

# Comparison of Allocation Strategies of Specialized Physicians

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**Abstract:** As an aging population increases the demand for medical care, hospitals will have to increase the number of medical equipment and examiners to cope with the surge in demand for medical tests due to increased dependence on diagnostic equipment. However, doctors have different abilities and patients have a variety of diseases. Therefore, less skilled physicians cannot serve patients with complex diseases and need to consult jointly with more skilled physicians. This paper considers the influence of setting the service ability of physician system separately, studies the waiting time and queue length of patients in two systems with and without specialized physicians, and optimizes the configuration mode of physicians. It is found that when the consultation rate is higher than the threshold value, the optimal decision under the condition of not overloading is not to set a specialist. When the consultation rate is below the threshold, a specialist is appointed.

**Keywords:** Queuing Model; Specialized Physicians; Fluid Model.

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## 1. Introduction

With the development of the domestic economy and the improvement of people's living standards, people have higher and higher requirements for the quality of health, and the demand for basic medical care is also increasing. Limited by the specialty of medical examination, the difference in the level of medical equipment inspection personnel under the imbalance of medical resources supply and demand has a gradual impact on the efficiency of medical examination. Compared with other departments, the examination of medical technology departments also depends on the skill level of doctors, because the examiners not only need to operate the machine to observe the diseased parts, but also need to judge the disease situation according to the data and images and issue relevant reports. Due to the uneven inspection level of examining physicians, expert physicians are highly qualified and have sufficient examination experience, and can usually complete the examination of various diseases independently. However, general physicians are inexperienced and inexperienced. Therefore, when general physicians encounter complicated or difficult or rare diseases involving other specialties and find it difficult to complete the examination, they may apply for joint consultation and invite expert physicians for consultation to jointly participate in the diagnosis and treatment in order to prevent the delay in diagnosis and treatment or even misdiagnosis or mistreatment. In this chapter, it is considered to set up special physicians to assist general doctors in examination, and compare with the consultation system without special doctors, so as to study whether setting special doctors will improve the service efficiency of the system.

## 2. Literature References

In the traditional mode, joint consultation can be divided into three types: inter-departmental consultation, multidisciplinary consultation and remote consultation [1]. Most of the literatures on joint consultation are empirical studies, among which telemedicine is the most researched

field. Based on tele-consultation research, most of the research is for a single disease, a single patient and a single specialist doctor tele-consultation system. Sun et al. [2] used big data to investigate whether telemedicine could improve the nursing service rate in emergency rooms, and found that through flexible resource invocation, the availability of telemedicine in emergency rooms significantly reduced the average length of patient stay. The impact of telemedicine on the average waiting time in emergency rooms is more significant when the demand for emergency care increases sharply or the hospital service capacity decreases. In addition, telemedicine in emergency rooms can also reduce the length of hospital stay for inpatients. For chronic care, Rajan et al. [3], considering the wide application of telemedicine, studied the difference in medical conditions and the heterogeneity of travel burden faced by patients in clinics on a regular basis, and the influence of the price and service fee of specialist physicians on their decision making, as well as the influence of the price of specialist physicians on the strategic behavior of specialist physicians, built a game model, and determined the price threshold and cost range. Liu et al. [4] considered the telemedical mode in which offline general physicians receive patients and request online assistance from expert physicians, and studied how to match patients with online expert physicians, as well as offline general physicians with online expert doctors, while satisfying the preferences of offline physicians. The author defined this problem as a bilateral matching problem involving three parties. The multi-objective optimization model is established and the optimal matching scheme is obtained. Shah et al. [5] studied the matching problem between tasks and experts in online platforms, considering the limitations of experts' abilities, and the results showed that it was suboptimal for each expert physician to be assigned a most appropriate task. In the field of multidisciplinary diagnosis and treatment, Chin et al. [6] compared patient outcomes between physician collaboration and physicians working alone. Compared with physicians working alone, physician collaboration can reduce hospital stay, hospitalization index and patient waiting costs. There are few researches in the field of intradepartmental consultation

in the hospital. Galimberti et al. [7] analyzed the effect of inpatient consultation in dermatology and found that most patients whose skin conditions were not treated before consultation were improved after consultation.

