

Modeling Research and Analysis of Multibeam Bathymetric Systems

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Abstract: This paper conducted research on the application of multi beam measurement system for underwater detection. Firstly, a mathematical model was established and determined for the coverage width of multi beam bathymetry and the overlap rate between adjacent bands; Secondly, based on the rectangular sea area to be measured, a mathematical model for the coverage width of multi beam bathymetry was established. Finally, in order to meet the requirement of 10% to 20% overlap between adjacent bands, a set of survey lines with the shortest measurement length and capable of fully covering the entire sea area to be measured was further studied and designed.

Keywords: Multi beam bathymetry; Coverage width; Overlap rate.

1. Introduction

The multi beam bathymetric system is a widely used water depth measurement system in recent years, which has the characteristics of high resolution, high accuracy, large coverage range, automated mapping, and high efficiency [1]. The application of this model can conveniently, efficiently, and accurately detect the spatial position, size, and elevation of underwater objects for coverage detection, and has good application prospects in the field of underwater engineering detection [2].

This paper focuses on the difficulties in detecting underwater spatial information caused by the complex underwater environment, as well as the low efficiency and accuracy of single beam survey lines. It conducts research on the application of multi beam measurement systems for underwater detection. Firstly, a mathematical model was established for the coverage width of multi beam bathymetry and the overlap rate between adjacent bands, and relevant indicator values were calculated; Secondly, based on a rectangular sea area to be measured, a mathematical model for the coverage width of multi beam bathymetry was established, and the coverage width of multi beam bathymetry at the specified location was calculated; Finally, in order to meet the requirement of 10% to 20% overlap between adjacent bands, a set of survey lines with the shortest measurement length and capable of fully covering the entire sea area to be measured was further studied and designed.

2. Model establishment and solution

2.1. Mathematical model for coverage width and overlap rate between adjacent bands in multi beam bathymetry

Based on the seabed depth variation map shown in Figure 1, the intersection line between the survey line direction and the seabed slope forms a diagonal line with an angle of α to the horizontal plane, called α as the slope.

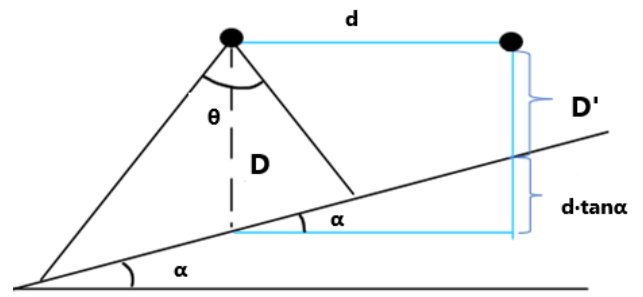


Figure 1. The seabed depth variation map

From the analysis in Figure 1, it can be concluded that the mathematical model of the seawater depth measured by the second measuring line can be expressed as:

$$D' = D - d \tan \alpha \quad (1)$$

Where, d is the distance from the center point of the sea area.

If we assume that the opening angle of the multi beam transducer is 120° , the slope is 1.5° , and the depth D of the seawater at the center point of the sea area is 70m , based on the total coverage width map of a single measurement point shown in Figure 2, we can obtain:

$$\begin{cases} \angle 1 = 30^\circ - \alpha = 28.5^\circ \\ \angle 2 = 30^\circ + \alpha = 31.5^\circ \end{cases} \quad (2)$$

From the sine theorem, it also can be concluded that:

$$\begin{cases} \frac{W_1}{\sin 60^\circ} = \frac{D}{\sin \angle 1} \\ \frac{W_2}{\sin 60^\circ} = \frac{D}{\sin \angle 2} \end{cases} \quad (3)$$

By analyzing the triangle beneath the seabed shown in Figure 2, the mathematical model of the total coverage width can be obtained:

$$W_{\text{总}} = (W_1 + W_2) \cos \alpha = D \sin 60^\circ \cos \alpha \left(\frac{1}{\sin \angle 1} + \frac{1}{\sin \angle 2} \right) \quad (4)$$

In the formula: $W_{\text{总}}$ represents the total coverage width of the detection area at this point.

