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An Optimization Method for Allocating Manufacturing Enterprises' Carbon Emission Quota

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Abstract: An optimization method for allocating manufacturing enterprises' carbon emission quota was proposed. A manufacturing enterprise's carbon emission quota allocation problem was divided into the carbon emission quota allocation problem from the group company to subsidiaries (problem 1) and the carbon emission quota allocation problem from a subsidiary to its main products (problem 2). For problem 1, a mathematical model was constructed with the optimization objective of maximizing the weighted average of the normalized values of the total profit of subsidiaries and the weighted importance of the carbon emission quota. For problem 2, a mathematical model was constructed with the optimization objective of maximizing the weighted average of the normalized value of the total profit of the products and the weighted importance of the carbon emission quota. Excel was taken as the platform to present the model in tabular form. A VBA program was designed to generate the Lingo script program and run the Lingo script program to realize the automated solution of the model. The case study showed that the method proposed in this paper can efficiently solve the problem of optimal carbon emission quota allocation for manufacturing enterprises.

Keywords: Manufacturing Enterprises; Carbon Quota Allocation; Excel VBA; Lingo.

1. Introduction

To actively address global warming, China, as the world's largest carbon emitter, has pledged to achieve the carbon peaking and carbon neutrality goals [1,2]. According to the Ministry of Industry and Information Technology 2022 data, China's manufacturing value-added has ranked first worldwide for 13 consecutive years [3]. Still, its long-standing model of sloppy development has also created serious carbon emission problems for society [4]. To realize the carbon peaking and carbon neutrality goals, promoting carbon emission reduction in the manufacturing industry has become a top priority, and manufacturing enterprises are bound to be subject to increasingly strict carbon emission constraints. Therefore, how to optimize the allocation of carbon emission quota under the constraints of carbon emissions has become an urgent problem for enterprises.

In recent years, academics have conducted exploratory research around the issue of carbon emission quota allocation, which can be mainly categorized into the following three aspects:

(1) Study on regional carbon emission quota allocation. With the gradual promotion of carbon market trading in China, the study of carbon emission quota allocation among provinces has received wide attention. Some scholars have studied the allocation of carbon emission quotas among provinces from the perspectives of fairness and efficiency, and have obtained some provincial carbon emission quota allocation schemes with a high degree of acceptance [5–7]; Chen et al. concluded that fewer studies consider the potential of carbon emission reduction, and proposed a provincial carbon emission quota allocation model that uses the DEA model to calculate the potential of emission reduction [8]. In addition, the allocation of carbon emission quotas in some key regions has also received greater attention. For example, Li et al. addressed the issue of carbon emission quota allocation in the Pearl River Delta (PRD) region from the perspectives of equality, efficiency, and feasibility [9]; To overcome the conflict between participants at different levels, Li et al. proposed a carbon emission quota allocation method that takes into account the cost of emission reduction and individual interests [10]. Li et al. argued that current research focuses mostly on the equity and efficiency perspectives and less on the other perspectives, and therefore proposed a carbon emission quota allocation method for Jiangsu, Zhejiang, and Shanghai regions that takes into account the dual constraints of total carbon emissions and intensity [11].

(2) Study on sectoral carbon emission quota allocation. As the largest source of carbon emissions in China, the electric power industry has been included in the first batch of the national carbon trading market, and some scholars have studied the allocation of carbon emission quotas to the electric power industry among the provinces in China [12–14]. Ma et al. proposed a method to optimize the allocation of carbon emission quotas in the power sector of the Yangtze River Economic Belt given the large differences in the distribution of carbon emissions among the provinces along the Yangtze River [15]. Some scholars have also studied the allocation of carbon emission quotas in other industries. Song et al. proposed an output-efficiency-based carbon emission quota allocation scheme for China's iron and steel industry [16]. Guo et al. proposed a model for carbon emission quota allocation in China's railroad industry from the perspectives of equity and efficiency under the constraint of total carbon emissions [17].

(3) Study on enterprise's carbon emission quota allocation. Optimizing the allocation of enterprise's carbon emission quotas is a key link in promoting carbon emission reduction. To address this issue, Xiao et al. took coal-fired generating units of power generation enterprises as the object and constructed a two-layer planning model targeting economic benefits and power generation intensity [18]. Wang et al.

constructed a carbon emission quota allocation model for coal power supply chain enterprises to minimize the fair deviation index and the production level of each link as the constraints [19].

In general, current carbon emission quota allocation research focuses mainly on the regional and industry levels, with relatively few studies at the enterprise level, especially for manufacturing enterprises. Based on this, this paper proposed a method for optimizing the allocation of carbon emission quotas in manufacturing enterprises, including the method for optimizing the allocation of carbon emission quotas from a group company to subsidiaries and the method for optimizing the allocation of carbon emission quotas from a subsidiary to products.

