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Analysis of Polynomial Interpolation Characteristics Based on 6DOF Manipulator Trajectory Planning

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Abstract: Aiming at the problems of choosing and using polynomials of different times and mixed polynomials in the process of manipulator trajectory planning, such as difficulty, wide range of light and low accuracy, in this paper, the application of different polynomials in manipulator trajectory planning is systematically analyzed by using MATLAB Robot Toolbox simulation platform and Polynomial interpolation algorithm. Using a 6-dof robotic arm Puma560 as a simulation object, the effects of three-, five-, and seven-Polynomial interpolation on joint position and pose in joint space trajectory planning were analyzed, a preliminary understanding of the application of different polynomials, and secondly, the more complex application of the fifth and seventh degree polynomials in Descartes space for the trajectory planning of the manipulator, the influence of trajectory planning with different degree polynomials on the position and pose of manipulator joint is clarified. The results show that the higher the number of polynomials, the smoother the transition of the joint at the critical path points, but the greater the maximum angular velocity and angular acceleration of the joint, the Polynomial interpolation of joint space and Descartes space trajectory planning are 14% and 23% lower than the average of absolute maximum angular velocity and absolute maximum angular acceleration of the Polynomial interpolation, respectively. Based on the analysis of the research results, this paper presents the applicable conditions and applications of Polynomial interpolation method in trajectory planning, and further summarizes the application conditions and methods of mixed polynomials.

Keywords: Mechanical arm; MATLAB; Polynomial interpolation; Track planning; Hybrid interpolation.

1. Introduction

In the control field of the manipulator, whether the trajectory planning is reasonable or not will directly affect the impact strength of each joint and the working accuracy of the end actuator. Reasonable trajectory planning can make the manipulator have good dynamic performance. Manipulator trajectory planning can be divided into joint space trajectory planning and Cartesian space trajectory planning.

In joint space trajectory planning, the operation path point is converted to joint angle variable by inverse kinematics solution, and the smooth change function formed by joint path point interpolation is used to describe the robot trajectory [1]. The control of the manipulator is carried out in the joint space, there is no singularity problem, and there is no need to calculate the inverse kinematics, so it is relatively simple to use. Cartesian space trajectory planning, under the condition that the working path of the robot and the position and posture of the end-effector are known, the inverse kinematics is used to solve the angle variation of each joint, so as to control the joint [2]. However, due to the need to solve the joint position and posture through inverse kinematics, the problem of joint singularity may occur.

With the study of manipulator trajectory planning, polynomial interpolation is more and more widely used in manipulator trajectory planning, and there are many ways of polynomial interpolation. Different combinations and different degrees of polynomials will produce different effects in trajectory planning [3-11]. Therefore, the application of polynomials is not accurate enough, resulting in inaccurate or not optimal trajectory planning results. Tension [12] proposed an acceleration and deceleration NURBS interpolation algorithm based on cubic polynomials, which requires data preprocessing, prediction and real-time planning. by finding the speed sensitive points to optimize the speed, it realizes the speed smoothing in NC machining and effectively controls the jitter problem when the speed changes rapidly. Shen et al. [13] compared the errors of quintic polynomial interpolation and cubic polynomial interpolation, and proved that quintic polynomial interpolation is better than cubic polynomial interpolation from the aspects of control precision error, speed error and so on. Zhang Jinming [14] proposed a new 3-segment "414" interpolation algorithm, which makes the acceleration of the joint more stable, combines the advantages of low-order polynomial acceleration and high-order polynomial easy to control, and effectively reduces the impact on the joint in the process of robot transmission. Yang Jing [15] put forward a "353" polynomial interpolation method. The algorithm effectively reduces the maximum speed and maximum acceleration of the robot, and makes the change of acceleration smoother, thus improving the stationarity of the robot. Wang Jun [16] uses B-spline curve for trajectory interpolation, which makes the joint velocity and acceleration curve of the robot smooth and continuous, and reduces mechanical vibration and shock. Pu Yasong [17] proposed a three-and four-order cross-mixed interpolation based on multi-segment, which avoids the jitter of the manipulator caused by too many constraints.

The method of polynomial interpolation is widely used, and most scholars only try many times to obtain better trajectory planning results, but there is a lack of systematic research on different polynomial characteristics. Based on this, this paper takes the 6-degree-of-freedom manipulator PUMA560 as the research object, applies the relevant theory of robot human kinematics, adopts cubic polynomial, quintic polynomial and seventh polynomial interpolation methods, carries on joint space trajectory planning and Cartesian space trajectory planning of the manipulator through MATLAB robot toolbox, and makes a systematic and comprehensive analysis of different polynomial interpolation characteristics through two kinds of trajectory planning formulas. To clarify the advantages and disadvantages of different interpolation methods, based on this, this paper further gives the application methods and suggestions of mixed polynomial interpolation, which provides a reference for the selection of manipulator trajectory planning using polynomial interpolation.

2. Virtual Prototyping of robotic arm

The research object of this paper is PUMA560, which can be used in assembly, handling and other production operations. The manipulator model can be generated by calling the robot toolbox (Robotics Toolbox) in MATLAB. The manipulator has six degrees of freedom, and its modified DH parameter table and connecting rod coordinate system are shown in Table 1 and figure 1. At present, the Dmurh parameter method is mostly used for the modeling of the manipulator, and the improved Dmurh parameter method can better solve the problem of ground singularity between adjacent parallel rods than the standard Dmurh parameter method, and is more widely used. Therefore, this paper uses the improved Dmurh parameter method to model and analyze the manipulator, and the Dmurh parameters are shown in Tab 1.

