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Research Progress on Vibration Reduction of Beam Structures Based on Piezoelectric Energy Harvesting

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Abstract: With the rapid development of the Internet of Things and intelligent structures, integrated piezoelectric energy harvesting and vibration reduction technologies aimed at self-powered monitoring and vibration suppression have become a research hotspot. Owing to mature structural mechanics, relatively low manufacturing cost and the ease of integrating piezoelectric elements, beam components have become the core load-bearing elements in piezoelectric energy harvesting and vibration control systems. Against this background, the fundamental theory of piezoelectric energy harvesting and beam-based vibration reduction is reviewed for typical prismatic and non-prismatic beam structures, including cantilever beams, L-shaped beams, U-shaped beams, and trapezoidal and tapered beams. The research progress regarding bandwidth extension, enhancement of energy conversion efficiency and improvement of vibration attenuation performance for these beam configurations is systematically summarized. Through a comparative analysis of the mechanisms and performance of different beam types, the key challenges faced by current beam-type piezoelectric energy harvesting–vibration reduction systems are identified, including nonlinear electromechanical coupling modelling, adaptability to broadband and multi-directional vibration environments, structural reliability and practical engineering application. On this basis, future development directions involving multi-field coupling, integrated co-design, introduction of smart materials and topological optimization are discussed. The review is intended to provide useful reference for structural design and engineering application of beam-type integrated piezoelectric energy harvesting and vibration reduction systems.

Keywords: Piezoelectric Energy Harvesting; Structural Vibration Reduction; Beam Structures; Cantilever Beam; L-shaped Beam; U-shaped Beam; Tapered Beam; Non-uniform Beam.

1. Introduction

Environmental vibration [1] is widely present in complex service scenarios such as transportation systems, bridge engineering, rotating machinery and marine structures. On the one hand, it may induce structural fatigue, shorten service life and cause noise and comfort problems; on the other hand, it contains weak mechanical energy that can be recovered and utilized. Vibration energy harvesting based on the piezoelectric effect can suppress structural vibration while converting mechanical vibration energy into electrical energy, thereby providing long-term, self-powered energy sources for wireless sensor nodes and low-power electronic devices, and thus serving as an important technical route to intelligent structures with “self-monitoring–self-sensing–self-powering” capabilities [2].

Among various structural forms, beam-type structures with bonded or embedded piezoelectric elements, especially cantilever beams [3], have been widely adopted due to their simple modelling, high tunability of parameters and ease of integration with existing engineering components. As research has progressed, beam-type piezoelectric energy harvesters have evolved from traditional straight beams to various topologies, including multi-degree-of-freedom, nonlinear and multi-directionally coupled configurations [4]. Previous studies have shown that such structures exhibit significant advantages in improving output power and extending working bandwidth. However, most existing investigations have focused on “maximizing energy output”, whereas systematic summaries of integrated design for “energy harvesting” and “vibration control” remain relatively limited, particularly in terms of unified evaluation and comparison of different beam configurations from the perspective of integrated performance.

In response to these issues, a review framework based on “beam structures + piezoelectric energy harvesting + vibration reduction” is adopted. Typical beam configurations and their applications in integrated energy harvesting–vibration reduction systems are examined, with emphasis on the geometric configurations of cantilever beams, L-shaped beams and U-shaped beams, as well as structural optimization strategies for prismatic and non-prismatic beams. The underlying mechanisms and performance characteristics are analyzed, and the key technical challenges and future research directions are discussed.

2. Fundamentals of Piezoelectric Energy Harvesting and Beam-Based Vibration Reduction

2.1. Piezoelectric Effect and Electromechanical Coupled Modelling

When subjected to external loads that induce elastic deformation, piezoelectric materials generate electric charges on the electrode surfaces, which is referred to as the direct piezoelectric effect [5]. Conversely, mechanical strain is produced when an external electric field is applied, which is known as the converse piezoelectric effect. Under small-strain conditions, piezoelectric materials can be described by linear piezoelectric constitutive relations, and their electro-mechanical coupling behaviour can be represented by a set of linear coupled equations among stress, strain, electric field and electric displacement [6].