2. The Optimization Allocation Model of Carbon Emission Quota

2.1. Description of the Problem

As shown in Figure 1, this paper divided manufacturing enterprises' carbon emission quota allocation problem into the carbon emission quota allocation problem from a group company to subsidiaries (Problem 1) and the carbon emission quota allocation problem from a subsidiary to products (Problem 2). For Problem 1, the group company obtains carbon emission quotas from the higher authority. Then it allocates them to subsidiaries reasonably, considering factors such as carbon emission, profit, and importance of the subsidiaries. For Problem 2, after obtaining carbon emission quotas from the group company, the subsidiaries allocate them to each product reasonably, considering factors such as profit, importance of the products, and market.

Hypothetical conditions: 1) Manufacturing companies use 1 year as a decision cycle for carbon emission quota allocation; 2) The upper and lower limits of carbon emission quotas required for each subsidiary and product year are known; 3) Profit on tons of carbon of subsidiaries and profit on tons of carbon of products is known; 4) The importance of subsidiaries to the manufacturing company and the importance of products to the subsidiary is known; 5) Upper and lower limits of carbon emission quotas, profit on tons of carbon and importance are fixed in the decision cycle.

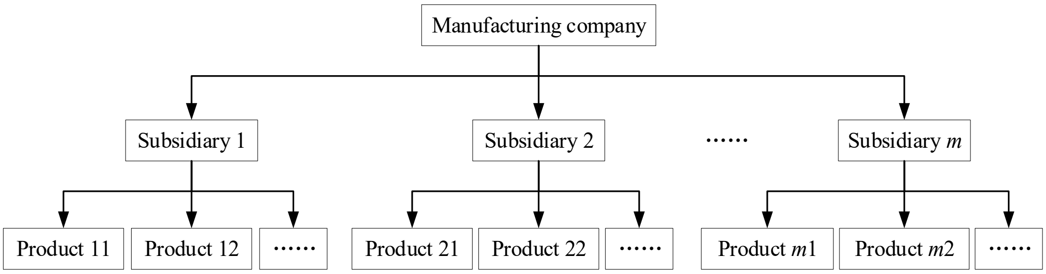

Figure 1. Carbon quota allocation process for enterprises.

2.2. Symbol Definition

Table 1 shows the symbols involved in the model.

Table 1. Description of symbols.

Symbol	Meaning	Symbol	Meaning
m	Number of subsidiaries of the group	$maxq$	Maximum carbon emission quota weighted importance for the group without constraints
gt	Objective function value for problem 1	$minq$	Minimum carbon emission quota weighted importance for the group without constraints
e	Minimum profit requirement for the group	U_i	Carbon emission quota cap for subsidiary i
c	Total carbon emission quotas of the group	L_i	Carbon emission quota floor for subsidiary i
C_i	Carbon emission quotas for subsidiary i	GZ_i	Objective function value for problem 2
P_i	Profit on tons of carbon of Subsidiary i	N_i	Number of products of subsidiary i
Q_i	Importance of subsidiary i	P_{ij}	Profit on tons of carbon of product j for subsidiary i
s	Group profit	Sg_i	The normalized value of the subsidiary i 's profit
r	The carbon quota weighted importance for the group	Rg_i	The normalized value of the weighted importance for the subsidiary i
sg	The normalized value of the group's profit	X_{ij}	Carbon emission quotas of product j for subsidiary i
rg	The normalized value of the weighted importance for the group	E_i	Minimum profit requirement for subsidiary i
α	The weighting of group profit	U_{ij}	Carbon emission quota cap of product j for subsidiary i
$maxp$	Maximum profit for the group without constraints	L_{ij}	Carbon emission quota floor of product j for subsidiary i
$minp$	Minimum profit for the group without constraints	β	The weighting of subsidiary i 's profit
g	Equity constraint parameters of the group	G_i	Equity constraint parameters of subsidiary i

2.3. The Mathematical Model of Problem 1

In Problem 1, the group company as the allocation subject allocates carbon emission quotas to subsidiaries, and its total profit and carbon quota weighted importance can be measured by equations (1) and (2).

$$s = \sum_{i=1}^{m} C_i P_i \tag{1}$$

$$r = \sum_{i=1}^{m} C_i Q_i \tag{2}$$

Since equations (1) and (2) have different units and value ranges, equations (3) and (4) are used to normalize the two.

$$sg = \frac{s - minp}{maxp - minp} \tag{3}$$

$$rg = \frac{r - minq}{maxq - minq} \tag{4}$$

According to the normalization results of equations (3) and (4), combined with the profit weight α , the objective function of carbon emission quota allocation from a group company to subsidiaries can be obtained, as shown in equations(5).

