Journal of Computing and Electronic Information Management
ISSN: 2413-1660 | Vol. 11, No. 3, 2023

A Programmatic Summary of Theoretical Basis and Methods of Micro-resistance Measurement

Shiyu Zhang

Jinan University - University of Birmingham Joint Institute at Jinan University, Jinan University, Guangzhou 510000, China

Abstract: The accuracy of micro-resistance measurements is an issue of widespread concern in industry. In this paper, we review the methods and main mathematical methods of tiny resistance measurement, firstly, we introduce the three basic mathematical knowledge of multiple regression model, the principle of least squares and the calculation of error, then we review the two-wire resistance measurement method and four-wire resistance measurement method. This paper presents a systematic description of the methods and mathematical theories related to the measurement and fitting of tiny resistors to provide a theoretical summary for future research in the area of tiny resistors.

Keywords: Multiple Regression; Least Square Principle; Error Calculation; Four-wire Resistance Measurement.

1. Introduction

Tiny resistors are the core components of precision instruments, and the accuracy of the micro-resistance value largely determines the performance of the precision instruments, however, the laboratory found that the measurement of micro-resistance has encountered a bottleneck, so it is hoped that the combination of mathematical methods in order to get the high-precision model of the measurement of micro-resistance. If mathematical modelling and numerical analysis of the experimental results of micro-resistance measurement can be carried out with mathematical methods for high-precision measurement, then it will have a greater positive impact on the practical engineering applications of micro-resistance measurement.

The basic idea of regression analysis was first introduced by F. Galton. At that time, Galton applied the regression idea to statistical analysis by considering the effect of only one independent variable on the dependent variable. However, when there are many independent variables, some of them do not have a great influence on the dependent variable, and some of them and other independent variables may be independent of each other, and there may be all kinds of interrelationships. In these cases, it is more effective to build a multiple regression model for statistical analysis.

The first asteroid, Ceres, was discovered in 1801 by Italian astronomer Giuseppe Piazzi. After 40 days of follow-up observations, Piazzi lost track of Ceres as it orbited behind the Sun. The orbit of Ceres, calculated by Gauss, then 24 years old, allowed the astronomical community to predict the exact position of Ceres from then on. The method Gauss used was the method of least squares, which was published in 1809 in his book A Treatise on the Motions of the Heavenly Bodies. In fact, the French scientist Legendre independently invented the method of least squares in 1806, but it fell into obscurity because it was not known to the world. Least squares method is a mathematical optimization technique. It is the most commonly used method for solving curve-fitting problems by minimizing the square of the error and finding the best function match for the data.

There are many sources of error, such as rounding error, truncation error, measurement error, etc. The use of error analysis in the process of curve fitting can be very helpful in determining how well the curve is fitted. However, there are many types of errors in the mathematical sense, which we will discuss further in the following. In addition, the method of tiny resistance measurement also has a great influence on the measurement accuracy, in this paper, we will mainly discuss the similarities and differences, advantages and disadvantages of the two-wire and four-wire methods of measurement.

2. Theoretical Basis for Micro-resistance Measurement

2.1. Multiple Regression Model

2.1.1. Definition and Basic Idea

The use of regression equations to quantitatively characterize the dependence between a dependent variable and multiple independent variables is known as multiple regression analysis [1]. The basic idea behind regression analysis is that although there is no strict, deterministic functional relationship between the independent and dependent variables, it is possible to find a mathematical expression that best represents the relationship between them. If this relationship is linear, then a linear multiple regression model can be used to describe it. Instead, we need to use a non-linear multiple regression model.

2.1.2. Modeling of Multiple Linear Regression Model

The multiple linear regression was modeled as $Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \varepsilon$, where Y is the dependent variable; $x_1, x_2, \ldots, x_p$ is the p independent variables; $\beta_0, \beta_1, \ldots, \beta_p$ is the unknown parameter, called the regression coefficient; and ε is the error term and a random variable, representing the influence of random factors on Y.

The model for multiple polynomial regression can be expressed as :

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p + \sum_{1 \le j \le k \le p} \beta_{jk} x_j x_k + \varepsilon$$

The basic assumptions of multiple linear regression are threefold. First of all, it is assumed that the error terms satisfy a normal distribution with zero mean and equal variance and are independent of each other; moreover, it is assumed that the independent variables are deterministic and not linearly correlated; and thirdly, it is assumed that the independent variables and the error terms are uncorrelated [2]. If these assumptions are not met, then there are a number of problems with the general least squares estimation of the model parameters.