In beam-type piezoelectric energy harvesters, piezoelectric layers are usually bonded or embedded in high-strain regions of metallic or composite beam substrates. By enforcing strain compatibility and internal force equilibrium, the piezoelectric layers and elastic substrate can be modelled as an electromechanically coupled beam with coupled stiffness and effective damping. For engineering analysis, Euler–Bernoulli or Timoshenko beam theory is commonly employed to model the beam body, and modal superposition or finite element methods are used to obtain equivalent mass, stiffness and damping parameters. These structural equations are then coupled with the governing equations of the external electrical circuit, such as resistive loads, rectifier circuits and energy storage circuits, to establish a complete electromechanical coupled dynamic model. In vibration reduction applications, the piezoelectric elements and their connected dissipative circuits can be regarded as "electromechanical dampers" in the sense of structural dynamics. When external excitation induces vibration of the beam, part of the mechanical energy is converted into electrical energy through electromechanical coupling and dissipated in the circuit, resulting in a reduction in structural vibration response; when the harvested electrical energy is stored or used to power loads, energy harvesting and vibration reduction can be realized simultaneously within the same system [7].

2.2. Vibration Reduction Mechanisms of Beam-Type Structures

The vibration reduction mechanisms of beam-type piezoelectric energy harvesting systems are mainly reflected in frequency tuning and modal control, nonlinear characteristics, and multimodal and multi-directional coupling [8]. By properly adjusting structural parameters such as beam length, cross-sectional shape and attached masses, the natural frequencies of the beam can be tuned to match the dominant excitation frequencies, so that efficient energy transfer and concentrated energy dissipation are achieved near resonance, leading to a significant reduction in the vibration amplitude of the protected structure. Furthermore, by exploiting geometric nonlinearity, externally applied magnetic fields, pre-buckled configurations or bi-/multi-stable structures, softening or hardening stiffness characteristics and multi-well potential landscapes can be introduced into the system, giving rise to rich nonlinear dynamic behaviour. In this way, large vibration responses can be maintained over a broader frequency range, and multiple energy dissipation paths can be formed through piezoelectric–circuit coupling, thereby enabling broadband operation.

Through structural designs involving multi-beam assemblies, branched beams and spatial bending–torsion coupling, multiple vibration modes can be excited simultaneously, which improves the adaptability of the system to multi-frequency and multi-directional excitations. This is beneficial not only for enhancing the capability of vibration energy harvesting, but also for achieving vibration reduction over multiple frequency bands. It is thereby indicated that the geometric topology and cross-sectional distribution of beams play a crucial role in exploiting these mechanisms. Targeted structural optimization and parameter design tailored to specific operating conditions are essential for improving the overall performance of beam-type piezoelectric energy harvesting and vibration reduction systems.

3. Research Progress on Beam-Type Piezoelectric Energy Harvesting–Vibration Reduction Systems

3.1. Cantilever Beam Structures

The cantilever beam is the most classical and widely used configuration for beam-type piezoelectric energy harvesting and vibration reduction, and has therefore occupied a dominant position in related research. In a conventional single cantilever beam, strain is highly concentrated near the clamped end, and piezoelectric patches are typically arranged in the high-strain region at the root to obtain large electrical output. However, such structures usually exhibit significant response only near the first bending mode, leading to a narrow operational bandwidth and strong sensitivity to excitation frequency variation. To overcome these limitations, modifications have been proposed in recent years from the perspectives of multimodal and multi-beam design, local stiffness control and surface constraints, as well as flow-induced vibration and multi-physics coupling.