Further, through the schematic diagram of adjacent lines shown in Figure 3, the coverage model of the previous line can be obtained as follows:

$$\frac{x}{\sin 150^\circ} = \frac{d}{\sin \angle 1} \Rightarrow X = \frac{d}{\sin \angle 1} \sin 150^\circ \quad (5)$$

Therefore, the overlapping part is: $a_1 - X$, the overlap rate with the previous line is $\frac{a_1 - X}{a_2} \%$.

Based on the mathematical model established above, the relevant indicator values were calculated in this section, as shown in Table 1.

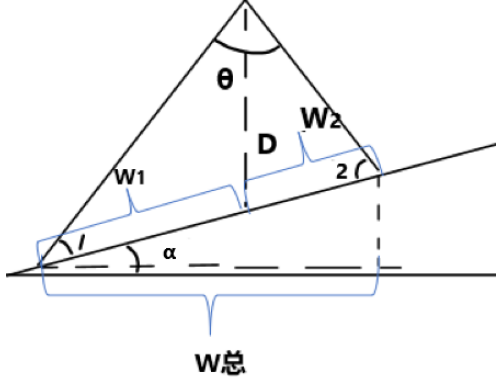


Figure 2. Total coverage width of a single measurement point

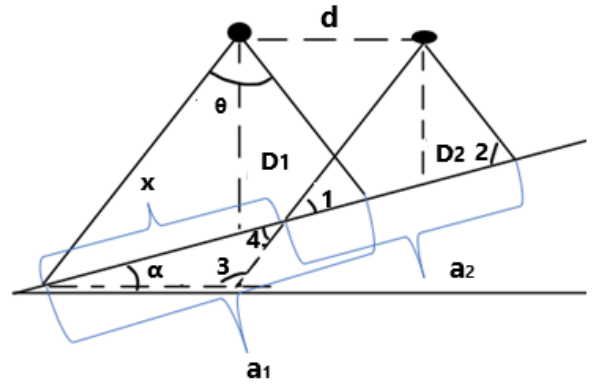


Figure 3. Schematic diagram of adjacent survey lines

Table 1. Calculation Results of Relevant Indicators

The distance between the measuring line and the center point/m	-800	-600	-400	-200	0	200	400	600	800
Depth of seawater/m	90.95	85.71	80.47	75.24	70.00	64.76	59.53	54.29	49.05
Coverage width/m	315.82	297.62	279.43	261.27	243.27	224.87	206.71	188.52	170.32
Overlap rate with the previous line/%	—	32.8	28.43	23.45	17.72	11.06	3.25	-6.09	-17.43

From Table 1, it can be seen that as the distance between the measuring line points and the center increases, the depth and coverage width of seawater show a decreasing trend, and the overlap rate with the previous measuring line also gradually decreases.

2.2. Mathematical model for coverage width of multi beam bathymetry

The line direction section schematic is shown in Figure 4.

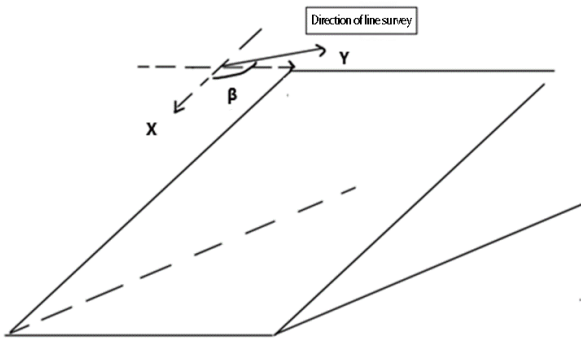


Figure 4. Exploded schematic diagram of line direction

Consider a rectangular sea area to be surveyed, where the angle between the direction of the survey line and the normal direction of the seabed slope projected on the horizontal plane is β . When moving, we decompose it into moving in the x and y directions [3]. The projection direction of the x-direction and the normal direction of the seabed slope on the horizontal plane is the same, and the y-direction is perpendicular to the x-direction. From the previous section, it can be seen that when moving in the y-direction, the seawater

depth of the measuring vessel remains unchanged. When moving in the x-direction, the seawater depth can be expressed as:

$$D = D_0 + s \cos \beta \tan \alpha \quad (6)$$

In the formula, s is the distance/nautical mile between the measuring vessel and the center point of the sea area.