### 3. Set Up the Model Construction of Specialized Physicians

#### 3.1. Model Section

This section sets up specialized physicians to assist general practitioners in examination. The consultation system will consist of three queues, the General practitioner queue, the Specialist practitioner Queue and the specialist physician queue (also known as the Specialist queue). The system has  $n$  specialist physician and  $m$  general physician. If the number of expert physicians in the specialized consultation queue is  $n_1$ , then the specialized consultation team is listed as a typical  $M / M / n_1$  queue, and the number of specialists in the specialist physician queue is  $n_2$ , then the specialist physician queue is  $M / M / n_2$  queue. The number of physicians in the queue of general physicians is  $m_2(t)$ . Since there is service interruption among general physicians and patients will be transferred to a special queue, both the arrival rate and the number of servers is related to the probability of consultation transfer of patients, so this queue is listed as a  $M / M / m_2(t)$  queue, as shown in Figure 1.

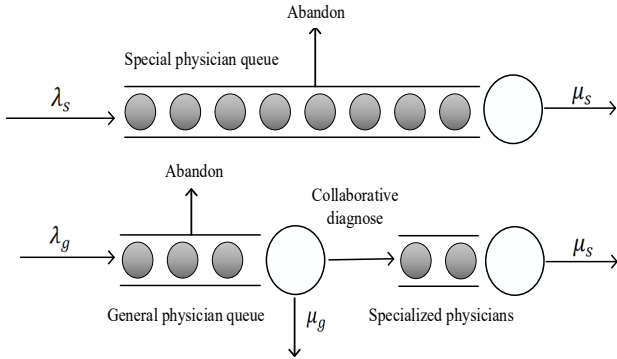


Figure 1. The queuing system for setting up specialized collaboration physicians

For these three queues, the arrival of patients met the Poisson distribution, the service rate of specialist physicians was  $\mu_s$ , and the service rate of general physicians was  $\mu_g$ , patients gave up in both the general physicians queue and the specialist physicians queue, but did not give up after entering the specialized queue.

The arrival rate of patients in the general physician queue includes the arrival rate of ordinary patients and the arrival rate of complex patients. The consultation rate of patients is the probability of complex disease. Therefore, the arrival rate of common patients in the general physician queue is  $(1-\gamma)\lambda_g$ . The arrival rate of complex patients was  $\gamma\lambda_g$ . The number of general physicians here refers to the total number of deployed, including those waiting for a consultation, those in consultation and those working independently. The dynamic equation of the general practitioner queue is:

$$\begin{aligned} Q_g(t) &= Q_g(0) + A_g(t) - S_g(t) - R_g(t) \\ &= Q_g(0) + \int_0^t \lambda_g(u) du - \int_0^t \gamma m \mu_s du - \int_0^t (1-\gamma) m \mu_g du - \theta \int_0^t (Q_g(u) - m)^+ du \end{aligned} \quad (1)$$

In the specialist physician cohort, the average waiting time for general and complex patients was consistent. The dynamic equation of the specialist physician cohort:

$$\begin{aligned} Q_s(t) &= Q_s(0) + A_s(t) - S_s(t) - R_s(t) \\ &= Q_s(0) + \int_0^t \lambda_s(u) du - \int_0^t n_1 \mu_s du - \theta \int_0^t (Q_s(u) - n_1)^+ du \end{aligned} \quad (2)$$

The dynamic equation of the specialized physician's cohort:

$$\begin{aligned} Q_c(t) &= Q_c(0) + A_c(t) - S_c(t) - R_c(t) \\ &= Q_c(0) + \int_0^t \lambda_c(u) du - \int_0^t n_2 \mu_s du - \theta \int_0^t (Q_c(u) - n_2) du \end{aligned} \quad (3)$$

#### 3.2. Evaluation Index in the System

##### 3.2.1. Set up a Queue System for Specialized Doctors

This chapter is based on a more realistic assumption that the system may be overloaded or not overloaded with different time. Therefore, it compares the system under different load levels whether special queues are configured or not.

At that time, that is  $\lambda_s < \gamma m \mu_s + (1-\gamma) m \mu_g$ , the arrival rate of patients in the general physician queue is less than the overall service capacity of physicians, and the arrival rate of patients in the special queue is  $\gamma \lambda_g$ , then the system load in the specialized physician queue is  $\rho_{q_3}^+$ :

$$\rho_{q_3}^+ = \frac{\gamma \lambda_g}{n_2 \mu_s} \quad (4)$$

$$p_{q_3}^+ = \left[ \sum_{k=0}^{n_2-1} \frac{1}{k!} \left( \frac{\gamma \lambda_g}{\mu_s} \right)^k + \frac{1}{n_2!} \cdot \frac{1}{(1-\rho_{q_3}^+)} \cdot \left( \frac{\gamma \lambda_g}{\mu_s} \right)^{n_2} \right]^{-1} \quad (5)$$

