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Research on Production Decision Modelling Based on Sequential Testing and Minimum Objective Planning

Yuan Wei *, Zhenjia Chang

College of Sciences, Liaoning Technical University, Fuxin, China
* **Corresponding author**: Yuan Wei (Email: 2604396348@qq.com)

Abstract: In this paper, the decision-making problem encountered by an enterprise in the production process is studied in depth by using the sequential test and the minimum objective planning method. The study begins with a sampling and testing model for spare parts, simplifies the model based on normal distribution, and applies the sequential test technique to reduce the number of necessary samples and effectively optimize the cost of sampling and testing. Immediately after that, we constructed a minimum objective planning model to model the production process of spare parts and finished products and their related costs in detail. We solved the optimal production cost for each of the six real production scenarios faced by the enterprise. The results show that our model can not only effectively reduce production costs, but also significantly improve decision-making efficiency.

Keywords: Binomial Distribution; Normal Distribution; Ordinal Test; Minimum Objective Planning.

1. Introduction

The purpose of this paper is to explore the decision-making problem in the production process of a company, especially for spare parts sampling and testing and production cost optimization. First, we focus on the sampling and testing of spare parts, in which the lack of an effective sampling strategy in traditional sampling and testing often leads to the waste of resources and increase in costs [1,2]. In order to solve this problem, this paper simplifies the sampling and testing model of spare parts based on normal distribution and introduces the sequential test method, which reduces the number of unnecessary samples and optimizes the cost of sampling and testing [3,4,5]. Immediately after that, we turn to the optimization problem of production cost. In this paper, we develop a minimum objective planning model that integrates the production processes of spare parts and finished products as well as the associated costs. The simulation results show that the proposed model can significantly reduce the production cost and improve the efficiency of decision-making [6,7,8].

2. Sampling and testing model based on normal distribution and sequential testing

In the product development and production process, sampling test is to ensure that the product meets the design requirements and its quality indicators to achieve the required level of essential means. In the reliability testing of electronic products, how to improve the efficiency of product sampling test, reduce the cost of product sampling test is the core issue in the design of sampling test program. For the design of a spare parts sampling and testing program, we investigated the quality of spare parts of an enterprise to obtain the relevant data. Supplier claims that a batch of spare parts defective rate will not exceed 10%, the enterprise is ready to use the sampling test method to decide whether to accept the purchase of this batch of spare parts from the supplier, the testing costs borne by the enterprise itself.

The quality inspection of spare parts is a discrete process and the results are only pass and fail, so we use binomial distribution $X \sim B(n,p)$ to model the inspection results. The binomial distribution $X \sim B(n,p)$ is used to describe the sampling and testing process, which shows that the defective rate of spare parts is:

$$p\{n=k\} = \binom{n}{k} p^k (1-p)^{n-k} \tag{1}$$

where p is the defective rate, n is the number of parts sampled, and k is the number of defective parts in the sampled parts.

Based on the sampling test to determine whether to accept or reject the spare parts, we assume that:

H_0: The actual defective rate of the spare parts θ_0 is greater than or equal to the nominal value θ, i.e. $\theta_0 \geq \theta$. If the test results H_0 is support, then we reject the part.

H_1: The actual defect rate of the parts θ_0 is less than the nominal value θ, i.e. $\theta_0 < \theta$. If the test results H_1 is support, then we accept this lot of parts.

Often, sampling is used because of the large sample size of spare parts. In the binomial distribution, when the sample size n is large, the binomial distribution can be approximated by the normal distribution to simplify the calculation. According to the central limit theorem, the binomial distribution can be approximated by the normal distribution:

$$SE = \sqrt{\frac{\theta_0(1-\theta_0)}{n}} \tag{2}$$

where the nominal value θ_0 is 10%, n is the number of spare parts taken, $\hat{p}$ is the defective rate obtained from the normal distribution.

According to the normal distribution approximation, the confidence interval of the defective rate can be expressed as:

$$\hat{p} \sim Z_{\frac{\alpha}{2}} \times SE \tag{3}$$

i.e.

$$\hat{p} \pm Z_{\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \tag{4}$$

where $Z_{\frac{\alpha}{2}}$ is the critical value corresponding to the confidence level $1-\alpha$ in the standard normal distribution.