Tab.1 Improved D-H parameter table of PUMA560

Connecting rod i	Torsion angle of connecting rod α_{i-1} /(°)	Connecting rod length a_{i-1} /m	Connecting rod offset d_i /m	Angle between connecting rods θ_i /(°)	Variable range θ_i
1	90	0.00000	0.00000	90	-160~160
2	0	0.43180	0.00000	0	-225~45
3	-90	0.02030	0.15005	-90	-45~225
4	90	0.00000	0.43180	0	-110~170
5	-90	0.00000	0.00000	0	-100~100
6	0	0.00000	0.00000	0	-226~266

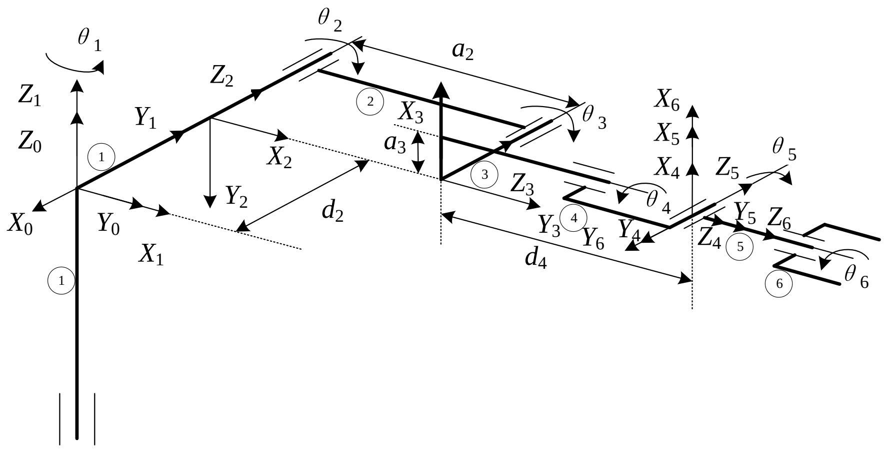

Fig.1 Link coordinate system

In MATLAB's robot toolbox, the virtual prototype of the PUMA560 manipulator invoked is shown in figure 2.

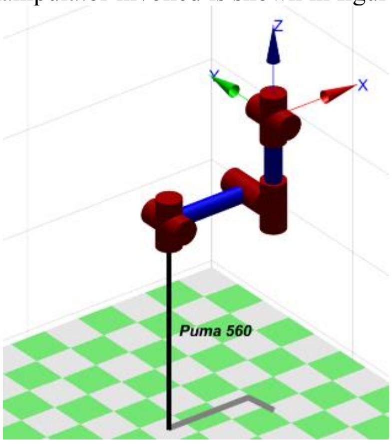

Fig.2 Virtual prototype of PUMA560

3. Kinematics Analysis of manipulator

For the control and optimization of robot trajectory, we need to use the relevant theory of robot kinematics as the basis of research. Robot kinematics describes and studies the variation of robot position, velocity and acceleration with time from the point of view of geometry. the problems involved in robot kinematics are mainly discussed in workspace and joint space. including forward kinematics and inverse kinematics [18,19]. In this paper, it is necessary to analyze the forward and inverse kinematics of the manipulator in Cartesian space trajectory planning.

3.1. Forward kinematics analysis of manipulator

The forward kinematics of the manipulator is to transform the joint space to Cartesian space through the homogeneous matrix. Through the homogeneous matrix transformation of each connecting rod, combined with the Dmurh parameter table, the pose of the manipulator relative to the base coordinate system can be finally determined. The homogeneous transformation matrix of adjacent joints is shown in formula (1).

$$
{}_{i-1}^{i}T = Rot(X,a_{i-1})Trans(X,a_{i-1})Rot(Z,\theta_i)Trans(Z,d_i)
= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i\cos\theta_{i-1} & \cos\theta_i\sin\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1}d_i \\ \sin\theta_i\sin\alpha_{i-1} & \cos\theta_i\sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)
$$

The general formula of the matrix is called Dmurh matrix, which is a 4 × 4 homogeneous transformation matrix. Four Dmurh parameters are used to represent the pose relationship or transformation relationship between the two adjacent links.

For a PUMA560 manipulator with six degrees of freedom, there are six Dmurh matrices as shown in formula (1), and the kinematic model of the manipulator is shown in formula (2).

$$ {}_0^6T = {}_0^1T \bullet {}_1^2T \bullet {}_2^3T \bullet {}_3^4T \bullet {}_4^5T \bullet {}_5^6T = \begin{bmatrix} {}_0^6R & {}_0^6P \\ 0 & 1 \end{bmatrix} \quad (2) $$

Above:

$$ {}_0^1T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad {}_1^2T = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} $$

$$ {}_2^3T = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_2 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}_3^4T = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -\sin\theta_4 & -\cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_4^5T = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}_5^6T = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_6 & -\cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

3.2. Inverse kinematics analysis of manipulator

In practical application, the trajectory of the robot is usually determined. in order to control the robot to achieve the predetermined trajectory, it is necessary to require its inverse kinematics solution, that is, to complete the transformation of the manipulator from Cartesian space to joint space. in order to control the operation of each joint motor, so that the end-effector can work normally according to the predetermined working requirements.

The inverse kinematics of the robot is to know the position and pose of the end of the robot and find out the problems of 1', 2', 3', 4', 5 'and 6' of the joint. So there is an equation:

$$ {}_0^6T = {}_0^1T \bullet {}_1^2T \bullet {}_2^3T \bullet {}_3^4T \bullet {}_4^5T \bullet {}_5^6T = \begin{bmatrix} n_x & o_x & \alpha_x & p_x \\ n_y & o_y & \alpha_y & p_y \\ n_z & o_z & \alpha_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3) $$

Because the inverse kinematics equations belong to nonlinear transcendental equations, they are often solved by Paul inverse transformation method. As shown in equation (3), the matrix on the right side of the equation is known, and the joint variables in the matrix on the left side of the equation are unknown. First, use the matrix ${}_0^1T^{-1}$ -left multiplication (3) matrix equation, then find and establish an equation with a single joint variable from the matrix elements on both sides of the equation, solve the variable, and then find and establish other single variable equations. if all joint variables cannot be solved, then left multiply matrix ${}_1^2T^{-1}$ -on both sides of the equation, and then find and establish a solvable single variable equation until all variables are solved.

$$ {}_0^1T \bullet {}_1^2T \cdots {}_{n-1}^{n}T = {}_0^nT \quad (4) $$

Left multiplication: ${}_0^1T^{-1}$: ${}_1^2T \cdots {}_{n-1}^{n}T = {}_0^1T^{-1} \bullet {}_0^nT$

Left multiplication: ${}_1^2T^{-1}$: ${}_2^3T \cdots {}_{n-1}^{n}T = {}_1^2T^{-1} \bullet {}_0^1T^{-1} \bullet {}_0^nT$

4. Trajectory planning of joint space polynomial interpolation for manipulator

The robot is composed of multiple joints, and the trajectory planning of joint space is to plan the smooth trajectory of each joint based on the joint motion constraint bar [17]. The motion constraints of the joint include its range of motion, velocity, acceleration and so on. By using the joint space trajectory planning, the path points (commonly called nodes) to be passed by the robot are determined in the robot operating space, and the corresponding joint values are obtained from the inverse kinematics solution. For each joint, to plan its transition trajectory between every two adjacent joint values is actually to use the smooth function to plan the stable variation curve of joint variables, and finally let each joint complete the planned joint trajectory in the same period of time. The expected motion of the robot in the operating space can be realized.