By introducing dual cantilevers or arrays of multiple cantilevers, coordinated participation of multiple low-order modes can be realized, so that prominent vibration responses are produced in several frequency regions. In this manner, the capability of capturing multi-frequency vibration energy is enhanced and the problem of amplitude concentration associated with a single resonance peak is alleviated [9]. By locally introducing stiffness discontinuities, thickness variations or surface constraints into the beam, the strain distribution along the beam length can be reshaped, thereby expanding the high-strain region, improving the utilization of piezoelectric material, increasing overall energy harvesting efficiency and enhancing vibration energy dissipation. In fluid environments, combinations of cantilever beams with tails, cylinders and other flow-sensitive structures can be adopted to induce vortex-induced vibration or flutter at relatively low flow velocities. By placing piezoelectric elements in high-response regions, flow-induced vibration energy can be effectively harvested, while appropriate structural and circuit designs contribute to the reduction of vibration levels of pipes or other structural components. In general, straight cantilever beams retain the advantages of simple modelling, convenient fabrication and easy engineering integration; however, inherent limitations in low-frequency adaptability and broadband response have motivated the continuous development and evolution of various new beam configurations.

3.2. L-Shaped Beam Structures

The L-shaped beam is obtained by adding a vertical branch to a straight beam, forming a folded configuration. This geometric feature significantly modifies the stiffness and mass distribution of the system, reduces the effective natural frequencies, and introduces bending–torsion coupling and internal resonance, thereby providing unique advantages for low-frequency and broadband energy harvesting and vibration reduction [10]. In early studies on vibration energy harvesting for unmanned aerial vehicles, electromechanically coupled models of L-shaped beams with tip masses were established, and the energy conversion mechanisms under specific internal resonance conditions were analyzed. The results demonstrated that, by virtue of internal resonance and multimodal coupling, L-shaped beams can maintain relatively high electrical output over a wide frequency range. Subsequently, a large number of broadband L-shaped piezoelectric energy harvesters based on 1:2 or 2:1 internal resonance have been proposed, and theoretical as well as experimental investigations have consistently indicated that, compared with linear straight beams, L-shaped beams can achieve significant improvements in energy harvesting efficiency and effective bandwidth.

In vibration reduction applications, combinations of L-shaped beams with bi-stable structures have been developed, where suitable geometric arrangements and pre-stress are employed to introduce double-well potentials. Under external excitation, the system exhibits snap-through motion between two stable equilibrium positions, thereby simultaneously enhancing energy dissipation and limiting the vibration amplitude. Experimental results have shown that such structures can markedly reduce peak vibration levels within the target frequency band. For ultra-low-frequency environments such as ocean waves, the natural frequencies of the system can be lowered by the L-shaped configuration and combined with other energy conversion mechanisms such as electromagnetic induction, thus forming hybrid piezoelectric–electromagnetic energy harvesting and vibration reduction devices. These devices are capable of improving overall energy utilization efficiency and at the same time suppressing low-frequency vibrations of platforms or structures subjected to wave excitation.

3.3. U-Shaped Beam Structures

U-shaped beams consist of a main beam and two side beams, and can be regarded as a combined configuration of bending and multiple beams. This topology exhibits remarkable performance in broadband, multimodal and multi-directional responses and is therefore suitable for energy harvesting and vibration reduction in multi-directional vibration environments [11]. Studies on multimodal bidirectional U-shaped piezoelectric energy harvesters have shown that, through the coupling between the main and side beams, multiple bending modes can be excited over a wide frequency range, enabling sensitive responses to both horizontal and vertical vibrations. Under identical material and volume constraints, U-shaped beams generally exhibit superior broadband characteristics and multi-directional energy harvesting capacity compared with L-shaped beams, whereas L-shaped configurations are more flexible in achieving ultra-low-frequency tuning and internal resonance design.

In fluid environments, U-shaped oscillators can be installed inside pipelines or coupled with fluid–structure systems. By optimizing geometric parameters and placement positions of the U-shaped bodies, flow-induced vibration amplitudes can be significantly amplified, which leads to increased harvestable piezoelectric energy and enhanced vibration dissipation. In practical vibration control, arrays of U-shaped beams can be used as multi-degree-of-freedom vibration absorbers and mounted at critical locations of host structures. By appropriately designing the parameters of such arrays, vibration suppression at multiple target frequency bands can be achieved. Experimental investigations have demonstrated that properly arranged U-shaped piezoelectric beam arrays can substantially reduce the acceleration response of the protected structure, while simultaneously providing electrical power output of engineering significance.