$$\max gt = \alpha sg + (1-\alpha) rg \tag{5}$$

Manufacturing enterprises are often affected by various factors in their production and operation, and this paper only considered the factors that impose constraints on the allocation of carbon emission quotas, which mainly involve the following constraints:

(1) The constraint of total carbon emission quota: The group company must allocate all carbon emission quotas for distribution to its subsidiaries, as shown in equation (6).

$$\sum_{\text{i}=1}^{m} C_i = c \tag{6}$$

(2) The constraint of profit floor: In emission reduction, profitability is still an important goal for manufacturing enterprises, so sufficient profit should be maintained when making carbon quota allocations, as shown in equation (7).

$$\sum_{i=1}^{m} C_i P_i \ge e \tag{7}$$

(3) The constraint of carbon emission quota cap: There is a carbon emission cap for each subsidiary due to factors such as production capacity or market demand, so the allocation of carbon emission quotas should be carried out in such a way as to ensure that this cap is not exceeded, as shown in equation (8).

$$C_i \le U_i, (i = 1, 2, \cdots, m) \tag{8}$$

(4) The constraint of carbon emission quota floor: To realize the reasonable distribution of carbon emission quotas among subsidiaries and prevent certain subsidiaries from not being able to fully develop due to the shortage of carbon emission quotas, it is necessary to set the lower limit of carbon emission quotas for each subsidiary, as shown in equation (9).

$$C_i \ge L_i, (i = 1, 2, \cdots, m) \tag{9}$$

(5) Joint constraint: There may be some special relationships among certain subsidiaries, such as industry chain relationships or resource sharing relationships. Therefore, the interdependence and influence between them should be considered comprehensively when formulating the carbon emission quota allocation program, i.e. there is a kind of joint constraint. Equation (10) indicates that the total amount of carbon emission quotas obtained by subsidiaries 1, 2 and 5 is not less than 2,000 tons and not more than 5,000 tons. In the actual problem, the number of joint constraints should be determined case by case.

$$2000 \le C_1 + C_2 + C_5 \le 5000 \tag{10}$$

(6) Fairness constraint: To prevent extreme cases of carbon emission quota allocation, such as too many subsidiaries obtaining the theoretical maximum or minimum carbon emission quota, it is necessary to ensure fairness in the allocation process, as shown in Equation (11), where the smaller g represents a greater concern for fairness.

$$\begin{cases} \left| \frac{C_1}{U_1} - \frac{C_i}{U_i} \right| \le g, (i = 2, 3, \cdots, m) \\ \left| \frac{C_2}{U_2} - \frac{C_i}{U_i} \right| \le g, (i = 3, 4, \cdots, m) \\ \cdots\cdots \\ \left| \frac{C_{m-1}}{U_{m-1}} - \frac{C_i}{U_i} \right| \le g, (i = m) \end{cases} \tag{11}$$

2.4. The Mathematical Model of Problem 2

In Problem 2, the subsidiary is the main body of allocation. Maximizing the weighted average of its total profit and the weighted significance of its product carbon emission allowances after normalization as the optimization objective, and taking the upper and lower limits of the carbon emission quotas of each product, the total carbon emission quotas of subsidiaries, the profit floor, and other actual demands as the constraints, we constructed a model for the allocation of the carbon emission quotas from subsidiaries to products. Subsidiary i is described as an example.

The objective function is shown in equation (12), where Sg_i and Rg_i are similar to the derivation of sg and rg in Problem 1.

$$\max GZ_i = \beta Sg_i + (1-\beta) Rg_i \tag{12}$$

The constraints are shown in equations (13) to (18).

$$\sum_{j=1}^{N_i} X_{ij} = C_i \tag{13}$$

$$\sum_{j=1}^{N_i} X_{ij} P_{ij} \ge E_i \tag{14}$$

$$X_{ij} \le U_{ij}, (j = 1, 2, \cdots, N_i) \tag{15}$$

$$X_{ij} \le L_{ij}, (j = 1, 2, \cdots, N_i) \tag{16}$$

$$X_{i1} + X_{i2} + X_{i3} \ge 450 \tag{17}$$

$$\begin{cases} \left| \frac{X_1}{U_1} - \frac{X_{ij}}{U_{ij}} \right| \le G_i, (j = 2, 3, \cdots, N_i) \\ \left| \frac{X_2}{U_2} - \frac{X_{ij}}{U_{ij}} \right| \le G_i, (j = 3, 4, \cdots, N_i) \\ \cdots\cdots \\ \left| \frac{X_{iN_i-1}}{U_{iN_i-1}} - \frac{X_{ij}}{U_{ij}} \right| \le G_i, (j = N_i) \end{cases} \tag{18}$$

In Problem 2, the joint constraint is similar to Problem 1. It should be determined case by case, e.g., equation (17) indicates that the sum of carbon emission quotas received by products 1, 2, and 3 of subsidiary i is not less than 450 tons.

