Teaching Design of Bayes’ Formula Based on Kolb's Learning Cycle Theory

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Abstract: This paper uses Kolb's learning cycle theory to develop an instructional design based on the Bayes’ formula for experiential learning. Students experience in their own trustworthiness calculations that trustworthiness declines rapidly after lying. Through reflection and observation, they understand how the Bayes’ formula uses information to correct the probability, and make students pay attention to collecting more information in life in order to obtain more accurate decisions. Experiential learning process improves students' understanding and application of Bayes’ formula.

Keywords: Experiential learning, Bayes’ formula, Kolb's Learning Cycle Theory, Teaching design.

1. Introduction

In the 18th century, the British scholar Thomas Bayes discovered the Bayes’ formula. By adding new information, it can be used to analyze how people's understanding of things transitions from prior probability to posterior probability [1]. When a stock falls one day, which factor is most likely to cause it? When a student loses his trust, how much will his personal integrity be affected? The core content of Bayesian statistics is the Bayes’ formula and the Bayesian inference and prediction theory. In terms of teaching content, the course focuses on theoretical knowledge and the understanding and application of relevant conclusions. From the perspective of teaching effect, it is difficult for some students to learn. Therefore, how to teach the theoretical knowledge is the key and difficult point in the teaching process.

The existing teaching discussion on Bayes’ formula is mainly carried out from the aspects of heuristic, inquiry and project-driven. Chen Zhongming used heuristic teaching method to discuss the teaching design of Bayes’ formula from the aspects of formula introduction, understanding and application [2]. Li Li introduced the inquiry-based learning method to the flipped classroom, where teachers create teaching situations and guide students to discover the objects of inquiry [3]. Zhang Fang, Wang Rong and Liang Huadong adopted the micro-project-driven teaching method, integrating teaching content and knowledge points into the project to guide students to learn independently[4]. Su Jialin, Hu Zhen and Chang Zaibin derived the idea of Bayes’ formula from examples [5]. These methods increase students' understanding of knowledge from different perspectives, but in order to achieve students' knowledge transfer, so that students can use previous learning to solve new problems, the teaching exploration of Bayes’ formula can be carried out through experiential learning[6].

Experiential learning involves engaging students in exploring issues that they find relevant and meaningful. Teachers act as facilitators to convince students that valid and meaningful conclusions can be drawn from their own experiences. This way of learning has been shown to be more rewarding than just listening to a teacher [7].

The representative of experiential learning theory is Kolb's learning cycle theory proposed by Kolb, which includes four stages of learning process: concrete experience, reflective observation, abstract generalization, and active inspection [8]. This theory specifically refers to that learners gain direct experience in practice; then, through thinking and reflection on direct experience, they explore the difference and connection between old and new experience; and then reflect and observe the results to form a logical concept And abstract the laws; finally, verify these concepts and laws in new scenarios, and use them to solve problems and formulate strategies [9].

The application of Kolb's learning cycle theory to this case is students use the Bayes’ formula to experience their own integrity, and through reflection and observation on the phenomenon of rapid decline in integrity, they summarize the negative impact of untrustworthiness on individuals, and How the Bayes’ formula corrects the probability through new information, and uses this method to actively test the events in daily life to improve the accuracy of decision-making.

We applies the Kolb's learning cycle theory to explain the Bayes’ formula based on experiential learning, further analyzes the impact of personal behavior on credit, guides students to establish correct values, and pays attention to the corrective effect of information on probability.

2. Instructional Design

Bayes’ formula belongs to the course of "Probability Theory and Mathematical Statistics". Its teaching goal is to master Bayes’ formula, understand prior probability and posterior probability, and improve students' ability to analyze problems. Before the course starts, we send the teaching objectives, teaching points and difficulties of this class to the students, so that the students can clarify the learning requirements and procedures, and guide the students. In class, students take part in a questionnaire, then explain the concepts, and then analyze the results of the students’ own questionnaires, draw conclusions and summarize and reflect, further emphasizing the theme of this class.
3. Teaching Process Based on Kolb’s Learning Cycle Theory Design

3.1. Concrete Experience

We use questionnaires to investigate the integrity of our students. The topic was "How many times have you broken your trust in the last week?" In order to improve the authenticity of the data, anonymously will be emphasized.

Table 1. Survey results

<table>
<thead>
<tr>
<th>Untrustworthy times</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>≥3</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>78%</td>
<td>19%</td>
<td>3%</td>
<td>0</td>
</tr>
</tbody>
</table>

3.2. Reflective Observation

Understanding the Bayes’ formula requires the use of Conditional Probability Formula and the Total Probability Formula. Therefore, on the basis of reviewing the relevant knowledge points, the Bayes’ formula is explained.

Conditional probability: If A and B are two events associated with the same sample space of a random experiment, provided \( P(A) > 0 \), the conditional probability of event B given that A has occurred is given by

\[
P(B | A) = \frac{P(AB)}{P(A)} \tag{1}
\]

Total Probability Formula: assumed \( B_1, B_2, ..., B_n \) to be a list of mutually incompatible events, \( \bigcup_{i=1}^{n} B_i = \Omega \) and \( P(B_i) > 0 \) \( (i = 1, 2, ..., n) \), then

\[
P(A) = \sum_{i=1}^{n} P(B_i)P(A | B_i) \tag{2}
\]

On the basis of understanding conditional probability and total probability formula, we can better understand the Bayes’ formula associated with them.