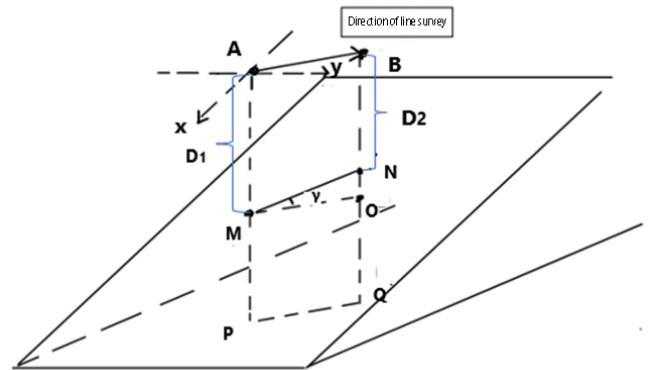


Figure 5. Schematic diagram of slope γ

Furthermore, the mathematical model of slope angle γ can be expressed as:

$$\tan \gamma = \frac{1}{\cos \beta \tan \alpha} \Rightarrow \gamma = \tan^{-1} \frac{1}{\cos \beta \tan \alpha} \quad (7)$$

Among them, ON represents the depth of seawater that changes when the measuring vessel moves on the measuring line; MO is parallel to the direction of the survey line, with a length of s at the center point of the sea area; MN is the straight line passing through the direction of the survey line

and perpendicular to the horizontal plane intersecting the seabed slope surface; Therefore, the angle between MN and MO is the calculated slope angle γ .

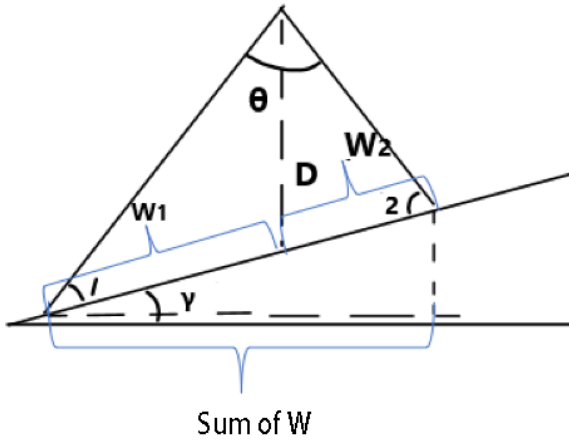


Figure 6. Schematic diagram of total coverage

If the opening angle of the multi beam transducer is assumed to be 120° , the slope angle γ is 1.5° , and the depth D of the seawater at the center point of the sea area is 120m, then based on the schematic diagram of total coverage shown in Figure 6, we can obtain:

$$\begin{cases} \angle 1 = 30^\circ - \gamma = 28.5^\circ \\ \angle 2 = 30^\circ + \gamma = 31.5^\circ \end{cases} \quad (8)$$

Therefore, the model for solving the total coverage width of slope angle γ is represented as:

$$W = D \sin 60^\circ \left(\frac{1}{\sin(30^\circ - \gamma)} + \frac{1}{\sin(30^\circ + \gamma)} \right) \cos \gamma = (D_0 + s \cos \beta \tan \alpha) \sin 60^\circ \left(\frac{1}{\sin(30^\circ - \gamma)} + \frac{1}{\sin(30^\circ + \gamma)} \right) \cos \gamma \quad (9)$$

Based on the mathematical model established above, the relevant indicator values were calculated in this section, as shown in Table 2:

Table 2. Calculation results of relevant indicators

Coverage width /m		Measure the distance between the ship and the center point of the sea area / mile							
		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
Angle between measuring line directions / $^\circ$	0	416.55	467.05	517.56	568.06	618.56	669.06	719.57	770.07
	45	416.12	451.79	487.47	523.14	558.82	594.49	630.16	665.84
	90	415.69	415.69	415.69	415.69	415.69	415.69	415.69	415.69
	135	416.12	380.45	344.77	309.10	273.42	237.75	202.08	166.40
	180	416.55	366.05	315.54	265.04	214.54	164.04	113.53	63.03
	225	416.12	380.45	344.77	309.10	273.42	237.75	202.08	166.40
	270	415.69	415.69	415.69	415.69	415.69	415.69	415.69	415.69
	315	416.12	451.79	487.47	523.14	558.82	594.49	630.16	665.84

2.3. Research on the Method of Line Layout

In combination with the reference to parallel layout and the shortest possible total length of survey lines in the "Research on Line Layout Method of Multi beam System in Yangtze River Channel Measurement" (1006-7973 (2017) 01-0052-04)[4], it is concluded that this set of north-south survey lines meets the shortest measurement length, can fully cover the entire sea area to be measured, and the overlap rate between adjacent strips meets the requirement of 10% to 20%.