3. Model Solution

Since the solutions to Problems 1 and 2 are similar, only the solution to Problem 1 is presented below.

3.1. Ideas for Solving the Problem

As shown in Figure 2, Excel and Lingo were combined to automate the solution of the model by generating and calling Lingo programs through Excel VBA code. The specific steps are as follows: 1) Excel was used as the input and output interface to design the parameter setting interface; 2) Excel VBA code was used to read the relevant data from the parameter setting interface to generate the data file (jttpefpdata.txt); 3) Excel VBA code was used to create the Lingo script file (jttpefp.ltf) for model solving; 4) Calling Lingo software with Excel VBA code to run the file (jttpefp.ltf), calculate the optimal carbon allowance allocation results, and store them in the file (jttpefpopt.txt); 5) Excel VBA code was used to read the file (jttpefpopt.txt) and output the results of carbon emission quota allocation to the variable value line of the parameter setting interface.

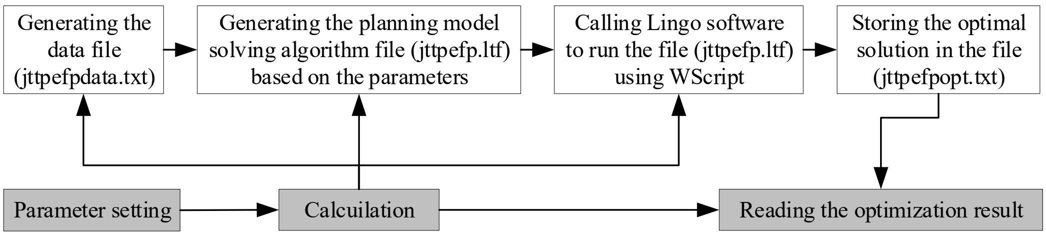

Figure 2. Algorithmic process

Calculation	Return to zero									
Auxiliary area	Total profit	0	Fairness parameter	0.25	Maximum importance	0.31	Stopping row of constraint	22	Number of variables	4
	Carbon emission quotas (tons/year)	90000	Maximum profit on tons of carbon	962	Minimum importance	0.2	Starting column of variable	F		
	Profit weighting	0.65	Minimum profit on tons of carbon	441	starting row of constraint	13	Stopping column of variable	I		
Target area	Solution type		Variable area	Variable name	C1	C2	C3	C4		
	Objective function value	0.000		Variable type	real	real	real	real		
				Variable value	0.00	0.00	0.00	0.00		
				Importance	0.31	0.23	0.26	0.2		
				Profit on tons of carbon	715	859	962	441		
Constraint area										
Serial number	Type of constraint	constraint value	Notation	Resource volume	C1 coefficient	C2 coefficient	C3 coefficient	C4 coefficient		
1	Profit floor	0	>=	72500000	715	859	962	441		
2	C1 quota floor	0	>=	26000	1					
3	C1 quota cap	0	<=	39000	1					
4	C2 quota floor	0	>=	17200		1				
5	C2 quota cap	0	<=	31000		1				
6	C3 quota floor	0	>=	19950			1			
7	C3 quota cap	0	<=	29500			1			
8	C4 quota floor	0	>=	7500				1		
9	C4 quota cap	0	<=	11500				1		
10	Joint quota	0	>=	48000	1		1			

Figure 3. Parameter setting interface.

3.2. Parameter setting

To achieve the "WYSIWYG" visualization effect, Excel was used as a platform to present the model as a table, as shown in Figure 3.

The parameter setting interface was divided into four areas: target area, variable area, constraint area, and auxiliary area. The target area was used to output the value of the model's objective function and the solution type. The variable area was used to set information about the decision variables, including variable name, variable type (real/integer), variable value, importance, and profit on tons of carbon. The constraint area was used to set constraints The auxiliary area was used to store information such as carbon emission quotas and profit weight to provide auxiliary data for the model's calculations.

3.3. Calculation

3.3.1. Generating Data File

Excel VBA code was used to read the profit on tons of carbon vector, the importance vector, the variable coefficient vector, and the constrained resource volume vector according to the parameter setting interface, and deposited them into the file (jttpefpdata.txt) with "~" as the separator. The content of the corresponding data file in Figure 3 is as follows:

```
715 859 962 441~
0.31 0.23 0.26 0.2~
715 859 962 441
1 1 1 1
1 0 0 0
1 0 0 0
0 1 0 0
0 1 0 0
0 0 1 0
0 0 1 0
0 0 0 1
0 0 0 1
1 0 1 0~
70000000 90000 26000 39000 17200 31000 19950 29500 7500 11500 48000
```