3.4. Trapezoidal, Tapered and Other Non-Prismatic Beam Structures

Compared with traditional prismatic beams, trapezoidal beams [12], tapered beams and other non-prismatic beams with varying cross-sectional width or thickness along the length [13] enable “customized design” of stiffness and strain distributions, thereby significantly improving the effective utilization of piezoelectric materials and energy conversion efficiency. For trapezoidal hollow cantilever beams, finite element simulations combined with experimental optimization of cross-sectional parameters have indicated that, under comparable total volume, the output power of trapezoidal-section beams is notably higher than that of rectangular prismatic beams. For tapered beams and more general non-prismatic beams, theoretical analyses based on polynomial cross-section variation functions have revealed that appropriate combinations of tapering and trapezoidal profiles can increase average strain levels while satisfying geometric constraints, thus achieving improved output performance.

From the standpoint of vibration reduction, non-prismatic beams can be utilized to control local mode shapes. For instance, by reducing the cross-sectional size at specific locations, local strain concentration can be induced to efficiently absorb vibration energy in designated regions. Cross-sectional discontinuities can also introduce nonlinear stiffness characteristics, which help extend the effective working bandwidth of the system and limit peak vibration amplitudes. Recent studies have further shown that joint optimization of the geometric profile of cantilever-like structures, including cross-sectional contours and segment lengths, can significantly improve both energy harvesting and vibration reduction performance without considerable mass increase [14].

3.5. Complex and Hybrid Beam Structures

In addition to simple geometric forms, various complex or hybrid beam structures have been proposed to further exploit dynamic characteristics and enhance functional integration. By attaching branch beams or auxiliary oscillators to a main beam, multi-degree-of-freedom systems can be constructed [15], which exhibit multiple resonance peaks over different frequency regions and thus enable wideband energy harvesting and vibration control. Bending–torsion coupled beams formed by asymmetric cross-sections and three-dimensional arrangements can simultaneously excite bending and torsional modes under external excitations, significantly enhancing the sensitivity of the system to multi-directional loading.

Through the incorporation of magnetic fields, shape memory alloy elements or electromagnetic induction devices into beam structures, multi-physics coupled systems with bi-stable or multi-stable potentials can be realized [16]. In large-amplitude vibration environments, these systems can achieve markedly higher energy harvesting efficiency while providing strong vibration suppression. Overall, such complex and hybrid beam configurations demonstrate that beam-type structures still possess substantial potential in topological innovation and functional integration, and offer new structural platforms and design concepts for building integrated systems with high-efficiency energy harvesting and high-performance vibration reduction.

4. Performance Comparison and Application Analysis of Different Beam Types

From the perspective of integrated energy harvesting and vibration reduction, distinct advantages and limitations are exhibited by different beam configurations due to their respective topologies and dynamic characteristics, and consequently they are suited to different application scenarios. Straight cantilever beams are characterized by simple topology and a small number of design parameters, which allows classical beam theory to be conveniently employed for modelling and analysis. Fabrication and assembly processes are relatively straightforward, and integration with conventional dampers and various engineering structures can be readily achieved. Therefore, this configuration is the most widely used in current engineering practice. However, the operational bandwidth of straight cantilever beams is generally narrow, and their response is highly sensitive to excitation frequency drift. In the absence of geometric or material optimization, strain tends to be concentrated in local regions, resulting in insufficient utilization of piezoelectric materials. Consequently, such structures are more suitable for engineering systems where excitation frequencies are relatively stable, structural space is limited and high accuracy of the analytical model is required.

L-shaped beams, relying on their folded geometries, can achieve low-frequency tuning at the structural level and obtain broadband responses through internal resonance and nonlinear effects. Strong energy capture capability for low-frequency and large-amplitude vibrations is thereby provided, which is advantageous in applications such as bridges, vehicles and marine platforms where low-frequency and broadband vibrations dominate. Nevertheless, the relatively large number of structural parameters leads to complicated design and optimization, and in multi-directional vibration fields, the response distribution is highly sensitive to geometrical details and mass allocation, which imposes higher requirements on analysis and fabrication. U-shaped beams, which are configured as frame-like structures consisting of a main beam and side beams, naturally possess multimodal and multi-directional response capabilities. These structures can realize multi-band energy harvesting and vibration reduction under complex multi-directional excitations, and are suited to engineering scenarios such as machinery foundations and pipeline systems where multi-directional excitations are pronounced. Furthermore, U-shaped beams generally exhibit better broadband and multi-directional adaptability than L-shaped beams, but demand higher precision in manufacturing, assembly and structural symmetry.

