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Two laws of Spatially Coherent Superposition of Eigenmodes

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Abstract: Unlike conventional light sources, lasers possess unique characteristics such as coherence and focused nature. Consequently, numerous applications of laser beams can be enhanced according to their spatial distribution characteristics. This paper presents two patterns of change in the intensity distributions of Hermite Gaussian beams regarding the impact of variables, including relative phase, intensity coefficient, and order of indexes. By conducting simulations in Matlab, we generate various patterns of Hermite Gaussian beams by adjusting parameters such as order indexes, relative phase, and intensity coefficients. The superposition principle is then applied to calculate the spatial intensity distribution of the combined laser beams. Our results indicate that as the order of indexes increases, the complexity of spatial distributions also increases. We analyze the symmetry and rotational properties of the Hermite Gaussian modes, revealing reflection and central symmetries, as well as rotational changes concerning the relative phase. Our discussions focus on exploring the potential for generating complex optical fields and theoretically validating experimental feasibility. We provide a schematic representation of the generation process for complex structured optical fields.

Keywords: Hermite Gaussian; Coherent Superposition; Eigenmode; Structured Beams.

1. Introduction

The description of the spatial structure of the different beam modes has been the object of theoretical and experimental analysis since the invention of the laser and is now a standard textbook topic in modern optics and laser physics courses [1-3] Such study has contributed to technological breakthroughs in several areas such as scientific research and communications.

Theoretically, several distinct types of laser beams can be generated, each with various properties, for instance: Hermite-Gaussian beams (HG) which exhibit complex spatial structure, and Laguerre-Gaussian beams (LG), known for its applications in the field of optical trapping.[4]

However, due to the complex setup of the optical machines needed to generate such beams, the major approaches of the generation and application of laser beams are based on the basic Gaussian mode due to its simple and symmetric spatial structure;[5,13] the basic Gaussian can be considered as a special case of the Hermite Gaussian mode in which the order of indexes equals to zero. [6,11]

Still recently, higher-order laser beams are no longer confined to the object of study of a restricted group of specialists. Thanks to several new applications, the interest concerning these beams has grown dramatically.[7] This paper first analyzed the effect of eigenmode, relative phase and intensity coefficient on the structure of beams. Based on that, it furthermore analyzes the patterns The further analysis encompasses the theoretical underpinnings of higher-order laser beams, experimental setup and data collection, as well as an in-depth discussion and conclusion regarding the experimental results.[8,9] The article provides a detailed introduction to the Helmholtz equation, Hermite Gaussian modes, and the principle of superposition, along with the process of simulating and encoding Hermite Gaussian modes using Matlab in the experiment.[10,12] A thorough analysis of the experimental results is presented, particularly focusing on the superposition effects, symmetry, and rotational characteristics of Hermite Gaussian modes with different order index values. In conclusion, the article summarizes the research findings through the discussion and conclusion sections, emphasizing the importance of Hermite Gaussian modes for structured laser beams and providing a schematic illustration of the complex structured optical field generation process.

2. Theories of high-order laser beams:

To generate high-order laser beams, it is necessary to follow several research principles.

2.1. The Helmholtz equation

Helmholtz equation, named after the German physicist Hermann von Helmholtz, has a variety of applications in physical science. It’s mainly used to predict the light field distribution in an empty cavity. Here’s the expression:

$$\nabla^2 f = -k^2 f$$

where ∇2 is the Laplace operator, k2 is the eigenvalue, and f is the (eigen)function. When the equation is applied to waves, k is known as the wave number.

2.2. The Hermite Gaussian Modes

Hermite Gaussian modes are the solution of Helmholtz equation in cartesian coordinate. After inputting certain parameters including the order of indexes, relative phase and the intensity coefficient, it will output the spatial intensity distribution of the Hermite Gaussian beam as a 3D model:

$$E_{l,m}(x,y,z) = E_0 \frac{\omega_0}{\omega(z)} H_l\left(\frac{\sqrt{(2)}x}{\omega(z)}\right) H_m\left(\frac{\sqrt{(2)}y}{\omega(z)}\right) \times \exp\left(-\frac{x^2+y^2}{\omega^2(z)}\right) \exp\left(-i\frac{k(x^2+y^2)}{2R(z)}\right) \times \exp\big(i\psi(z)\big)\exp(-ikz)$$

Through the multiplication of Hermite polynomials, Gaussian functions and phase factor, the Hermite Gaussian modes can be obtained.