Bayes’ formula is a way of finding a probability when certain other probabilities are known.

\[
P(B_i | A) = \frac{P(AB_i)}{P(A)} = \frac{P(B_i)P(A | B_i)}{P(A)} \tag{3}
\]

Substituting the total probability formula into the denominator of the formula (3):

\[
P(B_i | A) = \frac{P(AB_i)}{P(A)} = \frac{P(B_i)P(A | B_i)}{\sum_{i=1}^{n} P(B_i)P(A | B_i)} \tag{4}
\]

\( P(B_i) = \) how likely \( B_i \) happens (Prior knowledge);

\( P(A) = \) how likely \( A \) happens;

\( P(B_i | A) = \) how likely \( B_i \) happens given that \( A \) has happened (Posterior knowledge);

\( P(A | B_i) = \) how likely \( A \) happens given that \( B_i \) has happened.

Understanding Prior and Posterior Probabilities with Bayes’ Formula. Prior probability is the probability of an event based on established knowledge, before empirical data is collected. Posterior Probability is the revised probability of an event occurring after taking into consideration new information. It is the probability of Bi calculated after knowing the information A.

Taking student Zhang as an example, according to the number of dishonest behaviors he filled in within a week, how did Zhang’s credibility change?

Analysis: Record A as "Zhang lied", B as "Zhang is trustworthy". Assume that everyone's impression of college students in the past is \( P(B) = 0.9, P(\bar{B}) = 0.1 \) (the prior probability of a student’s credibility is 0.9). Assuming the possibility of a credible student lying is \( P(A | B) = 0.05 \), the possibility of an unreliable student lying \( P(A | \bar{B}) = 0.6 \), analyze the change in the credibility of Zhang after lying once. According to the meaning of the question

\[
P(B | A) = \frac{P(AB)}{P(A)} = \frac{P(B)P(A | B)}{P(AB) + P(\bar{B})P(A | \bar{B})} = 0.429
\]

Zhang’s credibility was adjusted from 0.9 to 0.429. It can be seen that after he lied once, his credibility dropped by more than half.

If Zhang has lied twice in the questionnaire, his credit has been renewed.

Since the prior probability has been corrected by the information of the first lie in the second calculation, the impression of the person has been corrected, the new prior probability is \( P(B) = 0.429 \).

\[
P(B | A) = \frac{P(AB)}{P(A)} = \frac{P(B)P(A | B)}{P(AB) + P(\bar{B})P(A | \bar{B})} = 0.059
\]

When Zhang loses his trust twice, the trust level of the insider in him is only 0.059 (new posterior probability), which is significantly lower than the prior probability of 0.429 at this time. When he broke his trust twice, the trust level of insiders dropped from 90% to less than 1%. Although this decline is related to the probability we assume, such a rapid decline also shows that it will be difficult for Zhang to gain the trust and cooperation opportunities of insiders. Bayes’ formula can quickly correct the probability of the Zhang’s credibility through the new information such as the Zhang’s lying.

Then each student analyzed the impact of dishonest behavior on himself based on the number of dishonest behaviors reported, guide students to understand the importance of maintaining honesty.

3.3. Abstract Generalization

Ask students to complete the following story on how Bayes’ formulas can be used to explain these events from a probabilistic perspective.

(1) In Aesop’s fable “The Wolf is coming”. The child lied that the wolf was coming. After the villagers were deceived twice, the wolf really came for the third time. Why didn't the villagers come to help fight the wolf?
(2) In order to please the beautiful Bao Si, King You of Zhou in ancient China lit lamps and beacon fires again and again to amuse the princes. Why did the princes not believe in the beacon, and gradually stopped coming? It should be understood that, whether it is a country or an individual, malicious deception will lead to a decline in its own credit rating, resulting in serious negative effects.

(3) Monty hall problem. There are three closed doors; behind the doors are a car, a sheep, and a sheep. The player wants to get a car. After the player chooses a door, the host opens it and finds that it is a sheep, and then asks the player if he wants to change his choice. That is, before and after changing the choice, will the probability of winning the lottery change?

3.4. Active Inspection

Using the Bayes’ formula, by adding new information, the prior probability can be modified to obtain a more accurate posterior probability. Remind students to collect as much information as possible when encountering problems in the future, and constantly revise the probability in order to make more correct judgments. Think twice before doing anything!

Guide students to seek truth from facts. When you act dishonestly, you can have serious consequences for yourself. Issues such as lateness, test integrity, credit card repayments, etc. can have a lasting impact on an individual, so take your reputation seriously.

4. Discussion

This article is based on the experiential learning of Kolb's learning cycle theory. Starting from the survey, it analyzes the changes in each person's integrity and helps students understand the operation principle of Bayes’ formula through the experience of the rapid decline of integrity, remind students to pay attention to personal credit, and to draw inferences from one case to another when encountering similar problems.

References

[1] Li Yiqing. The application of Bayes’ formula [J]. Science and Education Wenhui (Early Issue), 2019(11):75-76.