In the study of joint space trajectory planning of manipulator, the key is to determine the unknown coefficients of different degree polynomials, which need to be obtained according to different path point constraints.

4.1. Cubic polynomial interpolation function

Cubic polynomial function has the characteristics of first-order and second-order differential smoothness, so it is widely used in robot joint space planning. The starting point of a pair of joint values is defined as θ_0, and the ending point is defined as θ_f, and the task of trajectory planning is to construct a smooth trajectory function $\theta(t)$, which satisfies the constraint conditions of joint motion and passes through the starting point and ending point.

The general formula of cubic polynomial function is:

$$ \theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \quad (5) $$

In order to realize the smooth motion of the joint, the function $\theta(t)$ must satisfy at least four constraints, the angle and velocity constraints of the starting point and the ending point.

Angle constraint:

$$ \begin{cases} \theta(0) = \theta_0 \\ \theta(t_f) = \theta_f \end{cases} \quad (6) $$

Angular velocity constraint:

$$ \begin{cases} \dot{\theta}(0) = \dot{\theta}_0 \\ \dot{\theta}(t_f) = \dot{\theta}_f \end{cases} \quad (7) $$

By substituting four constraints into (5), can get the following four equations:

$$ \begin{cases} \theta_0 = a_0 \\ \theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 \\ \dot{\theta}_0 = a_1 \\ \dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 t_f^2 \end{cases} \quad (8) $$

The cubic polynomial coefficients can be obtained by solving the problem:

$$ \begin{cases} a_0 = \theta_0 \\ a_1 = \dot{\theta}_0 \\ a_2 = \dfrac{3}{t_f^2}(\theta_f - \theta_0) - \dfrac{2}{t_f}\dot{\theta}_0 - \dfrac{1}{t_f}\dot{\theta}_f \\ a_3 = -\dfrac{2}{t_f^3}(\theta_f - \theta_0) + \dfrac{1}{t_f^2}(\dot{\theta}_0 + \dot{\theta}_f) \end{cases} \quad (9) $$

By bringing the above coefficients into the cubic polynomial general formula, the trajectory function $\theta(t)$ between the two nodes can be obtained.

4.2. Quintic polynomial interpolation function

The general formula of quintic polynomial function is:

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \tag{10}$$

The trajectory function $\theta(t)$ curve needs to satisfy eight constraints, including angle constraint, velocity constraint, acceleration constraint and impact constraint of starting point and ending point.

Angle constraint:

$$\begin{cases} \theta(0) = \theta_0 \\ \theta(t_f) = \theta_f \end{cases} \tag{11}$$

Angular velocity constraint:

$$\begin{cases} \dot{\theta}(0) = \dot{\theta}_0 \\ \dot{\theta}(t_f) = \dot{\theta}_f \end{cases} \tag{12}$$

Angular acceleration constraint:

$$\begin{cases} \ddot{\theta}(0) = \ddot{\theta}_0 \\ \ddot{\theta}(t_f) = \ddot{\theta}_f \end{cases} \tag{13}$$

By bringing the above six constraints into the quintic polynomial general formula, six equations can be obtained:

$$\begin{cases} \theta_0 = a_0 \\ \theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 + a_4 t_f^4 + a_5 t_f^5 \\ \dot{\theta}_0 = a_1 \\ \dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 t_f^2 + 4a_4 t_f^3 + 5a_5 t_f^4 \\ \ddot{\theta}_0 = 2a_2 \\ \ddot{\theta}_f = 2a_2 + 6a_3 t_f + 12a_4 t_f^2 + 20a_5 t_f^3 \end{cases} \tag{14}$$

By solving the above equation, the coefficients of quintic polynomials are obtained:

$$\begin{cases} a_0 = \theta_0 \\ a_1 = \dot{\theta}_0 \\ a_2 = \dfrac{\ddot{\theta}}{2} \\ a_3 = \dfrac{20(\theta_f - \theta_0) - (8\dot{\theta}_f + 12\dot{\theta}_0)t_f - (3\ddot{\theta}_0 - \ddot{\theta}_f)t_f^2}{2t_f^3} \\ a_4 = \dfrac{30(\theta_0 - \theta_f) + (14\dot{\theta}_f + 16\dot{\theta}_0)t_f + (3\ddot{\theta}_0 - 2\ddot{\theta}_f)t_f^2}{2t_f^4} \\ a_5 = \dfrac{12(\theta_f - \theta_0) + (6\dot{\theta}_f + 6\dot{\theta}_0)t_f + (\ddot{\theta}_0 - \ddot{\theta}_f)t_f^2}{2t_f^5} \end{cases} \tag{15}$$

By bringing the above coefficients into the quintic polynomial general formula, the trajectory function $\theta(t)$ between the two nodes can be obtained.

4.3. Quartic polynomial interpolation function

The general formula of a quartic polynomial function is:

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6 + a_7 t^7 \tag{16}$$

The trajectory function $\theta(t)$ curve needs to satisfy eight constraints, including angle constraint, velocity constraint, acceleration constraint and impact constraint of starting point and ending point.