C. The Superposition Principle

For laser beams that have the same frequencies, same wavelength; according to the superposition principle, the distribution of light intensity of the superposition of 2 or more eigenmodes can be calculated by:

$$E = E_1 + E_2 + \cdots \ldots + E_k$$
$$= HG_{m1,n1} + HG_{m2,n2} + \cdots \ldots + HG_{mk,nk}$$

Where E represents the output, E1, E2......Ek represents each of the eigenmode individually.

3. Experimental Setup and data collections

3.1. Materials

In this project, the lack of available optical devices necessitated the utilization of Matlab for generating patterns of Hermite-Gaussian (HG) modes through simulations. HG modes are solutions to the paraxial wave equation in optics and find wide applications in laser resonators, optical communication systems, and beam shaping. By leveraging Matlab's computational capabilities, we were able to model the spatial distribution of these modes with precision and flexibility.

The simulation process involved defining the parameters of the HG modes, such as the order indexes (m, n), relative phase (Δϕ), and intensity coefficient (α). These parameters play crucial roles in determining the characteristics of the generated modes, including their spatial profiles and propagation properties.

Furthermore, the use of Matlab allowed us to impose constraints and explore various scenarios efficiently. For instance, we could simulate HG modes under different boundary conditions, investigate the effects of varying parameters on mode profiles, and analyze the interaction between multiple modes. By incorporating these considerations, we aimed to develop a comprehensive understanding of HG mode generation and behavior.

3.2. Experimental Procedures

1). HG Mode Coding in Matlab:

Implementing the mathematical expressions for Hermite and Gaussian functions to generate HG mode patterns.

Writing Matlab code to define the spatial distribution of HG modes based on user-specified parameters.

2). Parameter Selection and Input:

Choosing appropriate order indexes (m, n) to represent the desired mode shape and size.

Determining the relative phase (Δϕ) to introduce spatial modulation in the mode pattern.

Selecting the intensity coefficient (α) to control the overall brightness or energy distribution of the mode.

3). Plotting Spatial Distribution:

Executing the Matlab code to generate the spatial distribution of HG modes.

Visualizing the mode patterns using Matlab's plotting functions, such as surf or contour plots.

4). Data Organization and Analysis:

Collecting and organizing the generated patterns of HG modes for further analysis.

Examining the influence of parameter variations on mode characteristics.

Summarizing and interpreting the results through comparative analysis, statistical measures, and visualization of relevant graphs and charts.

Drawing conclusions regarding the effectiveness and limitations of the simulated HG mode generation process.

By following these experimental procedures and conducting thorough analysis, we aimed to gain insights into the generation and manipulation of HG modes using computational simulations, laying the groundwork for potential applications in optical systems and beyond.

4. Results

In the simulation, the effect of several parameters of Hermite Gaussian modes were tested, including relative phase, order indexes, generating a great quantity of patterns of Hermite Gaussian beams.

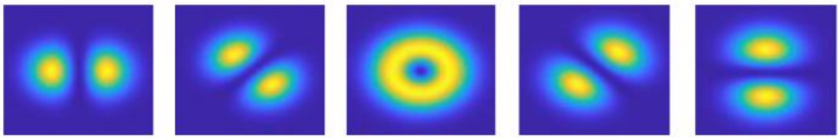

Fig.1. Superposition of two HG beams with order indexes equal to 1.

Fig.1 illustrates the superposition effect of two HG beams with order indexes both equal to 1. These characteristics result from the combined effects of the phase difference and spatial distribution properties of the two beams. For two HG beams with order indexes both equal to 1, their superposition creates a new pattern, displaying interference patterns different from those of individual HG modes.

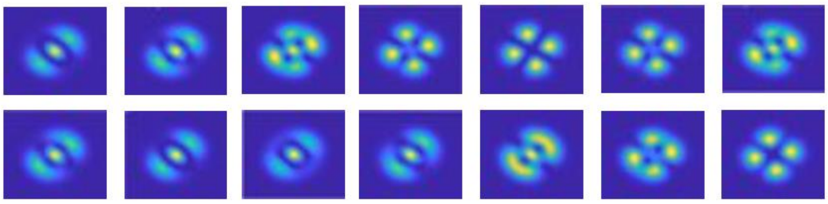

Fig.2 Superposition of two HG beams with order indexes equal to 2.