3.3.2. Generating Lingo Script File

Excel VBA code and the splicing technique were used to generate a model solution code that conforms to the syntax of the Lingo program and appends it to the string s. Then, the content in the string s was written to the file (jttpefp.ltf) based on the parameter settings and the relevant data file (jttpefpdata.txt). The content of the Lingo script program corresponding to Figure 3 is as follows:

```
set default
set echoin 1
set global 1
model:
data:
w=0.65;
gpzs=0.25;
blgs=4;
yss=10;
zgstpe=90000;
maxp=962;
minp=441;
maxq=0.31;
minq=0.2;
enddata
sets:
OB1/1..blgs/: X,P1,P2,E;
OB2/1..yss/:B;
OB3(OB2,OB1):XS;
endsets
data:
P1=@file('D:\model\jttpefpdata.txt');
P2=@file('D:\model\jttpefpdata.txt');
XS=@file('D:\model\jttpefpdata.txt');
B=@file('D:\model\jttpefpdata.txt');
@text('D: \model\jttpefpopt.txt')=@write(@status(),@newline(1));
@text('D: \model\jttpefpopt.txt')=@write(opt,@newline(1));
@text('D:\model\jttpefpopt.txt')=@writefor(OB1(i):X(i),' ');
enddata
opt=w*(@sum(OB1(i):X*P1)-zgstpe*minp)/(zgstpe(maxp-minp))+(1-w)*(@sum(OB1(i):X*P2)-zgstpe*minq)/
(zgstpe(maxq-minq));
max=opt
@sum(OB1(j):XS(1,j)*X(j))>=B(1);
@sum(OB1(j):XS(2,j)*X(j))=B(2);
@sum(OB1(j):XS(3,j)*X(j))>=B(3);
@sum(OB1(j):XS(4,j)*X(j))<=B(4);
@sum(OB1(j):XS(5,j)*X(j))>=B(5);
@sum(OB1(j):XS(6,j)*X(j))<=B(6);
@sum(OB1(j):XS(7,j)*X(j))>=B(7);
@sum(OB1(j):XS(8,j)*X(j))<=B(8);
@sum(OB1(j):XS(9,j)*X(j))>=B(9);
@sum(OB1(j):XS(10,j)*X(j))<=B(10);
@sum(OB1(j):XS(11,j)*X(j))>=B(11);
@abs(X(1)/E(1)-X(2)/E(2))<=gpzs;
@abs(X(1)/E(1)-X(3)/E(3))<=gpzs;
```

```
@abs(X(1)/E(1)-X(4)/E(4))<=gpzs;
@abs(X(2)/E(2)-X(3)/E(3))<=gpzs;
@abs(X(2)/E(2)-X(4)/E(4))<=gpzs;
@abs(X(3)/E(3)-X(4)/E(4))<=gpzs;
end
set terseo 1
go
quit
```

3.3.3. Calling Lingo Script File

In Excel VBA code, the "WScript.Sheet" object was used to call Lingo software to run the Lingo script program (jttpefp.ltf), and the results of the calculation will be written to the file (jttpefpopt.txt). Figure 3 corresponds to the output as follows:

Global optimum

35354.73 20352.48 26742.68 7550.11

The data in row 1 is the solution type, and the data in row 2 is the result of the optimal carbon quota allocation.

3.4. Reading the Optimization Result

Excel VBA code was used to read the optimization results from the file (jttpefpopt.txt). Then the results were displayed in the row of solution type and the row of variable value, respectively. Excel automatically updated the total profit and target value through the corresponding formulas. The optimized allocation results corresponding to Figure 3 are shown in Figure 4.

Calculation	Return to zero									
Auxiliary area	Total profit	72500000	Fairness parameter	0.25	Maximum importance	0.31	Stopping row of constraint	22	Number of variables	4
	Carbon emission quotas (tons/year)	90000	Maximum profit on tons of carbon	962	Minimum importance	0.2	Starting column of variable	F		
	Profit weighting	0.65	Minimum profit on tons of carbon	441	starting row of constraint	13	Stopping column of variable	I		
Target area	Solution type	Global optimum	Variable area	Variable name	C1	C2	C3	C4		
	Objective function value	0.657		Variable type	real	real	real	real		
				Variable value	30668.44	25217.43	26614.13	7500.00		
				Importance	0.31	0.23	0.26	0.2		
				Profit on tons of carbon	715	859	962	441		
Constraint area										
Serial number	Type of constraint	constraint value	Notation	Resource volume	C1 coefficient	C2 coefficient	C3 coefficient	C4 coefficient		
1	Profit floor	72500000	>=	72500000	715	859	962	441		
2	C1 quota floor	30668.44052	>=	26000	1					
3	C1 quota cap	30668.44052	<=	39000	1					
4	C2 quota floor	25217.42905	>=	17200		1				
5	C2 quota cap	25217.42905	<=	31000		1				
6	C3 quota floor	26614.13043	>=	19950			1			
7	C3 quota cap	26614.13043	<=	29500			1			
8	C4 quota floor	7500	>=	7500				1		
9	C4 quota cap	7500	<=	11500				1		
10	Joint quota	57282.57095	>=	48000	1		1			

Figure 4. Results of the allocation of carbon emission quotas.