Trapezoidal, tapered and other non-prismatic beams achieve refined control of stiffness and strain distributions through cross-section variation. Under constraints of limited volume and mass, such configurations can greatly increase the utilization of piezoelectric materials and improve frequency tuning characteristics, thereby enhancing both energy density and vibration attenuation efficiency. These beams are appropriate for applications requiring high energy density, superior vibration reduction performance and structural lightweighting, where more complex design and fabrication processes are acceptable. At the same time, a larger number of design parameters increases the difficulty of theoretical analysis and manufacturing, and higher requirements are imposed on cross-sectional accuracy and material uniformity. In summary, cantilever, L-shaped and U-shaped beams constitute the basic topological framework of current beam-type piezoelectric energy harvesting–vibration reduction systems, while trapezoidal, tapered and other non-prismatic designs provide geometric optimization tools for further performance enhancement. Through rational selection and combination of different beam types and cross-sectional forms at the system design stage, together with multi-objective parameter and topology optimization in accordance with target operating conditions, coordinated improvements in energy harvesting efficiency and vibration reduction performance can be achieved in a wider range of applications.

5. Key Technical Issues and Challenges

Despite considerable theoretical and experimental achievements in beam-type vibration reduction technologies based on piezoelectric energy harvesting, several key challenges remain for engineering application and large-scale deployment. First, electromechanical nonlinear modelling requires further development. Under strong excitation and broadband operating conditions, the effects of nonlinear stiffness, electrical nonlinearities introduced by rectification and power management circuits, and frequency- or amplitude-dependent damping on system dynamic response cannot be neglected. Existing models often fail to strike an appropriate balance between modelling accuracy and computational complexity, and a unified nonlinear electromechanical coupling framework that is both sufficiently general and convenient for engineering design and rapid prediction remains to be established.

Second, system robustness in broadband and multi-directional vibration environments is still inadequate. In practical engineering, vibration excitations typically exhibit frequency drift, superposition of multiple frequency components and multi-directional coupling. Single-topology or fixed-parameter beam-type structures generally cannot maintain high energy harvesting efficiency and good vibration reduction performance over all operating conditions. The realization of adaptive adjustment of modal characteristics and energy distribution at the structural level, enabling dynamic reconfiguration of effective modes and response paths in accordance with varying operating conditions, remains one of the primary research difficulties.

Third, co-optimization of vibration reduction performance and energy harvesting efficiency remains non-trivial. Many existing designs place greater emphasis on enhancing electrical output, while systematic evaluation of the vibration response of the protected structure is comparatively insufficient. In practical applications, vibration control and energy harvesting objectives often compete with or even conflict with each other. Consequently, multi-objective optimization frameworks are required in which vibration indices (such as displacement and acceleration), energy harvesting indices (such as average power and energy density) and engineering constraints (including mass, volume and cost) are incorporated simultaneously to achieve genuine performance synergy and global optimality.

In addition, long-term reliability and encapsulation design are critical constraints on engineering application. During long-term service, piezoelectric elements may suffer from fatigue cracking, interfacial debonding and depolarization, while environmental factors such as humidity–temperature cycling, temperature fluctuations and corrosive media can deteriorate structural and interfacial integrity, leading to performance degradation or even failure. This issue is particularly critical for harsh service environments such as marine structures and oil–gas pipelines, where high-reliability packaging, protection and maintainable structural integration of piezoelectric and circuit modules are essential for practical implementation.