Fig. 2 showcases the superposition effect of two HG beams with order indexes both equal to 2. By comparing the interference patterns in the figure, we can observe that the spatial structure of this superposition mode is more complex compared to the case with order indexes of 1 as shown in Fig. 3. In this superposition mode, we observe more interference fringes and interference nodes, reflecting the richer interactions between HG modes with different order indexes.

When the order of indexes equals 3, we studied the superposition effect of two HG beams. By closely observing the interference patterns, we observed denser interference fringes and nodes, indicating a closer interaction between HG modes with an order index of 3. Additionally, the interference patterns exhibited higher complexity, with secondary interference patterns and finer details appearing within the main interference fringes. The superposition of higher-order HG beams resulted in more intricate interference phenomena, with additional interference lobes and more pronounced intensity distribution variations across the beam profile.

It can be seen that as the number of order of indexes increases, the complexity of the spatial distributions of HG modes increases significantly accordingly. Among all the characteristics of Hermite Gaussian modes identified, two of them are worth noting:

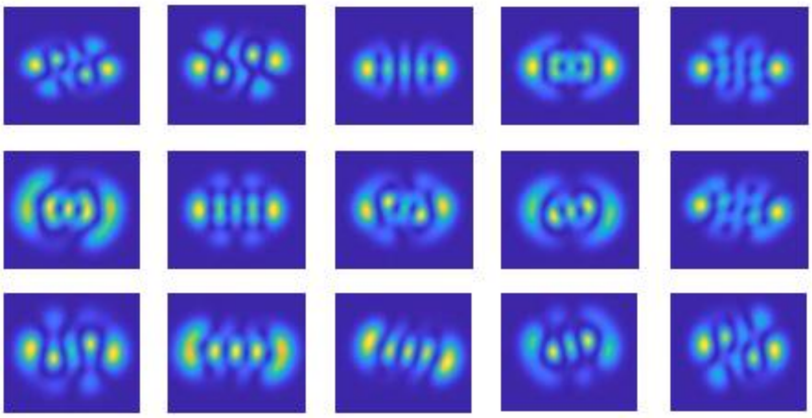

Fig.3 Superposition of two HG beams with order indexes equal to 3.

4.1. Properties of symmetries:

When two HG modes with order indexes (m,n) and (n,m) superimpose (m and n stands for 0 and any positive integer), reflection symmetry is achieved in the spatial distribution of the superposition of HG modes.

Noting that central symmetry specifically can be achieved when two beams with order indexes (m,0) and (0, n) overlap. (When m or n = 0)

However, when additional beams are added for superposition, the result might deviate from symmetrical structure.

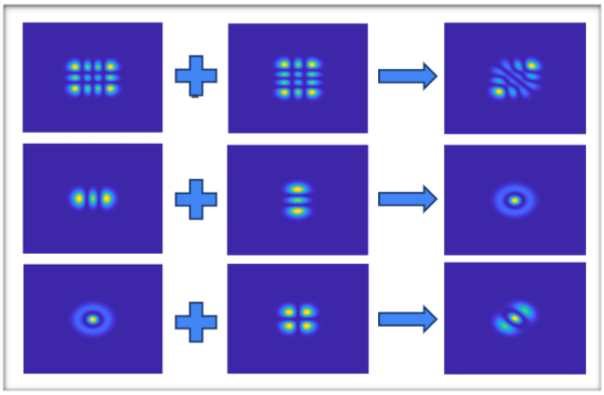

Fig.4 Symmetry properties of superimposed HG beams with various order indexes.

For the first row, one HG mode with order of indexes (2,3) is superimposed another HG mode with order of indexes (3,2) to generate a pattern that exhibits reflection symmetrical structure. For the second row, one HG mode with order of indexes (0,2) is superimposed with another HG mode with order of indexes (2,0) to generate a pattern that exhibits central symmetrical structure. For the third row, two HG modes with central symmetrical structure are superimposed, while the result deviates from central symmetrical structures.