Therefore, the average queue length  $L_{q_3}^+$  and average wait time for complex patients  $W_{q_3,s}^+$  in the specialized cohort are:

$$L_{q_3}^+ = \frac{\rho_{q_3}^+ (n_2 \rho_{q_3}^+)^{n_2}}{n_2! (1-\rho_{q_3}^+)^2} P_{q_3}^+ \quad (6)$$

$$W_{q_3,s}^+ = \frac{L_{q_3}^+}{\gamma \lambda_g} \quad (7)$$

The average queue length and average waiting time of patients in the general practitioner cohort are  $L_{q_2}^+$  and  $W_{q_2,c}^+$ , where, the arrival rate is, and the service rate is:

$$\rho_{q_2}^+ = \frac{\lambda_g}{m \mu_g} \quad (8)$$

$$p_{q_2}^+ = \left[ \sum_{k=0}^{(1-\gamma)m-1} \frac{1}{k!} \left( \frac{\lambda_g}{m \mu_g} \right)^k + \frac{1}{((1-\gamma)m)!} \cdot \frac{1}{(1-\rho_{q_2}^+)} \cdot \left( \frac{\lambda_g}{m \mu_g} \right)^{(1-\gamma)m} \right]^{-1} \quad (9)$$

$$L_{q_2}^+ = \frac{\rho_{q_2}^+ \left( (1-\gamma)m\rho_{q_2}^+ \right)^{(1-\gamma)m}}{\left( (1-\gamma)m \right)! \left( 1 - \rho_{q_2}^+ \right)^2} P_{q_2}^+ \quad (10)$$

$$W_{q_2,c}^+ = \frac{L_{q_2}^+}{(1-\gamma)\lambda_g} \quad (11)$$

The arrival rate of the expert physician queue is  $\lambda_1$ ; the average service rate is  $\mu_1$ ; the number of physicians is  $L_{q_1}^+$ ; the average queue length and average waiting time of patients in the queue  $W_{q_1}^+$  are respectively:

$$\rho_{q_1}^+ = \frac{\lambda_s}{n_1\mu_s} \quad (12)$$

$$P_{q_1}^+ = \left[ \sum_{k=0}^{n_1-1} \frac{1}{k!} \left( \frac{\lambda_s}{\mu_s} \right)^k + \frac{1}{n_1!} \cdot \frac{1}{(1-\rho_{q_1}^+)} \cdot \left( \frac{\lambda_s}{\mu_s} \right)^{n_1} \right]^{-1} \quad (13)$$

$$L_{q_1}^+ = \frac{\rho_{q_1}^+ (n\rho_{q_1}^+)^{n_1}}{n_1! (1-\rho_{q_1}^+)^2} P_{q_1}^+ \quad (14)$$

$$W_{q_1}^+ = \frac{L_{q_1}^+}{\lambda_s} \quad (15)$$

### 3.2.2. Non-preemptive Queue

Consider a  $M/M/C$  queueing system with  $M$  servers and an infinite waiting room with  $P$  priority customer. The arrival process of class  $P$  customer is independent and subject to Poisson process, and the rate is  $\lambda_p$ . The service time of each type of customer is subject to exponential distribution, and the parameter is  $1/\mu$ , if the level is not taken into account, the indexes of the first type and the second type of customer are respectively 1, 2, ...,  $P$ . The smaller the number, the higher the priority of the customer.

$$\lambda_p^+ := \sum_{k=1}^p \lambda_k, 1 \leq p \leq P, \lambda := \lambda_p^+ := \sum_{k=1}^P \lambda_k \quad (16)$$

$$\rho := \rho_p^+ := \sum_{k=1}^P \rho_k = \frac{\lambda}{m\mu} \quad (17)$$

$C(m, m\rho)$  represents the probability that all servers in the queue are busy:

$$C(m, m\rho) := \frac{m\rho^m}{m!} \left( (1-\rho) \sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!} + \frac{(m\rho)^m}{m!} \right)^{-1} \quad (18)$$