Angle constraint:

$$\begin{cases} \theta(0) = \theta_0 \\ \theta(t_f) = \theta_f \end{cases} \tag{17}$$

Angular velocity constraint:

$$\begin{cases} \dot{\theta}(0) = \dot{\theta}_0 \\ \dot{\theta}(t_f) = \dot{\theta}_f \end{cases} \tag{18}$$

Angular acceleration constraint:

$$\begin{cases} \ddot{\theta}(0) = \ddot{\theta}_0 \\ \ddot{\theta}(t_f) = \ddot{\theta}_f \end{cases} \tag{19}$$

Impact constraints:

$$\begin{cases} \theta^{(3)}(t_0) = j_0 \\ \theta^{(3)}(t_f) = j_f \end{cases} \tag{20}$$

By substituting the above eight constraints into the general formula of seventh degree polynomial, eight equations can be obtained:

$$\begin{cases} \theta_0 = a_0 \\ \theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 + a_4 t_f^4 + a_5 t_f^5 + a_6 t_f^6 + a_7 t_f^7 \\ \dot{\theta}_0 = a_1 \\ \dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 t_f^2 + 4a_4 t_f^3 + 5a_5 t_f^4 + 6a_6 t_f^5 + 7a_7 t_f^6 \\ \ddot{\theta}_0 = 2a_2 \\ \ddot{\theta}_f = 2a_2 + 6a_3 t_f + 12a_4 t_f^2 + 20a_5 t_f^3 + 30a_6 t_f^4 + 42a_6 t_f^5 \\ j_0 = 6a_3 \\ j_f = 6a_3 + 24a_4 t_f + 60a_5 t_f^2 + 1200a_5 t_f^3 + 210a_5 t_f^4 \end{cases} \tag{21}$$

By solving the above equation, the coefficients of the seventh degree polynomial are as follows:

$$\begin{cases} a_0 = \theta_0 \\ a_1 = \dot{\theta}_0 \\ a_2 = \dfrac{\ddot{\theta}}{2} \\ a_3 = \dfrac{j_0}{6} \\ a_4 = \dfrac{210(\theta_f - \theta_0) - t_f[(30\ddot{\theta}_0 - 15\ddot{\theta}_f)t_f + (4j_0 + j_1)t_f^2 + 120\theta_0 + 90\theta_f}{6t_f^4} \\ a_5 = \dfrac{-168(\theta_1 - \theta_0) + t_f[(20\ddot{\theta}_0 - 14\ddot{\theta}_f)t_f + (2j_0 + j_1)t_f^2 + 90\theta_0 + 78\theta_f}{2t_f^5} \\ a_6 = \dfrac{420(\theta_1 - \theta_0) - t_f[(20\ddot{\theta}_0 - 39\ddot{\theta}_f)t_f + (4j_0 + 3j_1)t_f^2 + 216\theta_0 + 204\theta_f}{6t_f^6} \\ a_7 = \dfrac{-120(\theta_1 - \theta_0) + t_f[(12\ddot{\theta}_0 - 12\ddot{\theta}_f)t_f + (j_0 + j_1)t_f^2 + 60\theta_0 + 60\theta_f}{6t_f^7} \end{cases}$$

By bringing the above coefficients into the general formula of the seventh degree polynomial, the trajectory function $\theta(t)$ between the two nodes can be obtained.

5. Simulation Analysis of polynomial interpolation trajectory Planning in Joint Space

5.1. Description of optimization mode

The purpose of polynomial interpolation based on joint space planning is to ensure the stationarity of the manipulator in the motion process and to ensure the execution accuracy of the end effector, but the manipulator has different motion requirements under different working conditions. at present, the main use of polynomial interpolation are cubic polynomial interpolation and quintic polynomial interpolation. However, when using polynomial interpolation in joint space trajectory planning, it often causes unnecessary joint impact due to improper selection, and at the same time makes the control of joint drive motor more difficult. There are also some literatures that use seventh-degree polynomials for joint space trajectory planning, but it is not that the higher the degree, the better, too high times will produce Runge phenomenon. although in the trajectory planning of joint space with seven-degree polynomial, the influence of the Runge phenomenon can be greatly reduced by setting the time interval of the manipulator passing through the path point, but it may also lead to other problems.

Based on the above different interpolation methods, this paper studies the influence of different polynomial interpolation on joint posture from single joint space trajectory planning and Cartesian space trajectory planning respectively.

5.2. Simulation Analysis of polynomial interpolation trajectory Planning in Joint Space

When using polynomial interpolation for trajectory planning, it is necessary to comprehensively consider the advantages and disadvantages of different interpolation methods, and the mixed interpolation method can also be used to maximize their advantages. In the working process of the manipulator, there are two cases, namely, through the specified path point motion and only through the starting point and end point movement. passing the specified path point means adding constraints to the polynomial interpolation. there will be changes in joint angle, angular velocity and angular acceleration, which is also the focus of path planning. Therefore, based on the difficult and easy order of verification, this section takes the posture and motion of a single joint as the research object, adopts the research method of multi-path points, and sets the position and attitude constraints of different path points on angle, angular velocity and angular acceleration in advance. As a known condition for finding the polynomial interpolation coefficient, different interpolation methods are used to plan the joint trajectory under this pose constraint.

As a result, the characteristics of different interpolation methods are preliminarily evaluated. In order to make the abrupt change of the manipulator joint at the path point more obvious, the pose constraint parameters of different path points are set as shown in Table 2.

Tab.2 Position and pose settings of joints at different path points

Path point	Time of arrival/ s	Joint angle/(rad)	Joint angular velocity/(rad/s)	Joint angular acceleration/(rad/s^2)	Joint impact/(rad/s^3)
starting point A	0	0	0	0	0
Intermediate point B	4	0.87	0	0	0
Intermediate point C	6	1.74	0	0	0
destination D	10	2.27	0	0	0

As can be seen from the content of the third section, in order to study the polynomial interpolation of a single joint, it is necessary to specify the pose of the joint at the path point in advance, that is, angle θ, angular velocity $\dot{\theta}$ and angular acceleration $\ddot{\theta}$, and there are three known constraints for one position. according to equations (5), (10) and (16), cubic polynomials have four unknown coefficients, which need to be solved by two position points. The quintic polynomial has six unknown coefficients, which needs to be solved by at least two position points, while the seventh degree polynomial needs at least three position points. Therefore, combined with the starting point and the end point, a total of four position points are set, that is, the starting point A, the intermediate point B, the intermediate point C and the end point D. The whole running time is set to 10 seconds, the interval between point An and point B is 4 seconds, the interval between point B and point C is 2 seconds, and the interval between point C and D is 4 seconds. The position and posture of each path point is set as shown in Table 2.

