2. Granules and Their Operations

A granule is a subset of a domain, and granule computation naturally satisfies the operational rules of sets. This section primarily introduces the concept of granules and their operational forms in information systems.

Definition 1

Formally, a quadruple $I=\left(U, At, V, \inf\right)$ is called an information system in granular computing. U is a non-empty set of objects, At is a finite set of attributes, $V=\bigcup_{a\in At}V_a$, where V_a is the value domain of attribute $a\in At$, $\inf: U\times At\rightarrow V$ is an information function, and $\inf\left(x,a\right)\in V_a$ for all $x\in U$.

A partition $U/R=\left\{X_1^R, X_2^R,\cdots,X_m^R\right\}\left(1\le m\le\left|U\right|\right)$ of

$R \subseteq At$ is called a granule layer based on the clustering criteria of V_R, where $X_i^R\left(1 \le i \le m\right)$ is called a granule in $I = \left(U, At, V, \inf\right)$.

$GD(R) = \sum_{i=1}^{m} \left|X_i^R\right|^2 / \left|U\right|^2$ is referred to as the granularity of U / R. $|*|$ denotes the number of elements contained in the granule.

A quadruple $I = \left(U, At, V, \inf\right)$ is abbreviated as $I = \left(U, At\right)$ in this paper.

A granule is a subset of objects and the fundamental unit of calculation; calculations involving granules naturally satisfy set operation rules.

$U / P = \left\{X_1^P, X_2^P, \cdots, X_m^P\right\}$ and $U / Q = \left\{X_1^Q, X_2^Q, \cdots, X_l^Q\right\}$ are defined in $I = \left(U, At\right)$. $X_i^P\left(1 \le i \le m\right)$ and $X_j^Q\left(1 \le j \le n\right)$ can perform $X_i^P \cap X_j^Q$ (intersection), $X_i^P \cup X_j^Q$ (union), $X_i^P \backslash X_j^Q$ (difference), and $C_U X_i^P$ (complement) operations.

These operations are promoted based on $X_i^P \cap X_j^Q$, $X_i^P \cup X_j^Q$, $X_i^P \backslash X_j^Q$, and $C_u X_i^P$ as follows:

a) Synthesis: $X_i^P \cup X_j^Q$ is synthesized from X_i^P and X_j^Q.

b) Decomposition: X_i^P is decomposed into $X_i^P \cap X_j^Q$ and $X_i^P \backslash \left(X_i^P \cap X_j^Q\right)$ by X_j^Q.

c) Inheritance: $X_i^P \cap X_j^Q$ and $X_i^P \backslash \left(X_i^P \cap X_j^Q\right)$ are sub-granules of X_i^P; X_i^P is the super-granule of $X_i^P \cap X_j^Q$ and $X_i^P \backslash \left(X_i^P \cap X_j^Q\right)$.

d) Jaccard Similarity: $J(X_i^P, X_j^Q) = \dfrac{\left|X_i^P \cap X_j^Q\right|}{\left|X_i^P \cup X_j^Q\right|}$ is the Jaccard similarity of X_i^P and X_j^Q.

e) Symmetric Difference: $X_i^P \Delta X_j^Q = \left(X_i^P \cup X_j^Q\right) \backslash \left(X_i^P \cap X_j^Q\right)$ is called the Symmetric Difference of X_i^P and X_j^Q.

3. Isomorphic Mapping

When an algebraic operation is defined on a set, the set and the operation together constitute an algebraic system. $\left\langle P\left(U\right), \cap, \cup, \backslash\right\rangle$ is an algebraic system on U, where intersection($\cap$), union($\cup$), and difference($\backslash$) are set operations, and $P\left(U\right)$ is the power set of U. From the perspective of Boolean algebra, a set can be represented as a Boolean vector, and the operations on sets can be realized through logical operations on Boolean vectors. These Boolean logical operations can be directly implemented by binary operations within computers.

To establish the correspondence between sets and Boolean vectors, this paper defines a mapping on the object set U.

Definition 2

$U = \left(x_1, x_2, \cdots, x_{|U|}\right)$ is a set of objects, $X \in P\left(U\right)$ is a subset of U. The mapping $\nu : X \to \boldsymbol{\nu}\left(\boldsymbol{X}\right)$ is defined on X, $\boldsymbol{\nu}\left(\boldsymbol{X}\right) = \left(\nu^X\left(x_1\right), \nu^X\left(x_2\right), \cdots, \nu^X\left(x_{|U|}\right)\right)$ is called the eigenvector of X, where for any $x \in U$, $\nu^X\left(x\right) = \begin{cases} 1, & x \in X \\ 0, & x \notin X \end{cases}$.