(1) Parallel deployment

The main survey lines within the survey area should be in a parallel relationship, and the direction of the survey lines should be consistent with the direction of the isobaths. This can maximize the coverage of the river bottom and maintain the effective scanning width within a certain range.

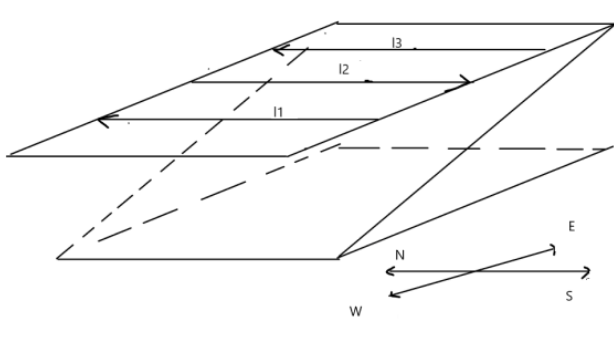


Figure 7. Schematic diagram of line group direction

As shown in Figure 7, the depths of lines I1, I2 and I3 in the north-south direction are consistent, and the isobath direction

is the north-south direction in Figure 7. It can be determined that the direction of the line group is north-south.

(2) The total length of the measuring line should be as short as possible

The total length of the measuring line should be as short as possible. The total length of the measuring line is one of the standards for measuring the complexity of the measurement work. On the premise of meeting the measurement requirements, the total length of the measuring line should be kept as short as possible. The cross-section of the Yangtze River waterway is in a "U" shape, and the water depth is characterized by "deep in the middle and shallow on both sides". The layout of multi beam survey lines should fully utilize the characteristics of water depth distribution, and the spacing between survey lines may not be equal. The survey lines in areas with deeper water depths can be slightly sparse, which is manifested in the practical application of the Yangtze River waterway: the survey lines at the center of the Yangtze River are slightly sparse, and the areas close to both banks are slightly dense. Although the spacing between survey lines in areas with deeper water depths can be appropriately increased, it cannot exceed 80% of the effective survey width to ensure 10% coverage of adjacent survey lines. Reasonable setting of survey line density can reduce the number of survey lines to a certain extent, shorten the total length of survey lines, avoid blind spots in scanning and avoid excessive overlap.

Due to the same depth of the seabed in the same survey line when running north-south, it can be transformed into a cross-sectional view, as shown in Figure 8:

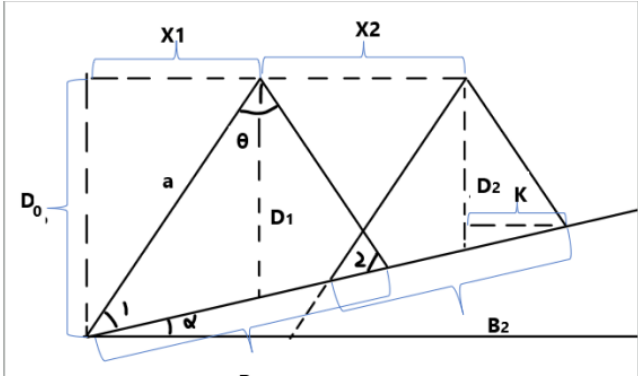


Figure 8. Cross Section of the Line Group

According to Figure 8, it can be concluded that:

$$\begin{cases} \angle 1 = 30^\circ - \alpha = 28.5^\circ \\ \angle 2 = 30^\circ + \alpha = 31.5^\circ \end{cases} \quad (10)$$