The MATLAB program is written according to the definitions and formulas of cubic, quintic and heptic polynomials, and the different simulation results are shown in figure 3-5.

Figure 3 shows the angle change of the joint. When different interpolation methods are applied to the joint, the change trend of the angle is almost the same, but at the path points tweak 4s and tweak 6s, the angle transition is smoother with the increase of the number of polynomials.

Figure 4 shows the change of the angular velocity of the joint. The maximum angular velocities of cubic, quintic and seventh polynomial interpolation are 0.6525rad/s, 0.8156rad/s and 0.9516rad/s respectively, as shown in Table 3. At the path points tweak 4s and tweak 6s, the transition of joint angular velocity of cubic polynomial interpolation is not stable, and the transition of cubic polynomial interpolation is more stable than that of quintic polynomial interpolation. In the working process of the manipulator, the larger the angular speed is, the more difficult it is to control the joint motor and the lower the control accuracy of the end-effector. in this simulation case, the maximum angular velocity of cubic polynomial interpolation is 20% lower than that of quintic polynomial, and the maximum angular velocity of quintic polynomial interpolation is 14% lower than that of quintic polynomial.

Figure 5 shows the joint acceleration curve. From the results, it can be seen that the acceleration mutation occurs directly at the path points tweak 4s and tweak 6s, resulting in the impact of qz nodes. Compared with quintic polynomial interpolation, the transition at path points is more stable. According to Table 3, the maximum angular acceleration of quintic polynomial interpolation is 23% lower than that of quintic polynomial interpolation.

From the above analysis, we can see that the higher the number of polynomials, the smoother the transition at the critical path point, but it will increase the maximum velocity and angular velocity of the joint of the manipulator, while if the number of times is too low, it will lead to a sudden change in joint acceleration, causing the manipulator to vibrate and affect its working performance. therefore, quintic polynomial interpolation is compared with the method of seventh degree polynomial interpolation on the premise of reducing joint impact at the path point. It can effectively reduce the maximum acceleration.