X and Y are represented by the Boolean eigenvectors $\boldsymbol{\nu}\left(\boldsymbol{X}\right)$ and $\boldsymbol{\nu}\left(\boldsymbol{Y}\right)$. $X \cap Y$, $X \cup Y$, and $X \backslash Y$ can be transformed into the Hadamard product (□), Plus (+), and Minus (−) of the Boolean eigenvectors $\boldsymbol{\nu}\left(\boldsymbol{X}\right)$ and $\boldsymbol{\nu}\left(\boldsymbol{Y}\right)$.

(1) $\boldsymbol{\nu}\left(\boldsymbol{X} \cap \boldsymbol{Y}\right) = \boldsymbol{\nu}\left(\boldsymbol{X}\right) \square\, \boldsymbol{\nu}\left(\boldsymbol{Y}\right)$, where $\boldsymbol{\nu}\left(\boldsymbol{X}\right) \square\, \boldsymbol{\nu}\left(\boldsymbol{Y}\right) = \left(\nu^X\left(x_1\right)\nu^Y\left(x_1\right), \nu^X\left(x_2\right)\nu^Y\left(x_2\right), \cdots, \nu^X\left(x_{|U|}\right)\nu^Y\left(x_{|U|}\right)\right)$.

(2) $\boldsymbol{\nu}\left(\boldsymbol{X} \cup \boldsymbol{Y}\right) = \boldsymbol{\nu}\left(\boldsymbol{X}\right) + \boldsymbol{\nu}\left(\boldsymbol{Y}\right) - \boldsymbol{\nu}\left(\boldsymbol{X}\right) \square\, \boldsymbol{\nu}\left(\boldsymbol{Y}\right)$,

(3) $\boldsymbol{\nu}\left(\boldsymbol{X} \backslash \boldsymbol{Y}\right) = \boldsymbol{\nu}\left(\boldsymbol{X}\right) - \boldsymbol{\nu}\left(\boldsymbol{X}\right) \square\, \boldsymbol{\nu}\left(\boldsymbol{Y}\right)$.

After the new operation rules are defined in $P\left(U\right)$ based on the eigenvectors $\boldsymbol{\nu}\left(\boldsymbol{X}\right)$ and $\boldsymbol{\nu}\left(\boldsymbol{Y}\right)$, the new algebraic system $\left\langle \boldsymbol{\nu}\left(\boldsymbol{P}\left(\boldsymbol{U}\right)\right), \square, +, -\right\rangle$ is obtained.

Theorem 1: $\left\langle P\left(U\right), \cap, \cup, \backslash\right\rangle$ and $\left\langle \boldsymbol{\nu}\left(\boldsymbol{P}\left(\boldsymbol{U}\right)\right), \square, +, -\right\rangle$ are two algebraic systems over the object set U; ν is an isomorphic mapping from $\left\langle P\left(U\right), \cap, \cup, \backslash\right\rangle$ to $\left\langle \boldsymbol{\nu}\left(\boldsymbol{P}\left(\boldsymbol{U}\right)\right), \square, +, -\right\rangle$.

Proof: (1) For any $X, Y \in P(U)$, $\boldsymbol{\nu}\left(\boldsymbol{X} \cap \boldsymbol{Y}\right) = \boldsymbol{\nu}\left(\boldsymbol{X}\right) \square\, \boldsymbol{\nu}\left(\boldsymbol{Y}\right)$, $\boldsymbol{\nu}\left(\boldsymbol{X} \cup \boldsymbol{Y}\right) = \boldsymbol{\nu}\left(\boldsymbol{X}\right) + \boldsymbol{\nu}\left(\boldsymbol{Y}\right) - \boldsymbol{\nu}\left(\boldsymbol{X}\right) \square\, \boldsymbol{\nu}\left(\boldsymbol{Y}\right)$, and $\boldsymbol{\nu}\left(\boldsymbol{X} \backslash \boldsymbol{Y}\right) = \boldsymbol{\nu}\left(\boldsymbol{X}\right) - \boldsymbol{\nu}\left(\boldsymbol{X}\right) \square\, \boldsymbol{\nu}\left(\boldsymbol{Y}\right)$ hold, ν is a homomorphic mapping from $\left\langle P\left(U\right), \cap, \cup, \backslash\right\rangle$ to $\left\langle \boldsymbol{\nu}\left(\boldsymbol{P}\left(\boldsymbol{U}\right)\right), \square, +, -\right\rangle$.