4. Case Analysis

To verify the effect of the carbon emission quota allocation model and related algorithms, the relevant data of the E manufacturing enterprise was used for case analysis.

4.1. Relevant Parameters and Analysis Result

There are 4 subsidiaries of E manufacturing enterprise, namely A, B, C, and D, and the total carbon emission quota is expected to be 90000 tons/year. By analyzing the production data of the four subsidiaries and the market demand in recent years, combined with the enterprise's strategic objectives, the relevant parameters obtained are shown in Table 2.

Each of the four subsidiaries produces different products. Subsidiary A mainly produces 3 products, namely A-1, A-2, and A-3, and the relevant parameters are shown in Table 3. Subsidiary B mainly produces 4 products, namely B-1, B-2, B-3, and B-4, and the relevant parameters are shown in Table 4. Subsidiary C mainly produces 4 products, namely C-1, C-2, C-3, and C-4, and the relevant parameters are shown in Table 5. Subsidiary D mainly produces 2 products, D-1 and D-2, and the relevant parameters are shown in Table 6.

According to the relevant parameters of the subsidiaries of E manufacturing enterprise given in Table 2, the Excel parameter interface was set up to obtain the optimal subsidiaries-based carbon emission quota allocation scheme using the designed carbon emission quota allocation model and the solution algorithm. Then, the allocation result at the subsidiary level was used as the total carbon emission quota constraints at the product level, and the optimal product-based carbon quota allocation scheme was obtained by refining the parameter setting interface and executing the solving operation based on the data in Tables 3 to 6. The optimal programs based on subsidiaries and products were combined to obtain the total carbon emission quota allocation program for E manufacturing enterprise as shown in Table 7.

In the above case, the solution type of the model was "Global optimum ", but in some cases, the solution type may also be "Local optimum ", "Infeasible" and "No solution". This requires the operator to be familiar with the influence of some key parameters on the model so that the parameters can be adjusted according to the actual situation of the enterprise to get the optimal solution of the carbon emission quota allocation problem.

Table 2. Parameters related to subsidiaries.

Parameter	Subsidiary			
	A	B	C	D
Profit on tons of carbon	￥715	￥859	￥962	￥441
Importance	0.31	0.23	0.26	0.2
Carbon emission quota interval (tons)	26000~39000	17200~31000	19950~29500	7500~11500
Joint carbon emission quotas (tons)	$C_1+C_3 \geq 48000$			
Profit weighting	0.65			
Fairness parameter	0.25			
profit limit	￥70000000			

Table 3. Parameters related to each product of Subsidiary A.

Parameter	Product		
	A-1	A-2	A-3
Profit on tons of carbon	￥721	￥658	￥766
Importance	0.38	0.29	0.33
Carbon emission quota interval (tons)	9500~14000	8000~11500	8500~13500
Joint carbon emission quotas (tons)	None		
Profit weighting	0.6		
Fairness parameter	0.2		
profit limit	￥25000000		

Table 4. Parameters related to each product of Subsidiary B.

Parameter	Product			
	B-1	B-2	B-3	B-4
Profit on tons of carbon	￥913	￥871	￥901	￥756
Importance	0.32	0.2	0.25	0.23
Carbon emission quota interval (tons)	5700~10000	4500~7000	4000~8000	3000~6000
Joint carbon emission quotas (tons)	$X_{21}+X_{22} \leq 13500$			
Profit weighting	0.7			
Fairness parameter	0.25			
profit limit	￥17000000			

Table 5. Parameters related to each product of Subsidiary C.

Parameter	Product			
	C-1	C-2	C-3	C-4
Profit on tons of carbon	￥961	￥983	￥1017	￥893
Importance	0.33	0.26	0.22	0.19
Carbon emission quota interval (tons)	5350~8000	4000~8,000	7600~9000	3000~4500
Joint carbon emission quotas (tons)	$X_{32}+X_{33} \geq 15000$			
Profit weighting	0.75			
Fairness parameter	0.28			
profit limit	￥26000000			

Table 6. Parameters related to each product of Subsidiary D.