The average wait time for customers of different priority levels can be obtained:

$$W_p^* = \frac{1}{C(m, m\rho) B_k B_{k-1}} \quad (19)$$

$$B_k = 1 - \frac{\sum_{k=1}^P \lambda_k}{s\mu} \quad (20)$$

In this paper, when the arrival rate of patients in the specialist physician queue does not exceed the service rate  $\bar{\lambda}$ , if the arrival rate of general physician queue is less than  $\gamma\lambda_2$ , the consultation arrival rate is; If the arrival rate of general practitioners is higher than  $\bar{\lambda}$ , the arrival rate of consultation is  $\gamma\bar{\lambda}$ . Since the consultation patients start service immediately after waiting for the doctor to be free, the priority is higher than that of the specialist queue patients. Therefore, this queue is a non-preemptive queue, in which there is  $n$  specialist physicians and two types of priority patients, the specialist queue patients and the consultation patients, both of which meet the Poisson process arrival, and the parameters are  $\lambda_1$  and  $1/\mu_1$ , respectively. The service time of each type of patients is subject to exponential distribution, and the mean is, the total arrival rate of patients in the priority queue is:

$$\lambda = \lambda_1 + \gamma\lambda_2 \quad (21)$$

System load  $\rho_s$  and busy probability  $C(n, n\rho)$  are respectively expressed as:

$$\rho_s = \frac{\lambda}{n\mu_1} \quad (22)$$

$$C(n, n\rho) := \frac{n\rho^n}{n!} \left( (1-\rho) \sum_{k=0}^{n-1} \frac{(n\rho)^k}{k!} + \frac{(n\rho)^n}{n!} \right)^{-1} \quad (23)$$

Mean waiting time for patients in the specialist physician cohort:

$$W_1 = \frac{C(n, n\rho)}{\left( 1 - \frac{\lambda_1}{n\mu_1} \right)} \quad (24)$$

Average waiting time for consultation:

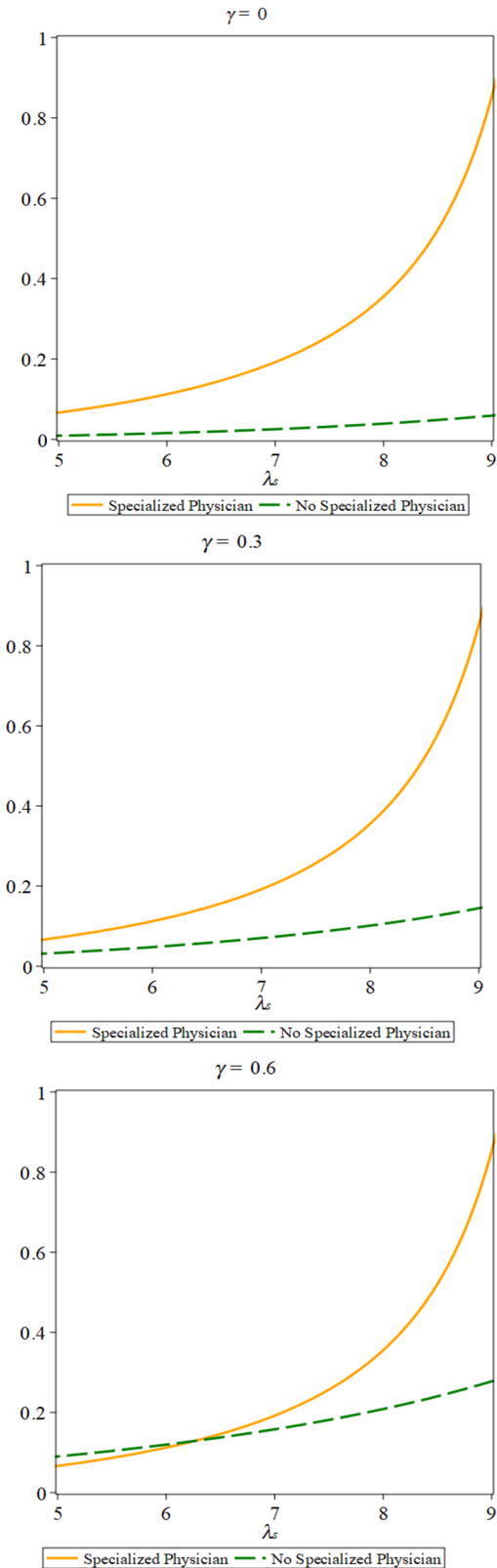
$$W_2 = \frac{C(n, n\rho)}{\left( 1 - \frac{\lambda_1}{n\mu_1} \right) \left( 1 - \frac{\lambda_1 + \gamma\lambda_2}{n\mu_1} \right)} \quad (25)$$