2.2. Least Square Principle

2.2.1. Basic Concept

In other words, the main idea of least squares is to solve for the unknown parameters such that the sum of the squares of the differences (i.e. errors) between the theoretical and observed values is minimized [3]:

$$E=\sum_{i=1}^{n}e_i^2=\sum_{i=1}^{n}(y_i-\hat{y})^2$$

Where the measured values y_i are our n groups of samples and the predicted values $\hat{y}$ are our hypothesis fit function. e_i is the difference between the predicted values and the measured values? E is the objective function and our goal is to obtain the parameters that minimize the objective function.

2.2.2. Weakness

(1) Sensitive to outliers

General least squares are sensitive to outliers, and just one strange outlier may change the final result. In terms of the final results of its algebraic solution, general least squares only takes information about the mean and variance of the points, so it is only applicable in the presence of ordinary noise. When outliers are present, the general least squares method cannot help and other methods are needed to improve or resolve the situation.

(2) Error in the independent variable is not considered

The general least squares method only takes into account the existence of errors in the dependent variable, and does not take into account the errors in the independent variable, so its application conditions are somewhat restricted. It is only more applicable when there is no bias in the independent variable, or when the bias in the independent variable is negligible within a certain range. When there is bias in the measurement of both the independent and dependent variables, the general least squares method is less suitable.

For the problem discussed in this paper: fitting tiny resistance values to the locations of voltage and current measurement points. In practice, however, there may be small errors in the position of the voltage and current measurement points, which may have an impact on the accuracy of the results.

(3) The existence of unsolvable cases

When the normal matrix XTX of the coefficient matrix of a curve fit is singular, its exact least squares solution cannot be found. And when the matrix XTX is close to singular, a direct inverse can also lead to large deviations. According to the specific data set used in this paper, we discuss the shortcoming of least squares method and puts forward an effective solution.

2.3. Error Calculation

2.3.1. Absolute Value

If x^* is an approximation of exact value x, then $x-x^*$is the absolute error of approximate value x^* [4].

2.3.2. Relative Error

In many cases, the magnitude of the absolute error does not yet portray the accuracy of the approximation. This is because the accuracy of an approximation is not only related to the magnitude of the absolute error, but also to the magnitude of the accurate value itself. Therefore, the concept of relative error needs to be introduced. We assume x^* .is an approximation of exact value x, and $e^*(x)$ is the absolute error between x^* and x, then $\frac{e^*(x)}{x}$ is the relative error of approximate value x^* [4].

2.3.3. Mean Squared Error

In mathematical statistics, the mean squared error is the expected value of the squared difference between the estimated value of a parameter and the true value of the parameter, denoted as MSE. MSE is a convenient way to measure the 'mean error', which evaluates the degree of variation in the data, and the smaller the value of MSE, the better the accuracy of the predictive model in describing the experimental data.

The mathematical expression for calculating the equation MSE is

$$MSE=\frac{1}{n}\sum_{i=1}^{n}(y_i-\hat{y}_i)^2$$

where y_i is the true data, $\hat{y}_i$ is the fitted data and n is the number of samples?

2.4. Gaussian Perturbation

Gaussian perturbation, also known as Gaussian noise, is a type of random noise. It is a random noise whose probability density function follows a Gaussian distribution (also known as a normal distribution), and a Gaussian perturbation is a random perturbation distributed according to a Gaussian probability. Gaussian noise mainly contains thermal noise, cosmic noise, undulating noise and scattered grain noise, etc. In addition to conventional noise reduction methods, mathematical and statistical methods are often used to eliminate Gaussian noise. The probability density of a Gaussian noise disturbance is Gaussian distributed (normal distribution). The formula for the Gaussian distribution is

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

There are occasions when we must add appropriate Gaussian noise to the standard data to produce some error and thus test the stability of the model, and therefore have great application in experiments, and what we do next in this paper is to add Gaussian perturbations to the multivariate curve fitting.

3. Resistance Measurement Method

3.1. Two-wire Resistance Measurement

A common method of resistance measurement is to use a voltmeter and ammeter on a digital multimeter and then obtain the resistance value according to Ohm's Law [5]. The two-wire method is one of the most common resistance measurement methods used in industry today, but the resistance measurement results obtained also include the resistance of the test leads. When the resistance under test is large, the lead resistance is negligible. However, when the test resistance is a tiny resistance, especially when the lead resistance even exceeds the resistance under test, the two-wire resistance measurement method is no longer applicable and we need to use the four-wire resistance measurement method to measure tiny resistances when the two-wire resistance measurement method is not applicable.