Parameter	Product	
	D-1	D-2
Profit on tons of carbon	￥417	￥465
Importance	0.43	0.57
Carbon emission quota interval (tons)	3500˜5000	4000˜6500
Joint carbon emission quotas (tons)	None	
Profit weighting	0.7	
Fairness parameter	0.15	
profit limit	￥3000000	

Table 7. Carbon quota allocation program for E manufacturing enterprise

Subsidiary	Subsidiary carbon emission quotas (tons)	Product	Product carbon emission quotas (tons)
A	35355	A-1	13517
		A-2	8803
		A-3	13035
B	20352	B-1	7500
		B-2	4500
		B-3	5352
		B-4	3000
C	26743	C-1	7496
		C-2	7496
		C-3	8434
		C-4	3317
D	7550	D-1	3500
		D-2	4050

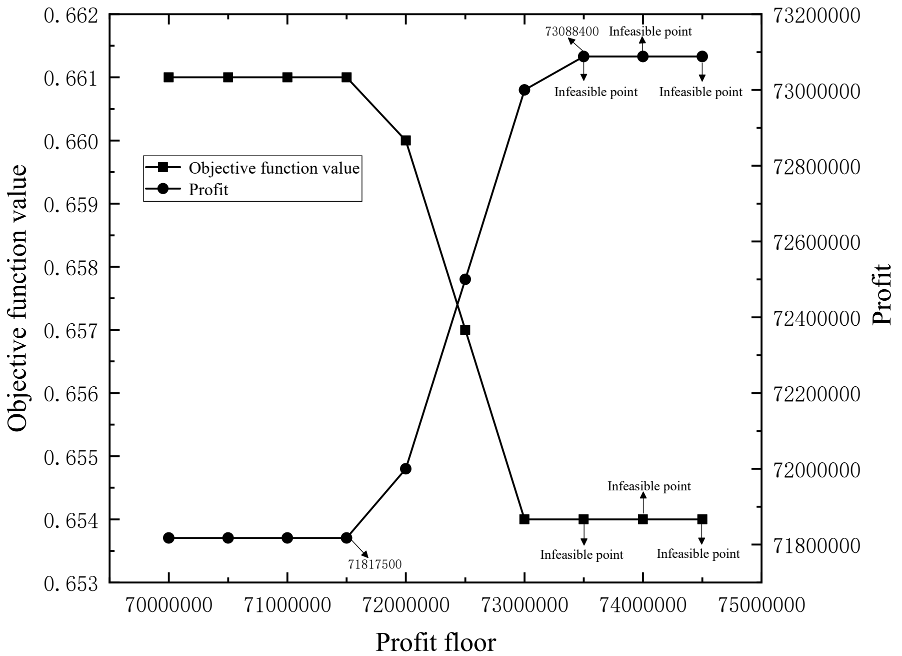

Figure 5. Impact of changes in profit floor on target value and profit.

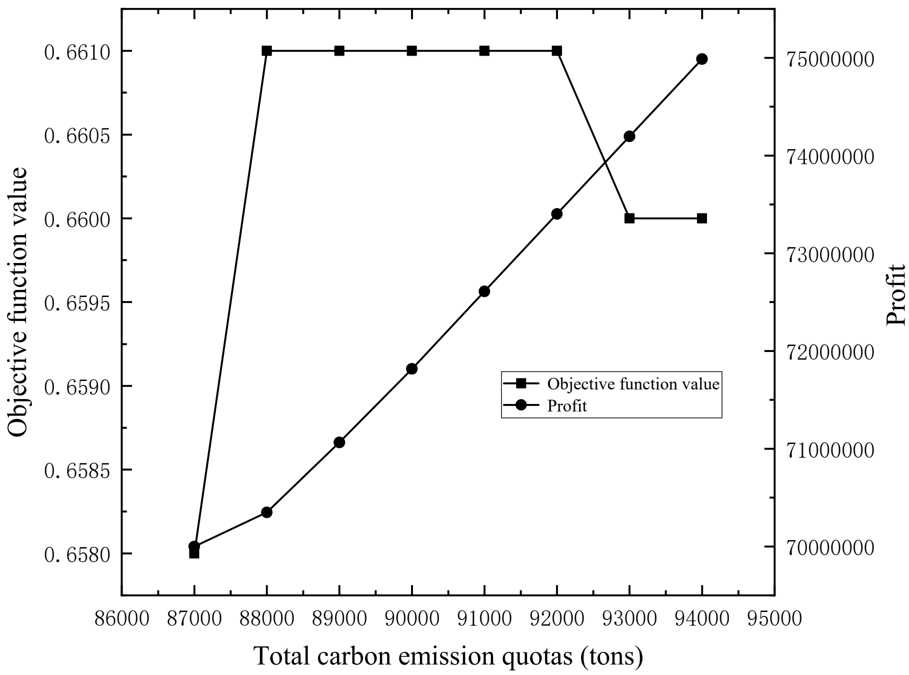

Figure 6. Impact of changes in total carbon emission quotas on target value and profit

4.2. Parameter Sensitivity Analysis

Taking the allocation of carbon emission quotas to subsidiaries A, B, C, and D as an example, the sensitivity analysis of some key parameters was conducted to understand the impact of different parameter changes on the model.