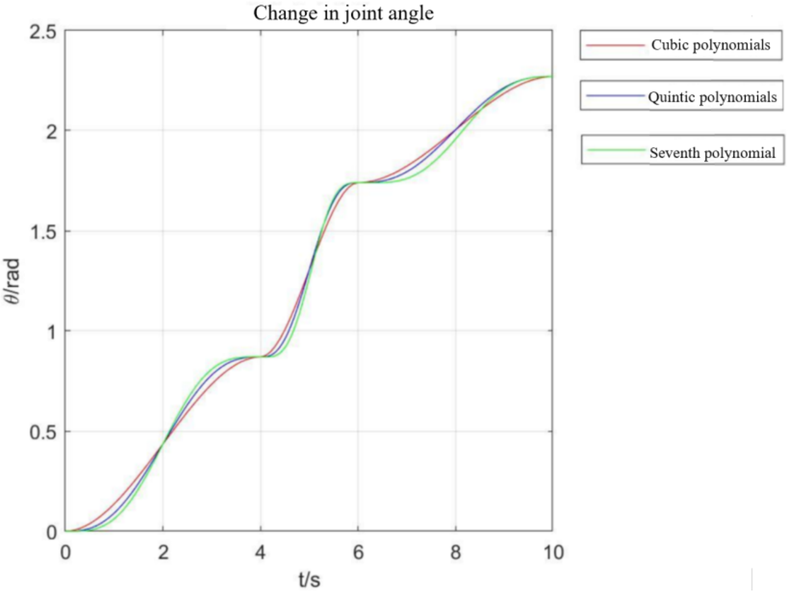

Fig.3 Simulation of Polynomial interpolation joint angle

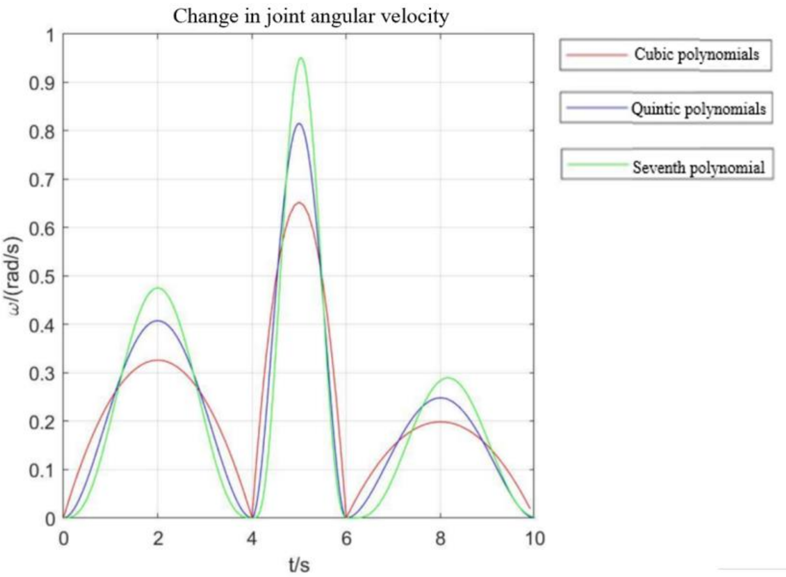

Fig.4 Simulation of Polynomial interpolation joint angular velocity

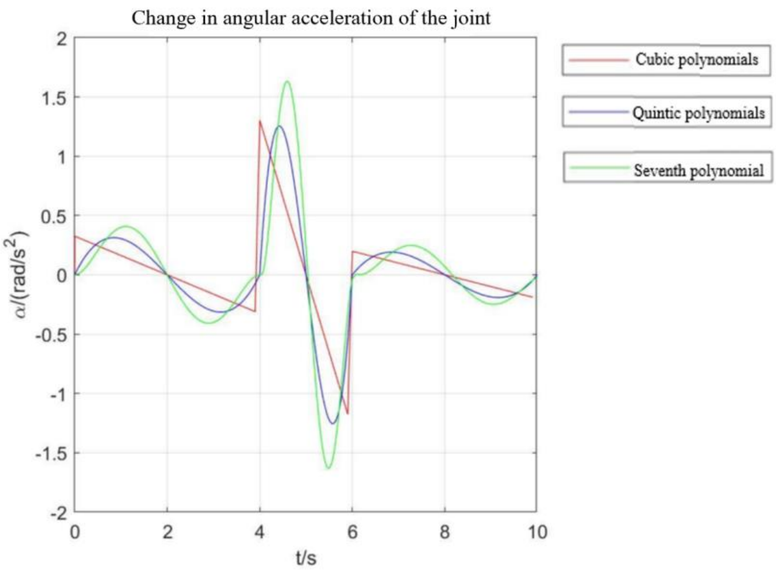

Fig.5 Simulation of Polynomial interpolation angular acceleration

Tab.3 Key simulation results on joint space Polynomial interpolation

	Cubic polynomial interpolation	Quintic polynomial interpolation	Seven-degree polynomial interpolation
Transition of angle curve	smoothness	smoothness	smoothness
Transition of angular velocity curve	There is a break point at the path point	smoothness	smoothness
Transition of angular acceleration curve	There is a mutation at the path point.	There is a mutation at the path point	smoothness
Maximum angular velocity /(rad/s)	0.6525	0.8156	0.9516
Maximum angular acceleration /(rad/s^2)	1.3037	1.2528	1.6303

5.3. Simulation Analysis of polynomial interpolation trajectory Planning in Cartesian Space

The simulation results of joint space trajectory planning can preliminarily understand the influence of different polynomial interpolation methods on joint motion, but it may have some particularity. This section is based on the Cartesian space trajectory planning of 6-degree-of-freedom manipulator PUMA560 path point, and also carries on the path point constraint to the end effector trajectory, and then interpolates the path through different polynomials. In the case of obtaining the position and attitude of the end-effector, the angles, angular velocity and angular acceleration of the six joints are solved by the inverse kinematics of the robot, and the application effects of different polynomials are analyzed.

Because the application of cubic polynomial interpolation is relatively simple and has obvious shortcomings, that is, the angle cubic curve can transition smoothly at the path point, joint space trajectory planning has been able to show all its characteristics that the angle and angular acceleration will not be smooth and abrupt at the path point curve, so this section only carries on the trajectory interpolation analysis of quintic and seventh degree polynomials.

(1) Set the simulation path

For trajectory planning in Cartesian space, it is also necessary to assume two path points in advance, the starting point E and the end point F, set corresponding constraints at the two path points, and observe the posture changes of the six joints.

Tab.4 Attitude settings of actuators at different path points

Path point	arrival time /s	Angular velocity of actuator /(rad/s)	Angular acceleration of actuator /(rad/s^2)	End effector impact /(rad/s^3)
starting point E	0	0	0	0
destination F	10	0	0	0

Based on the start and end points pre-set in Cartesian space, different polynomial interpolation will form different trajectories at the near point of the path, thus planning two kinds of trajectories, as shown in figures 6 and 7.

(2) Analysis of simulation results

The same path points are interpolated with quintic and seventh degree polynomials respectively, and the simulation results are observed and analyzed. The simulation results are shown in figure 8-13.

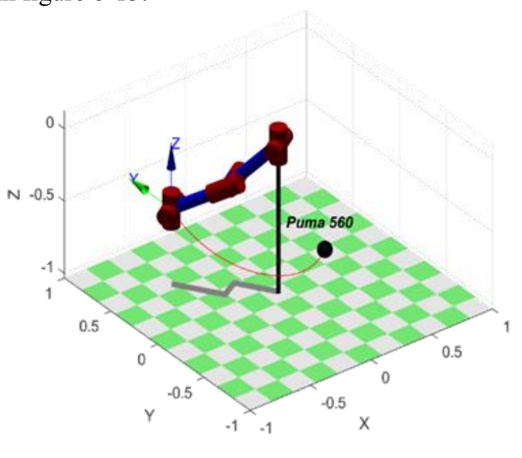

Fig.6 Quintic polynomial trajectory planning

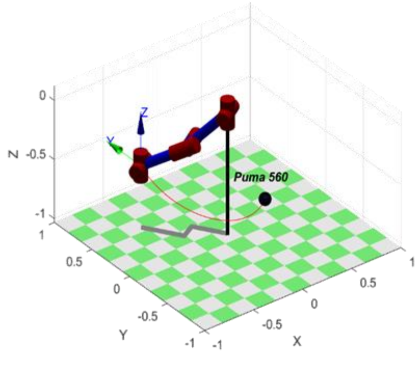

Fig.7 Seventh-degree polynomial trajectory planning

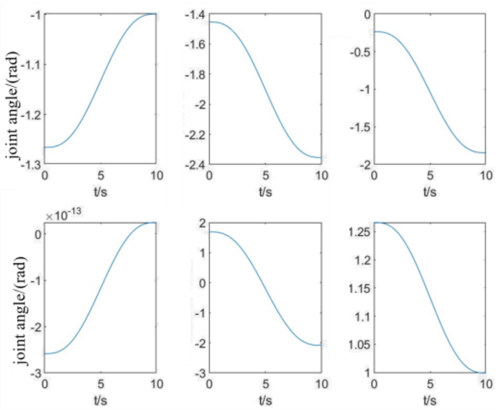

Fig.8 Five-degree polynomial planning of the posterior joint 1 ~ 6 angle position

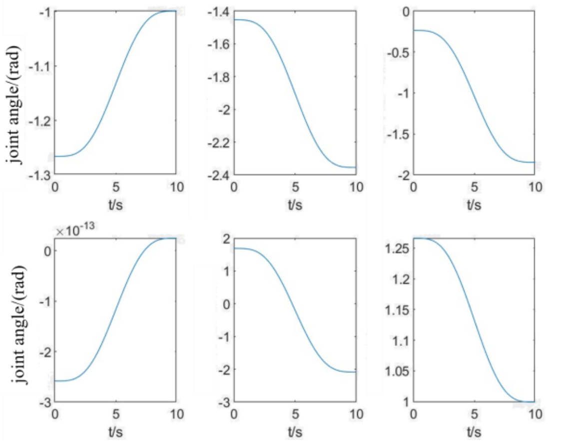

Fig.9 Seven-degree polynomial planning of the posterior joint 1 ~ 6 angle position