(2) If $X \ne Y$, then $\nu\left(X\right) = \boldsymbol{\nu}\left(\boldsymbol{X}\right) \ne \boldsymbol{\nu}\left(\boldsymbol{Y}\right) = \nu\left(Y\right)$, and ν is an injection.

(3) For $\forall \boldsymbol{\nu}\left(\boldsymbol{X}\right) \in \boldsymbol{\nu}\left(\boldsymbol{P}\left(\boldsymbol{U}\right)\right)$, $\exists X \in P\left(U\right)$, satisfying $\nu\left(X\right) = \boldsymbol{\nu}\left(\boldsymbol{X}\right)$, ν is a surjection.

As the analysis above shows, ν is an isomorphic mapping from $\left\langle P\left(U\right), \cap, \cup, \backslash\right\rangle$ to $\left\langle \boldsymbol{\nu}\left(\boldsymbol{P}\left(\boldsymbol{U}\right)\right), \square, +, -\right\rangle$.

Two algebraic systems with an isomorphism share identical algebraic structures and operational properties.

4. Boolean Vector Computation Method of Granules

By applying the isomorphism mapping (ν) to granules, the operations of granules are transformed from sets to Boolean vectors in granular computing.

Definition 3

A granule layer $U / R = \left\{X_1^R, X_2^R, \cdots, X_m^R\right\}\left(1 \le m \le |U|\right)$ of $R \subseteq At$ is given in $I = \left(U, At\right)$, where

$U=\left(x_1,x_2,\cdots,x_{|U|}\right)$. The mapping $\nu: X_i^R \to \boldsymbol{\nu}\left(\boldsymbol{X}_i^R\right)$ is defined on X_i^R, $\boldsymbol{\nu}\left(\boldsymbol{X}_i^R\right)=\left(\nu_i^R\left(x_1\right),\nu_i^R\left(x_2\right),\cdots,\nu_i^R\left(x_{|U|}\right)\right)$ is called the eigenvector of X_i^R, where for any $x\in U$, $\nu_i^R\left(x\right)=\begin{cases}1, & x\in X_i^R\\ 0, & x\notin X_i^R\end{cases}$.

$U/P=\left\{X_1^P,X_2^P,\cdots,X_m^P\right\}$ and $U/Q=\left\{X_1^Q,X_2^Q,\cdots,X_l^Q\right\}$ are given in $I=\left(U,At\right)$. $X_i^P\cap X_j^Q$, $X_i^P\cup X_j^Q$, and $X_i^P\setminus X_j^Q$ can be transformed into $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$, $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)+\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)-\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$, and $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)-\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$; where $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)=\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\cap\boldsymbol{X}_j^Q\right)$, $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)+\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)-\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)=\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\cup\boldsymbol{X}_j^Q\right)$, and $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)-\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)=\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\setminus\boldsymbol{X}_j^Q\right)$.

Synthesis, decomposition, inheritance, Jaccard similarity, and symmetric difference between X_i^P and X_j^Q are represented by $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$, $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)+\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)-\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$, and $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)-\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$.

a) Synthesis: $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)+\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)-\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$ is synthesized from $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)$ and $\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$.

b) Decomposition: $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)$ is decomposed into $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$ and $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)-\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$ by $\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$.

c) Inheritance: $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$ and $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)-\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$ are sub-eigenvectors of $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)$; $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)$ is the super-eigenvector of $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$ and $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)-\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$.

d) Jaccard Similarity: $J\left(\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right),\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)\right)=\dfrac{\left|\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)\right|}{\left|\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)+\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)-\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)\right|}$ is called the Jaccard similarity of $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)$ and $\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$.

e) Symmetric Difference: $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\Delta\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)=\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)+\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)-2\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)\odot\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$ is called the symmetric difference of $\boldsymbol{\nu}\left(\boldsymbol{X}_i^P\right)$ and $\boldsymbol{\nu}\left(\boldsymbol{X}_j^Q\right)$.

5. Conclusion

Granular Computing (GrC) is a new computational paradigm in the field of intelligent information processing. Granularization and granule computation are the two core issues in granular computing theory. Currently, granule operations based on set intersection, union, and difference have, to a certain extent, limited the development of fine-grained computation. This study constructs an isomorphic mapping between granules and Boolean characteristic vectors, and utilizes Boolean vector addition, subtraction, and the Hadamard product to reconstruct the computational methods for granule composition, decomposition, inheritance, similarity, and difference in Granular Computing, thereby providing a novel and systematic approach to granule computation.

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