4.2.1. Sensitivity Analysis of the Profit Floor

In the carbon emission quota allocation process, choosing the appropriate profit floor is crucial for decision-makers. In this paper, several profit floors were selected for analysis, and the results are shown in Figure 5.

As shown in Figure 5, the objective function value and profit show different changes in three profit lower bound intervals. When the profit floor is in the interval 1: (0, 71,817,500], the objective function value and profit remain stable, which are 0.661 and 71,817,500, respectively, and at this time, carbon quota allocation program 1 can be obtained, as shown in Table 8. When the profit floor is in the interval 2: (71,817,500, 73088400], with the increase of the profit floor, the objective function value shows a straight-line downward trend, while the profit shows a straight-line upward trend, and at this time, we can obtain the carbon quota allocation program 2 (with the profit floor = 72500000 as an example) as shown in Table 9.

Table 8. The program 1 of carbon emission quota allocation.

Subsidiary	A	B	C	D
Carbon emission quota (tons)	35355	20352	26743	7550

Table 9. The program 2 of carbon emission quota allocation

Subsidiary	A	B	C	D
Carbon emission quota (tons)	30669	25217	26614	7500

4.2.2. Sensitivity Analysis of Total Carbon Emission Quotas

Under the carbon trading mechanism, carbon emission quotas have become one of the important factors influencing corporate decision-making. The impact of the change in the total carbon emission quota on the objective function value and the profit was analyzed through the case data, and the result is shown in Figure 6. It can be found that there exists a total carbon quota interval [87,500, 92,000] that makes the objective function value optimal. When the total carbon emission quota is lower than this interval, the enterprise can choose to increase the total carbon emission quota, and when it is higher than this interval, the enterprise can choose to decrease the total carbon emission quota to optimize the objective function value. Profit roughly follows a linear upward trend with the increase of the total carbon emission quota. Therefore, when making carbon emission quota trading decisions, enterprises should fully consider their actual needs to maximize the overall benefits.

4.2.3. Sensitivity Analysis of Profit on Tons of Carbon

Based on the results of carbon emission quota allocation in Tables 8 and 9, different profits on tons of carbon are selected for the study to verify the impact of changes in profit on tons of carbon of subsidiaries on the results of carbon emission quota allocation in the case of different profit floors, as shown in tables 10 and 11. If the changed program is consistent with the original program, it is marked "Y", if not, it is marked "N", and if it is not feasible, it is marked "NF".

As shown in Tables 10 and 11, with the upward and downward fluctuation of the profit on tons of carbon, the possibility of changing the carbon quota allocation scheme is small when the profit floor is located in interval 1, while the possibility of changing the carbon quota allocation scheme or making it infeasible is very high when the profit floor is located in interval 2. Therefore, when the profit floor is located in interval 2, enterprises should pay great attention to the impact of the change in the profit on tons of carbon.

Table 10. Impact of changes in profit on tons of carbon of subsidiaries on the allocation of carbon emission quotas (the profit floor is located in interval 1)

Value-added profit on tons of carbon	Subsidiary			
	A	B	C	D
-￥30	Y	Y	Y	Y
-￥20	Y	Y	Y	Y
-￥10	Y	Y	Y	Y
￥10	Y	Y	Y	Y
￥20	Y	Y	Y	Y
￥30	Y	N	Y	Y

Table 11. Impact of changes in profit on tons of carbon of subsidiaries on the allocation of carbon emission quotas (the profit floor is located in interval 2

Value-added profit on tons of carbon	Subsidiary			
	A	B	C	D
-￥30	NF	NF	NF	N
-￥20	N	N	N	N
-￥10	N	N	N	N
￥10	N	N	N	N
￥20	Y	N	N	N
￥30	Y	N	Y	N

5. Conclusion

In this paper, the carbon emission quota allocation problem for a manufacturing enterprise was divided into the carbon emission quota allocation problem from the group company to subsidiaries and the carbon emission quota allocation problem from a subsidiary to its main products. Based on considering the weighted importance of carbon quota and profit, an allocation model is established with the maximization of the weighted average value after normalization of the two as the optimization objective, and with the total carbon emission quota, the profit floor, the upper and lower limits of the carbon emission quota of each subsidiary (product), and the joint quota, etc. as the constraints. Combining Excel with Lingo software, the automated solution of the model was achieved by generating and calling the Lingo script program through Excel VBA code. The relevant data of E manufacturing enterprise was analyzed as an example to verify the feasibility of the method of this paper. A sensitivity analysis of relevant parameters was carried out to obtain the effects of changes in the profit floor, total carbon emission quota of the enterprise, and profit on tons of carbon of subsidiaries on the results of the model.

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