Optimal Design of Heliostat Field Power Based on Whale Optimization Algorithm and Gravitational Search Algorithm

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Abstract: With the deterioration of global climate and the worsening of energy crisis, the construction of a new power system mainly based on new energy is an important measure for China to realize the goal of "carbon peak" and "carbon neutrality". In order to maximize the power generation efficiency of the mirror field, the layout design of the heliostat is particularly important. In this paper, we constructed a model to calculate each optical efficiency loss component, cosine efficiency, atmospheric transmittance, and discretized the points on the heliostat, and calculated the shadow masking efficiency by coordinate transformation, and finally obtained the collector truncation efficiency. Next, the optimal design of the heliostat field in different cases is obtained using the whale optimization algorithm and the gravitational search algorithm.

Keywords: Optical Efficiency; Optimal Fixed-sun Mirror Field; Whale Optimization Algorithm; Gravitational Search Algorithm.

1. Introduction
The construction of a new power system mainly based on new energy is an important measure for China to realize the goal of "carbon peak" and "carbon neutrality". With the deterioration of the global climate and the worsening of the energy crisis, solar power generation stands out for its high-power generation efficiency and low environmental pollution, and the construction of a fixed-sun mirror field and the use of tower-type solar thermal power generation is a new clean energy technology. It is planned to build a circular heliostat field in a circular area centered at 98.5°E, 39.4°N, 3000 m above sea level and with a radius of 350 m. The field will be located at the center of the area. In order to facilitate the characterization of the orientation of each fixed-sun mirror in the fixed-sun mirror field, as well as to describe the orientation and angle of the sun and the fixed-sun mirror, in this paper, we will take the center of the circular area as the origin, the due east direction as the x-axis positive, the due north direction as the y-axis positive, and perpendicular to the ground upward direction as the z-axis positive to establish a coordinate system, known as the mirror field coordinate system, as shown in Schematic Figure 1 the following figure.

![Figure 1. Schematic diagram of mirror field coordinates](image)

2. Modeling of solar altitude and azimuth angles

The solar altitude and azimuth angles are the main parameters affecting the efficiency and power of the mirror field, influencing the position and shape of the shadow as well as the intensity of the sunlight.

Formula for calculating solar altitude angle:
\[
\cos \alpha_s = \cos \delta \cos \phi \cos \omega + \sin \delta \sin \phi
\]  
where \(\delta\) is the angle of solar
deep night, \(\phi\) is the local latitude 39.4°; \(\omega\) is the solar time angle:
\[
\omega = \frac{\pi}{12} (ST - 12)
\]  
where ST is the local time and \(\delta\) is the angle of solar
declination:
\[
\sin \delta = \sin \frac{2\pi D}{365} \sin \left( \frac{2\pi}{360} \times 23.45 \right)
\]
Where D is the number of days counted from the vernal equinox as day 0.

After obtaining the sun's altitude angle, the sun's azimuth angle can be obtained:
\[
\cos \gamma = \frac{\sin \delta - \sin \alpha \sin \varphi}{\cos \alpha \cos \varphi}
\]  

2.3. Modeling of cosine efficiency

For a certain fixed-sun mirror, because the smaller the angle between the incident sunlight and the normal, the greater the cosine efficiency, so the cosine efficiency reaches the theoretical maximum when and only when the sunlight is directed to the fixed-sun mirror, the sun, the top of the absorber tower, the mirror constitutes a straight line. Cosine efficiency can be expressed as:
\[
\eta_{\cos} = \cos \gamma = \mathbf{e}_s \cdot \mathbf{n}_i
\]
Where \(\mathbf{e}_s\) is the incident light vector and \(\mathbf{n}_i\) is the normal vector.

Based on the solar altitude angle and azimuth angle modeled previously, the incident vector light can be expressed as:
\[
e_x = (-\sin y \cos x - \cos y \cos x, \sin x)
\]
According to the law of reflection, the normal vector can be expressed as:
\[
n_i = \frac{\mathbf{e}_s + e_{\text{tow}}}{||\mathbf{e}_s + e_{\text{tow}}||}
\]
where \(e_{\text{tow}}\) is the unit vector pointing from the center of the heliostat to the center of the heat absorber.