Due to the depth of 110m at the center of the sea area, it is shallow in the west and shallow in the east. Therefore, the depth of seawater in the west is:

$$D_0 = 110 + 2 \times 1852 \times \tan 1.5^\circ \quad (11)$$

If the westernmost side of the first survey line intersects with the exact sea area, assuming the distance of the first measuring line D_0 is X_1 , then both satisfy:

$$X_1 = D_0 \tan 60^\circ \quad (12)$$

By analyzing the triangle on the left side of Figure 8, it can be concluded that:

$$a = 2D_0 \quad (13)$$

Further based on the sine theorem, it can be concluded that:

$$\frac{a}{\sin 91.5^\circ} = \frac{D_1}{\sin 28.5^\circ} \quad (14)$$

So,

$$B_1 = D_1 \sin 60^\circ \left(\frac{1}{\sin \angle 1} + \frac{1}{\sin \angle 2} \right) W_1 = B_1 \cos \alpha \quad (15)$$

To achieve maximum coverage area, the overlap rate of adjacent lines must be minimized, which is 10% to ensure the integrity of measurements and data.

Assuming D_i is the water depth of the i -th measuring line; B_i is the coverage distance of the i -th survey line on the seabed slope; X_i is the distance between the i -th measuring line and the $(i-1)$ -th measuring line; k is the horizontal distance between the intersection point of the right line of the i -th survey line and the seabed and the central survey line. From the above content, it can be inferred that:

$$D_i = D_{i-1} - X_i \tan 1.5^\circ$$

$$\begin{aligned} B_i &= D_i \sin 60^\circ \frac{1}{\sin \angle 1} \\ &+ \frac{1}{\sin \angle 2} \left(B_{i-1} - \frac{X_i}{\sin 28.5^\circ} \sin 150^\circ \right) \cos \alpha = 10\% \\ &B_i \cos \alpha \end{aligned} \quad (16)$$

$k = \sin 60^\circ \frac{D_i}{\sin 31.5^\circ} \sin 88.5^\circ$

In the formula: i starts from 2, and each time one X_i can be obtained, it stopped at $X_1 + X_2 + \dots + X_i + k \geq 4 * 1852$.

Assuming to stop when $i=n$, the total length of this set of measuring lines is $(l_1+l_2+l_3+\dots+l_n+X_2+\dots+X_n)m$, as shown in Figure 9

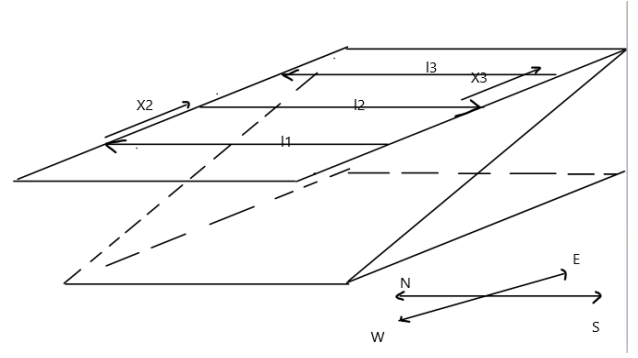


Figure 9. Schematic diagram of some lines in this line group

The length of this group of measuring lines is $(2*n*1852+X_2+\dots+X_n)m$.

In summary, based on the established model, it can be concluded that there are 34 north-south survey lines, so the total length of this group of survey lines is 132979.05m.

3. Conclusions

This paper focuses on modeling and solving analysis of multi beam bathymetry systems. Firstly, a mathematical model for the coverage width and overlap rate between adjacent bands of multi beam bathymetry is established, and relevant indicator data is calculated based on the relevant data; Secondly, based on a rectangular sea area to be measured, a mathematical model for the coverage width of multi beam bathymetry was established, and the coverage width of multi beam bathymetry at the specified location was calculated; Finally, in order to meet the requirement of 10% to 20% overlap between adjacent bands, a set of survey lines with the shortest measurement length and capable of fully covering the entire sea area to be measured was further studied and designed. Through the above modeling and analysis, the following conclusions were obtained:

(1) The mathematical model for the coverage width of multi beam bathymetry and the overlap rate between adjacent bands can clearly express the logical relationship between the distance from the center of the survey line point and the depth of seawater, the coverage width, and the overlap rate with the previous survey line, which has a significant solving effect.

(2) The mathematical model of the multi beam bathymetry system established in this article clearly represents the relationship between the distance between the measuring vessel and the center point of the sea area and the angle between the measuring line direction, which has certain practical application significance.

(3) This article further studies and designs a set of survey lines with the shortest measurement length and can fully cover the entire sea area to be measured, and the overlap rate between adjacent strips meets the requirement of 10% to 20%, which has good solving effect.

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