## 4. Numerical Analysis

### 4.1. Average Waiting Time for Patients in the Specialist Cohort

First, the situation of the queue is not overloaded is analyzed. Suppose the number of expert physicians  $n = 3$ , the number of general physicians  $m = 5$ , and the arrival rate of patients  $\lambda_g = 6$  in the general physician queue. And in the case, make the specialist physicians  $n_2 = 1$ , the specialist physicians' queue of physicians  $n_1 = 2$ . And in the case of

specialist physicians  $n_2 = 1$ , make the specialist physicians  $n_1 = 2$ . Figure 2 plots the average waiting time for patients in the specialist physician cohort. The solid line represents the change trend of the average waiting time in the cohort with specialized physicians.



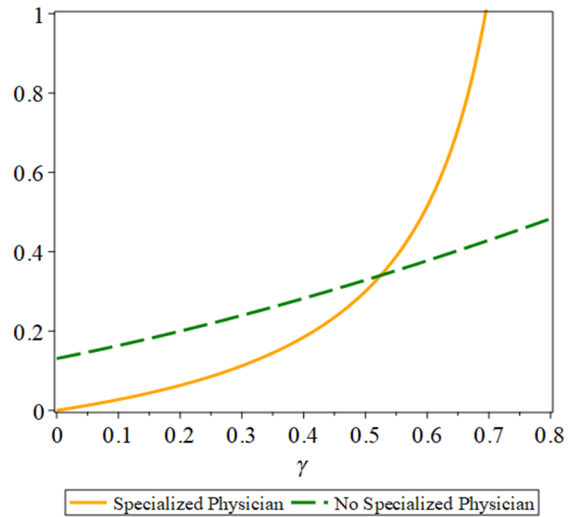
**Figure 2.** Average wait time in two specialized physician queue and no specialized physician queue

As can be seen from Figure 2, as the consultation rate increases, the gap between the mean waiting time of patients with specialized physicians and those without specialized

physicians decreases, and the mean waiting time of patients without specialized physicians decreases.

#### 4.2. Average Waiting Time for Consultation in the General Practitioner Cohort

The dotted line indicates that there is no specialized physician queue, and the solid line indicates that there is one specialized physician queue. It can be seen that when the consultation rate is lower than the intersection value, the average waiting time with specialized physicians is less than that without specialized physicians. However, with the increase of the consultation rate, the average waiting time of one specialist is gradually higher than that of the non-specialist queue. This is because with the increase of the consultation rate, the specialist cannot meet the consultation demand, resulting in the increase of the queue. Figure 3 plots the change of the average waiting time with two specialist physicians. Although increasing the number of specialist physicians can reduce the waiting time of patients, it is necessary to consider whether the remaining physicians can meet the needs of the patients in the specialist cohort.



**Figure 3.** Average waiting time of collaboration patients in two specialized physician queue and no specialized physician queue

For the patients of general doctors, when the patients do not exceed the service capacity of the system, if the consultation is higher than the threshold, no special physicians can effectively reduce the waiting time of patients. However, when the consultation rate is low, the waiting time of specialized physicians is less. This is because when there are no specialized physicians, the consultation patients jump the queue and enter the queue of specialist doctors to get more examination opportunities, but the waiting time of the patients in the queue of specialist doctors will be increased. Therefore, when the consultation rate is high, a separate queue of specialized physicians should not be set up.

### 5. Summary

In order to relieve the pressure caused by consultation waiting for general physicians, this paper sets specialized physicians separately based on the original consultation queuing model, and establishes the corresponding queuing model with specialized physician queue and the corresponding queuing model without specialized physician queue. By comparing the system waiting number and waiting time under different system loads, it is found that when the

system is not overloaded and the consultation rate is higher than the threshold, there should be no specialized physician. In the system overload system, it is the optimal configuration decision not to set a specialist when the consultation rate is lower than the threshold value and setting a specialist when the consultation rate is higher than the threshold value. However, there are some limitations in this paper. For example, if the patient arrives in line with Poisson's arrival, which is generally assumed in the medical system, inpatients and patients with appointments are regarded as outpatients. However, the average waiting time of these two types of patients is different from that of outpatients, and the patient threshold and abandonment probability are also different. Therefore, the input of patient arrival rate can be simulated according to the actual data, simulation tools can be used to simulate the patient arrival process, or a fitted patient arrival distribution function can be established.

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