2.4. Calculation of atmospheric transmittance

In the process of sunlight irradiation to the heliostat and reflection from the heliostat to the absorption tower, the heat
\[
V_B \left( l \frac{w}{2}, 2.0 \right); V_B \left( -l \frac{w}{2}, 2.0 \right); V_B \left( -l \frac{w}{2}, -2.0 \right); V_B \left( l/2, w/2 \right)
\]

To facilitate the statistical calculation of its shadow area on another mirror, this shadow coordinate system is first transformed (rotated clockwise along the pitch axis and then counterclockwise along the azimuth axis) to the global coordinate system, and the light equation is obtained through
\[
V_B^{\text{Gl}}(\cos \gamma \sin \beta \sin \alpha_B, \cos \gamma \sin \beta \sin \alpha_B, -\sin \gamma \cos \alpha_B \cos \beta, -\cos \gamma \cos \alpha_B \cos \beta, \sin \alpha_B)
\]
The second coordinate transformation is similar, again after translation rotation, and is expressed as:
\[
V_B^{\text{Gl}}(\cos \gamma \sin \beta \sin \alpha_A, \cos \gamma \sin \beta \sin \alpha_A, -\sin \gamma \cos \alpha_A \cos \beta, -\cos \gamma \cos \alpha_A \cos \beta, \sin \alpha_A)
\]

The four vertices (shaded quadrangles) of PB can be obtained. By writing a code to determine whether the discrete points on the A heliograph are located in the shaded quadrangles, the number of discrete points that meet the condition \(n_1\) is calculated and the above steps are repeated for the surrounding heliographs, and all the shaded points are summed to obtain \(N_1\).

2.7. Modeling of collector truncation efficiency

Approximate the sunlight as parallel light, according to the law of specular reflection can be seen, the projection of the sunlight in the absorption tower is an ellipse, as shown in energy will be scattered by the atmosphere or blocked by small particles of impurity particles in the air, resulting in a certain amount of energy loss. In this problem, the radius of the heliostat field is 350m, the area is small, so only consider the correlation with the distance, the atmospheric transmittance can be expressed as:
\[
\eta_{\text{at}} = 0.99321 - 0.0001176 d_{HR} + 1.97 \times 10^{-8} \times d_{HR}^2
\]
where \(d_{HR}\) denotes the distance from the center of the mirror to the center of the collector.

2.5. Modeling of shadow occlusion efficiency

According to the model assumptions, the heliostat is regarded as an ideal rectangular plane, set its length (l) and width (w), divide the plane into I × J (rows × columns) discrete points, in the heliostat plane to establish a coordinate system of discretized mirrors, with the center of the heliostat as the origin, the point can be expressed as:
\[
\begin{align*}
(x_{i,j}) &= \left( -\frac{l}{2} + (j - 1/2) \times w/l \right) \\text{if} \quad x \geq 0 \\
(y_{i,j}) &= \left( w/2 - (i - 1/2) \times w/l \right) \\text{if} \quad y \geq 0
\end{align*}
\]

2.6. Analyze the shadows and perform a transformation of the mirror coordinate system to the global coordinate system

If there are shadows on the two fixed-sun mirrors, their projections are also approximated as a quadrilateral shape, then it is only necessary to determine the parameters of that quadrilateral, including position, shape, area, etc., based on the four vertices of the other fixed-sun mirror. One of the selected fixed-sun mirrors B produces a shadow on the fixed-sun mirror A. The coordinates of the four vertices of its quadrilateral can be expressed separately in the mirror coordinate system:

Figure 2 below:

If we let this ellipse be A, the long semiaxis a is the diameter of the fixed heliograph, and the short semiaxis b can be found by the three-cosine theorem:
\[
b = a/2 \cdot \cos (\alpha_r - \alpha_t) \cdot \cos (\alpha_t - h_t)
\]
where \(\alpha_r\) is the azimuthal angle of the heliostat, \(\alpha_t\) is the azimuthal angle of the heat-absorbing tower with respect to the heliostat, \(h_t\) is the elevation angle of the heliostat, and \(h_t\) is the apparent altitude angle of the tower to the heliostat. As the solar beam through the reflector to reach the absorption tower formed by the elliptical spot energy distribution in line with the Gaussian distribution, if the sun
mirror energy center (ellipse center) and the absorption tower window center coincides, the maximum truncation efficiency is obtained. If the error caused by the solar rays considered as parallel light is set to, the truncation efficiency can be expressed as:

$$\eta_{\text{trunc}} = \frac{1}{2\pi\sigma^2} \int \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \, dx \, dy$$

where the point \((x, y)\) is the elliptical spot and the point inside the window of the absorber tower.

![Figure 2. Schematic diagram of the truncation phenomenon](image)

### 3. Whale optimization algorithm model
#### 3.1. Model building

Whale optimization algorithm is an optimization algorithm summarized based on the feeding characteristics of humpback whales, in this algorithm, each feasible solution represents a whale, which has two different behaviors in its feeding process, namely, swimming randomly and repelling prey with bubble nets [4].

In order to simulate the hunting behavior of whales with both encircling constriction and bubble net attack, we designed a stochastic strategy to select the position update method. When the stochastic probability \( p \geq 0.5 \), the whale will use the bubble net spiral position update; when \( p < 0.5 \), the whale will update the position based on the position information of the current optimal individual. In the iterative process of constantly approaching the target, \(|A| \) decreases, and when \(|A| = 0\), the iteration of the algorithm ends, and the optimal individual position found in the last iteration is the feasible solution of the problem, which is close to the theoretical optimal value.