4.2. Properties of rotations

When two HG beams overlap and the order equals to 1, the changes in the pattern rotational angles of the spatial distribution of the two HG beams can be determined:

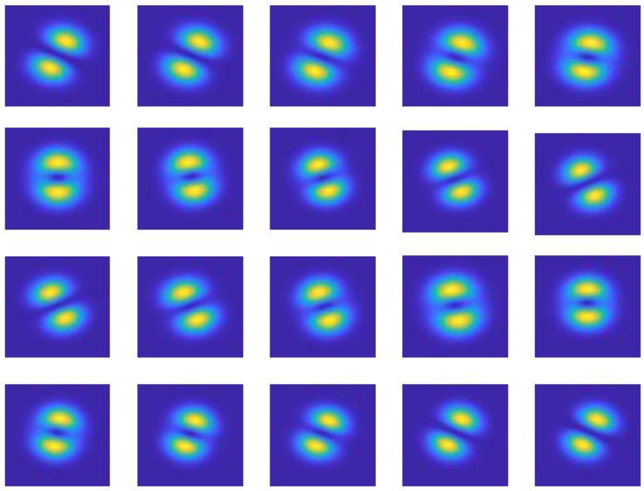

Fig.5 Superpositions of 2 HG modes when the order of indexes = 1

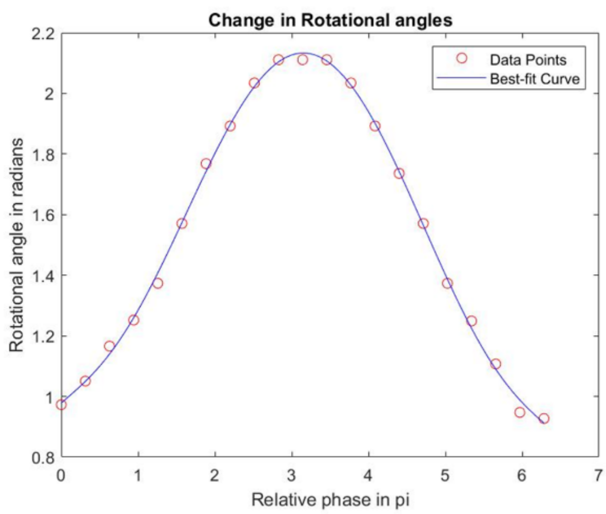

Fig.6 Change in rotational angles with respect to time

In Fig.6, the independent variable = relative phase in radians, the dependent variable= change in rotational angles in radians. The change in rotational angles reaches maximum value when relative phase = 1pi and regain its original value when relative phase = 2pi.

5. Discussions

Our focus in this study is to explore the potential for generating complex structured optical fields from both theoretical and simulation perspectives. The methods for generating these complex optical fields have been validated in previous academic research in terms of experimental feasibility. Therefore, our emphasis lies in further investigating the unknown optical fields theoretically. Once we identify valuable optical field structures, they can be obtained using existing experimental methods. Demonstrated below is an example of such:

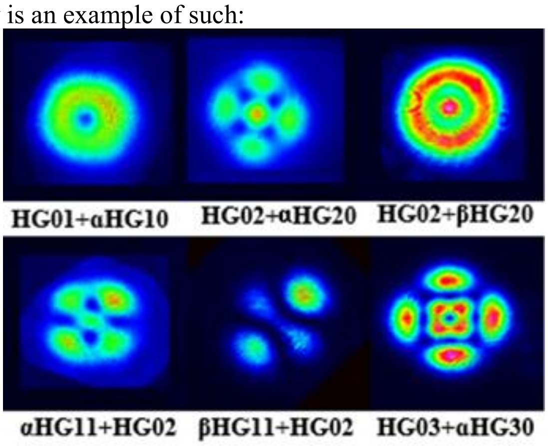

Fig.7 Schematic Representation of the Generation Process for Complex Structured Optical Fields

6. Conclusion

Based on the Hermite-Gaussian modes as well as the superposition principle, a large quantity of novel patterns of laser beam are obtained through experimental simulation. After comparing and graphing the data through line graph to make it easier to visualize, it can therefore be concluded that the spatial intensity distribution of structured lights is based on several parameters including order indexes, relative phase and that the Hermite Gaussian beam does possess properties such symmetry and rotation.

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