Finally, current technologies still exhibit deficiencies in standardization and system integration. Most studies remain at the prototype or small-scale validation stage, and unified structural design codes, experimental procedures and performance evaluation standards have not yet been established. The integration level with sensing, communication and energy management modules is relatively low, and system-level design and engineering packaging solutions are limited. These factors have significantly restricted the transition of beam-type piezoelectric energy harvesting–vibration reduction technologies from laboratory prototypes to mature engineering products.

6. Future Trends and Prospects

Considering the existing research foundation and future application demands, beam-type vibration reduction technologies based on piezoelectric energy harvesting are expected to evolve along several directions, including multi-level and multi-scale topology optimization, unified multimodal and multi-directional design, integration of smart materials with tunable structures, and system-level co-design of structure, circuits and energy management. At the structural level, different beam types such as cantilever, L-shaped, U-shaped and tapered beams can be combined with various cross-sectional forms, including rectangular, trapezoidal and hollow sections, as well as array layouts. Topology optimization carried out at multiple scales—from global structure to substructures and material level—has the potential to produce “customized” beam-type energy harvesting–vibration reduction devices tailored to specific vibration and environmental conditions.

At the dynamical and control level, multi-degree-of-freedom systems and bending–torsion coupled structures can be exploited. Multiple low-order modes and multi-directional responses can be taken into account simultaneously during the design phase, and appropriate matching of modal frequencies and damping ratios can be employed to achieve robust energy harvesting and vibration control in broadband and multi-directional vibration environments. Existing L-shaped and U-shaped beam configurations can be further extended to three-dimensional beam-lattice structures, thereby forming more complex and tunable dynamical characteristics.

At the material and structural levels, integration of piezoelectric materials with smart materials such as electroactive polymers, magnetostrictive materials and phase-change materials can be implemented to develop beam-type structures with tunable stiffness and damping. In this way, natural frequencies and damping ratios can be adjusted online to accommodate varying operating conditions, so that near-optimal energy harvesting and vibration reduction performance can be maintained over a relatively wide range.

At the system level, design concepts are expected to shift from isolated structural optimization to fully integrated “structure + circuits + energy management” co-design. Simultaneous optimization of beam structural parameters, rectification and power management circuits, and energy storage and load strategies can maximize energy utilization and vibration attenuation at the system scale. In parallel, long-term demonstration projects in representative engineering scenarios—such as bridge health monitoring, vibration reduction in rail transit, fault monitoring of rotating machinery and vibration control of marine engineering structures—combined with the establishment of comprehensive design workflows, experimental methodologies and evaluation metrics, are expected to promote standardization and engineering deployment of these technologies.

7. Conclusion

A systematic review has been presented on vibration reduction technologies of beam-type structures based on piezoelectric energy harvesting, with emphasis on the theoretical foundations and research progress of cantilever beams, L-shaped beams, U-shaped beams and non-prismatic beams such as trapezoidal and tapered configurations. The analysis shows that the cantilever beam remains the fundamental configuration in current beam-type piezoelectric energy harvesting–vibration reduction systems due to its simple structure, convenient modelling and ease of integration, although multimodal design, local stiffness control and geometric optimization are necessary to further improve performance under low-frequency and broadband vibration conditions. L-shaped beams, exploiting internal resonance and geometric nonlinearity, exhibit outstanding characteristics for low-frequency broadband energy harvesting and vibration reduction and constitute an effective structural solution for low-frequency vibration environments. U-shaped beams, relying on inherent multimodal and multi-directional response capabilities, are more suitable for complex vibration scenarios in which multi-band energy harvesting and vibration control are required. Trapezoidal, tapered and other non-prismatic beams can substantially enhance the utilization of piezoelectric materials through refined control of stiffness and strain distributions, thereby improving energy density and vibration attenuation performance without significant mass increase.

Complex hybrid beam configurations and multi-stable structures provide new platforms for achieving high performance and multifunctional integration, yet further research is needed in nonlinear modelling, long-term reliability and engineering implementation. Overall, with continuous advances in smart materials, topology optimization techniques and multi-physics simulation tools, beam-type integrated piezoelectric energy harvesting and vibration reduction technologies are expected to find increasingly widespread and in-depth applications in high-reliability self-powered monitoring, intelligent infrastructure and marine engineering.

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