Consider optical efficiency as the leading whale; the flow of the mixed-strategy whale optimization algorithm used to solve the EB layout heliostat mirror field is as follows:

1. Input data are traversed, including information on the height of the heliostat dimensions, heat absorber-related parameters, minimum mirror ring radius, and solar altitude and azimuth angles corresponding to the point in time.
2. The control variables are coded such that the azimuthal distance between neighboring heliostats on the same mirror ring and the distance between two neighboring mirror rings are used as the individuals of the algorithm.
3. Initialize the population and set the number of evolutionary generations \( t \) to 0.
4. Calculation of the objective function and simultaneous selection of the candidate part of the leading individual.
5. Update the position of the individual whales with the mutation operation and then calculate the corresponding objective function values.
6. The old population, the new population and the mutated individuals are sorted together in a non-dominated ordering, and then the elite particles are retained as the result of the current iteration and the set of globally optimal solutions is updated.
7. Determine whether the number of evolutionary generations \( t \) is greater than or equal to the set maximum number of iterations \( t_{\text{max}} \), if so, proceed to step 9; otherwise, proceed to step 8.
8. Iterative process: \( t = t + 1 \), return to step 4.
9. Output the optimal solution (or Pareto set of optimal solutions) and the corresponding individual information to end the execution of the algorithm.

#### 3.2. Model results

The above multi-objective whale optimization model is implemented in code to obtain two sets of design parameters for the heliostat field with different layouts before and after optimization, as shown in Figure 3 below:

![Figure 3. EB layout (left) No blocking-dense layout (right) Optimization parameters](image)

Observation of the above figure can be found that the two layout shapes are more different, considered because the EB layout is relatively single, only one layout mode, and ring-shaped distribution, cannot take into account the maximum solar utilization.

### 4. Gravitational search optimization algorithm model

#### 4.1. Model building

According to the No blocking-dense layout, its principle is mainly based on the combination of Campo layout and EB layout, which keeps the azimuthal spacing of neighboring heliostats on each ring equal, and restarts a ring to continue the arrangement when its distance increases to a certain degree. Using the characteristics of this layout, the optimal solution can be achieved by using the gravitational search optimization layout, and the specific steps are as follows [5]:

First, fix the size of the heliostat and start laying it out according to the No blocking-dense layout, and set a range of optical efficiencies based on the first few questions, mainly the range of shadow blocking losses and cosine losses.

The initial heliostat is traversed from the lowest height (2m) specified in the title, and the height that meets the range of optical efficiency is the preferred height for this heliostat.
The search for the optimal solution is carried out circle by circle. In particular, the height of the heliostat in the outer circle will theoretically be higher than that in the inner circle. The obtained parameters for each heliostat are stored in a table.

4.2. Optimization of heliostat dimensions

In the above algorithm, the optical efficiency of the heliostat field has been optimized to a large extent, but the calculation of the optical efficiency of the field found that there is still a lot of room for improvement, considering that it is due to the efficiency of some heliostats is too low; or the location with a higher efficiency of the land utilization is very low, so consider changing the size of the heliostat, using the gravitational search to optimize the layout of the specific steps are as follows:

Calculate the optical efficiency of each heliostat at a defined position and height based on the optical efficiency model established earlier.

Develop a range based on the average optical efficiency of the mirror field, set the less efficient heliostats to 0 and those with efficiencies above that range to 1, and then visualize the results.

The optimal solution obtained by applying the gravitational search algorithm to further expand the size of the more efficient heliostat to improve its land utilization and average optical efficiency.

The obtained parameters for each heliostat are stored in a table.

4.3. Model results

Implementing the above gravitational search algorithm in code, the optical efficiency difference map obtained based on the optical efficiency model of the first question is shown in Figure 4 below, where:

![Figure 4. Optical Efficiency Difference Chart](image1)

Applying the optimization algorithm, the optimal solution obtained is visualized in Figure 5 below, where it can be seen that the optically efficient heliostat with a larger size is shown in red:

![Figure 5. Visualization results after optimizing the size of the heliostat](image2)

5. Conclusion

In this paper, we first construct a complete model for calculating the loss components of each optical efficiency, calculate the cosine efficiency by the angle between the incident light vector and the normal vector of the mirror surface, calculate the atmospheric transmittance by the distance from the center of the mirror surface to the center of the collector according to the given formula, and discretize the points on the heliostat by establishing a discretized specular coordinate system, calculate the shadow blocking efficiency by two coordinate transformations, and calculate the collector truncation efficiency by using the integrals. The collector truncation efficiency is calculated by integrating the points on the heliostat. Secondly, the whale optimization algorithm is used to obtain the optimal design of the heliostat field under the same condition of the height and size parameters of the heliostat. Finally, the gravitational search algorithm is used to make the size and mounting height of the heliostat variable, so as to further improve the optical efficiency of the mirror field and the output thermal power per unit area.

References