Figure 1 is the schematic of two-wire resistance measurement. In Figure 3-1, VM is the voltage measured by meter, VR is the voltage across resistor, and the measured resistance is equal to VM/I.

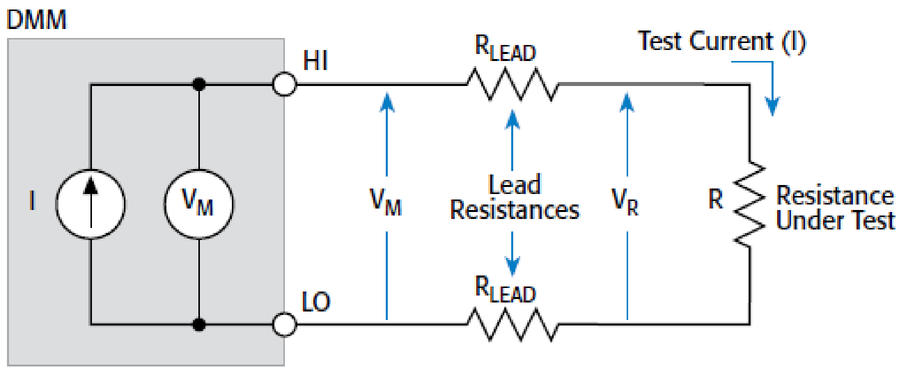

Figure 1. Schematic of two-wire resistance measurement

3.2. Four-wire Resistance Measurement

The principle of the four-wire resistance measurement method is that the voltmeter and ammeter of the multimeter are connected to the resistor under test, and since the voltmeter has a high impedance, there is no current in the leads connected to the voltmeter [6]. Therefore, the voltage value measured by the voltmeter is the true voltage across the resistor.

Following labels are used in the Figure 2: VM = Voltage measured by meter, VR = Voltage across resistor (R) and measured resistance = VM/ I = VR/ I.

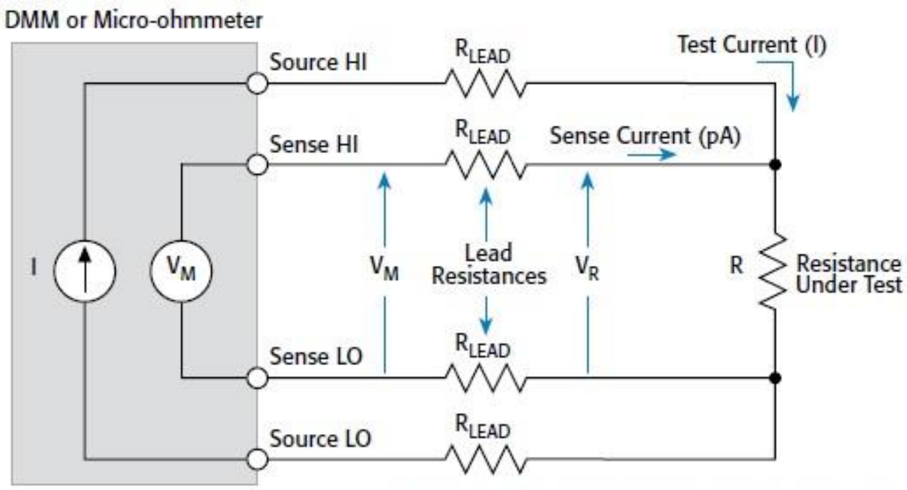

Figure 2. Schematic of four-wire resistance measurement

4. Conclusions

In this paper, mathematical methods are used to further investigate the measurement of tiny resistors, and statistical modelling software is used to carry out mathematical modelling and numerical analysis of high-precision measurements of tiny resistors, which makes it possible to substantially increase the production possibilities and accuracy of practical engineering applications. Multiple regression, least squares and error analysis are the most commonly used and basic mathematical methods in regression analysis problems, which are easy to implement with programming languages such as MATLAB. When the research accuracy is insufficient, scholars can consider introducing the idea of segmented fitting. In addition, there is a certain degree of disconnection between the development of mathematics and the practical application of engineering, and it is believed that the problem will be further developed with the development of more mathematical methods and methodological tools combined with practical engineering research.

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