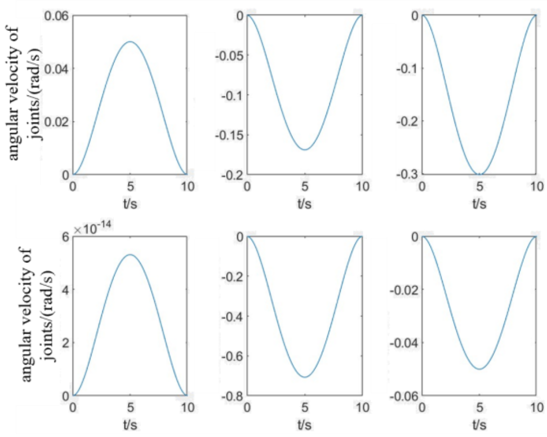

Fig.10 Five-degree polynomial programming for posterior joint 1-6 angular velocities

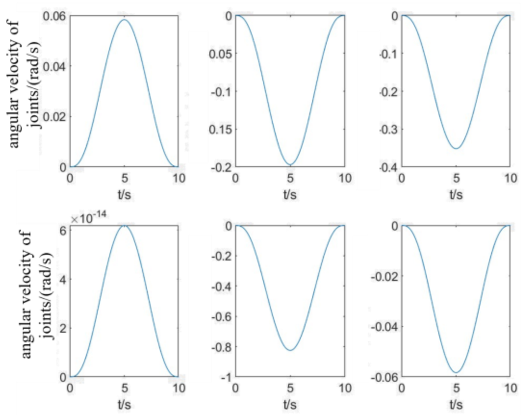

Fig.11 Seven-degree polynomial programming for posterior joint 1-6 angular velocities

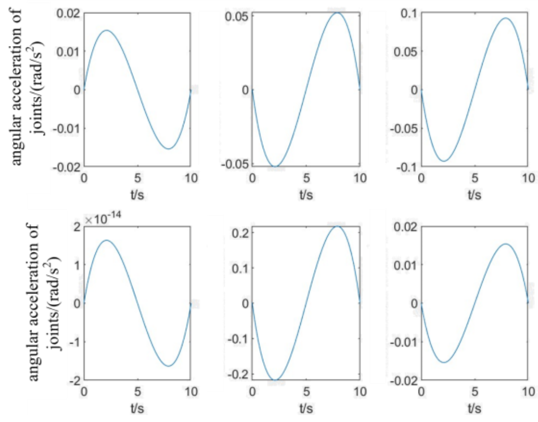

Fig.12 Five-degree polynomial programming for 1-6 angular acceleration of the posterior joint

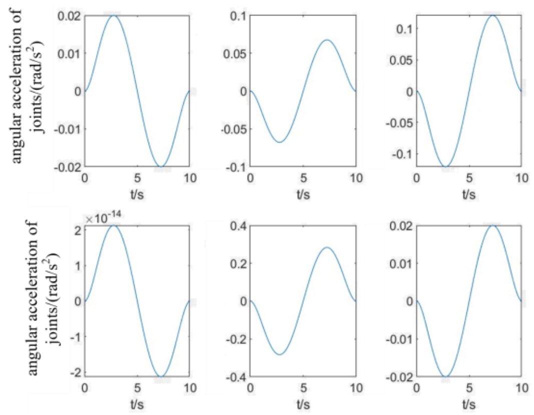

Fig.13 Seven-degree polynomial programming for 1-6 angular acceleration of the posterior joint

Tab.5 Descartes, Polynomial interpolation key simulation results

	Five Polynomial interpolation	Seven Polynomial interpolation
Transition of angle curve	smoothness	smoothness
Transition of angular velocity curve	smoothness	smoothness
Transition of angular acceleration curve	Smoother	smoothness
Average absolute maximum angular velocity of 6 joints /(rad/s)	0.2130	0.2485
Average absolute maximum angular acceleration of 6 joints/(rad/s^2)	0.0656	0.0853

The simulation results show that both quintic polynomial interpolation and quartic polynomial interpolation can make the angular velocity and angular acceleration of six joints transition smoothly. According to the image curve, the transition of velocity and acceleration of quartic polynomial at the starting point and end point is smoother, while the quintic polynomial interpolation method can effectively reduce the impact at critical path points. According to the results of table 5, the average absolute maximum angular velocity and absolute maximum angular acceleration of quintic polynomials are 14% and 23% lower than those of quartic polynomials, respectively, which are consistent with the data of two kinds of interpolation in joint space. it comprehensively reflects that the seven-degree polynomial interpolation is more difficult to control.

Through the trajectory planning simulation of the manipulator in Cartesian space, for the same path, the seventh polynomial interpolation trajectory planning can reduce the impact on the critical path points, but it will increase the angular velocity and angular acceleration of each joint during the motion of the manipulator, and increase the difficulty of control. Although the transition of quintic polynomial interpolation trajectory planning is slightly less smooth than that of quartic polynomial interpolation at critical path points, it can make the overall angular velocity and angular acceleration of each joint lower than that of quartic polynomial interpolation, and reduce the difficulty of control. The conclusion is consistent with the trajectory planning of joint space.

6. Selection and Analysis of polynomial interpolation method for trajectory Planning

6.1. Single-class polynomial interpolation trajectory planning and selection

Cubic polynomial: because the angle is a cubic polynomial about time, for angular velocity, it is a quadratic polynomial about time, which is a parabola. For acceleration, it is a first-degree polynomial about time, which is a straight line, so it will make the angular velocity and angular acceleration of the joint change unsteadily and the acceleration abrupt at the critical path point, resulting in an impact. therefore, the application of cubic polynomials in trajectory planning is only suitable for situations where there are no special requirements in the working environment and low motion accuracy.

Quintic polynomial: the use of quintic polynomial can make the joint motion process more stable, and the joint motion process can be easily controlled, but the acceleration at the key points will produce slight irregularity.

Quintic polynomial programming is suitable for trajectory planning with relatively long paths and few designated path points, and the motion accuracy is relatively high. Seventh degree polynomial: the use of seventh degree polynomial can make the joint motion process stable, and the joint motion at the key point will not produce acceleration mutation, but because it will lead to relatively high angular acceleration and angular velocity, the joint motion process can be easily controlled. Therefore, it is suitable for the situation of short path, more path points and high motion accuracy. The summary is shown in Table 6.

Tab.6 Characteristics and application of different interpolation methods

	Features	Application scenarios
Cubic polynomial interpolation	The transition of the joint angle is smooth, and the angular velocity and angular acceleration of the path point will have an impact.	There are no special requirements in the working environment and the motion accuracy is not high.
Quintic polynomial interpolation	The transition of joint angle and angular velocity is stable, the transition of angular acceleration of path point is stable, and the process of joint motion is easy to control.	In the trajectory planning with relatively long paths and fewer designated path points, the motion accuracy is relatively high.
Seven degree polynomial interpolation	The transition of joint path point angle, angular velocity and angular acceleration is stable, and the joint motion process of larger angular velocity and angular acceleration can be easily controlled.	In the trajectory with shorter path and more path points, the motion accuracy is high.

6.2. Selection of mixed polynomial trajectory planning

After the systematic analysis of different polynomial interpolation, the mixed polynomial interpolation can be carried out according to the actual application scene and the respective characteristics of the polynomial.

Taking the angular velocity simulation results of 4.2 joint space trajectory planning as an example, using the seventh degree polynomial for a long time will make the joint control more difficult, so the seventh degree polynomial can be used in the middle path and the interpolation of other paths. for higher accuracy requirements, quintic polynomials can be used, and cubic polynomials can be used. The mixed interpolation results are shown in figure 14.

Joint space trajectory planning with mixed polynomial interpolation of 5-7-5 is used. As shown in figure 15, 0-4s and 6-10s are quintic polynomial interpolation, 4-6s is 7 degree polynomial interpolation. The curve transition of angle, angular velocity and angular acceleration at the critical path points t =0s, 4s, 6s and 10s is obviously better than that of single-class polynomial interpolation, and there is no obvious curve break and sudden change, which reduces the difficulty of joint control and reduces the impact of joints at the path points.

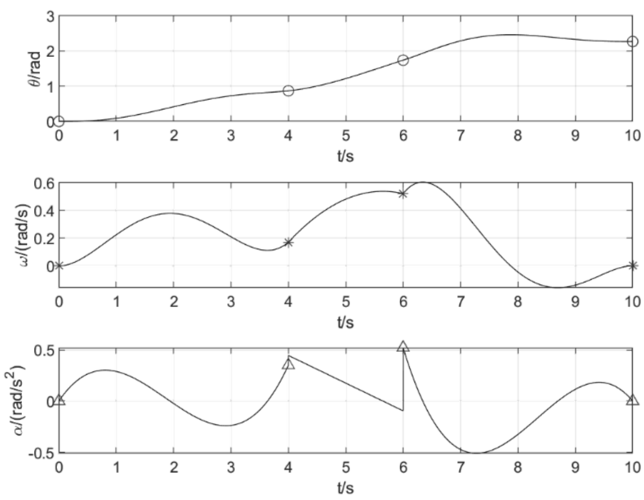

Fig.14 Joint Space 5-7-5 polynomial mixed interpolation joint pose changes

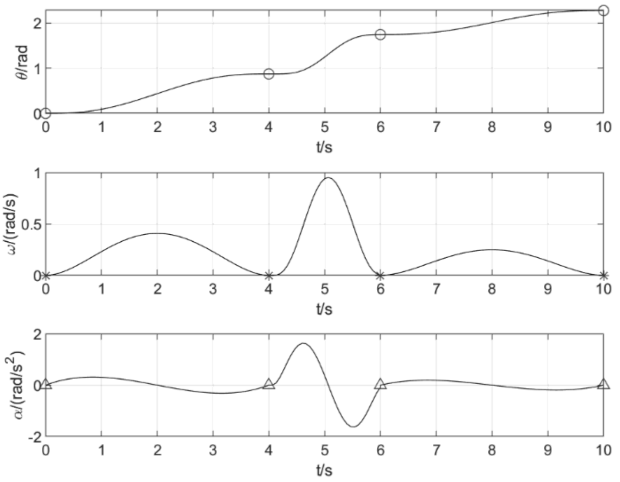

Fig.15 Joint Space 5-3-5 polynomial mixed interpolation joint pose changes

As another example, 5-3-5 polynomial mixed interpolation, such as figure 15, compared with 5-7-5 interpolation, although the maximum angular velocity and maximum angular acceleration are reduced, but the joint will produce vibration, but its vibration is obviously weaker than cubic polynomial interpolation, so it can be used in the case of low precision.

Similarly, according to the different accuracy and the requirements of critical path points, 7-5-7, 3-5, 5-3 and even four-stage interpolation such as 7-5-3-7 can be used, but the specific simulation results will not be described one by one.

The key of polynomial mixed interpolation lies in the position constraint and number of key points in spatial planning, the length of motion path and the execution accuracy of the end actuator. Therefore, the high number of interpolation is used in the critical path points, and the lower number of interpolation can be used in other path points, which can avoid joint impact as much as possible, reduce the difficulty of operation, and improve the execution accuracy of the end-effector.

7. Conclusion

This paper takes the 6-degree-of-freedom manipulator PUMA560 as the simulation object, and studies the application characteristics of cubic, quintic and seventh polynomial interpolation through joint space trajectory planning and Cartesian space trajectory planning. The simulation results show that the higher the degree of polynomial, the more stable the joint transition at the critical path point, but it will increase the joint maximum angular velocity and angular acceleration, for the same path. Compared with the average absolute maximum angular velocity and absolute maximum angular acceleration of joints interpolated by quintic polynomials, the average absolute maximum angular velocity and absolute maximum angular acceleration of joints are reduced by 14% and 23% respectively, although quartic polynomials can be used in high precision working environments. however, the joint angular velocity and angular acceleration are too large, which makes it more difficult to control the joint motor, which can be used in the situation of low speed and high precision. Quintic polynomial trajectory planning and control is easy, and the transition of joint position and posture is smooth, so it can be used in situations with high working speed and low precision. The mixed polynomial has a wide range of applications and great selectivity, and its core lies in the full understanding of a single polynomial and the actual trajectory planning application of different polynomials in different paths and under different conditions according to the different demands. make the trajectory of each joint of the manipulator smooth, stable and shock-free.

Through the above research, a comprehensive analysis of the characteristics of polynomial interpolation, better use of single-class polynomial interpolation and mixed polynomial interpolation in trajectory planning to make the manipulator move more smoothly, and then improve the control accuracy of the end-effector.

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