We calculate the critical values for different confidence limits according to the

$$\sigma_{\hat{p}} = \sqrt{\frac{\theta_0(1-\theta_0)}{n}}, Z = \frac{\hat{p}-\theta_0}{\sigma_{\hat{p}}} \tag{5}$$

where $\hat{p}$ is the defective rate in the sample, θ_0 is the nominal value 10%, $\sigma_{\hat{p}}$ and is the standard error of the defective rate in the sample. It can be derived $Z_{0.95} = 1.96$, $Z_{0.90} = 1.645$.

Using the cumulative distribution function in the binomial distribution, we can calculate the critical values for rejection and acceptance in both the 95% confidence and 90% confidence cases:

Rejection threshold: If the number of defective products detected k is greater than a certain value k_{reject} at a certain confidence level 95%, we can reject the supplier's stated defect rate, i.e. the assumption H_0 holds. i.e:

$$P(k \geq k_{reject} \mid \theta = \theta_0) \geq 0.95 \tag{6}$$

Corresponding to the criterion of rejection, the upper confidence limit of the substandard rate of the sample is:

$$\hat{p}_{upper} = \theta_0 + Z_{0.95} \times \sqrt{\frac{\theta_0(1-\theta_0)}{n}} \tag{7}$$

If the defect rate $\hat{p}$ in the sample exceeds this limit, the lot is rejected.

Acceptance Threshold: At a confidence level 90%, if the number of defective parts detected k is less than a certain value k_{reject}, we have reason to believe that the percentage of defective parts in the lot is less than the nominal value, and we can accept the lot of parts. i.e:

$$P(k < k_{reject} \mid \theta = \theta_0) \geq 0.90 \tag{8}$$

For the acceptance criteria, the lower confidence limit for the sample defective rate is:

$$\hat{p}_{lower} = \theta_0 - Z_{0.90} \times \sqrt{\frac{\theta_0(1-\theta_0)}{n}} \tag{9}$$

If the percentage of defective parts in the sample $\hat{p}$ is below this lower limit, the lot of parts is accepted.

Therefore, in the process of calculating the confidence limit, the sample size n directly affects the cost of testing and the accuracy of the acceptance or rejection decision. If the sample size is too large, the cost of testing is too high; if the sample size is too small, there is no guarantee of sufficient confidence in the decision. Our goal is to find a minimum sample size that satisfies the rejection and acceptance criteria while detecting the critical value condition. In order to determine the sample size n, we are based on the desired range error and confidence level, we want to limit the error in this confidence interval to the allowable range error E:

$$Z_{\frac{\alpha}{2}} \sqrt{\theta_0(1-\theta_0)} \leq E \tag{10}$$

Squaring both sides to eliminate the square root and shifting the terms respectively solves for:

$$n \geq \frac{Z_{\frac{\alpha}{2}} \sqrt{\theta_0(1-\theta_0)}}{E})^2 \tag{11}$$

where E is the allowed error and $Z_{\frac{\alpha}{2}}$ is the corresponding critical value of the standard normal distribution, i.e.

According to the nominal value $\theta_0 = 10\%$, we can calculate the sample size under different confidence levels. Assuming the allowable error $E = 0.02$, according to the above formula, we can calculate the corresponding sample size n:

$$n = (\frac{1.96 \times \sqrt{0.1 \times 0,9}}{0.02})^2 = 865 \tag{12}$$

This means that approximately 865 parts need to be sampled to determine if the defect rate exceeds 10%, at the confidence level 95%.

To further minimize the number of inspections and reduce inspection costs, we can use sequential inspection. Sequential testing can be in the inspection process more bureaus each time the results of the inspection dynamically adjusted whether to continue sampling, until the decision to reject or accept. Assuming that the defective rate has two hypothetical values $p_0 = 0.12, p_1 = 0.08$, then according to the SPRT judgment rule:

(1) The condition H_0 is valid for $865 \times 0.12 \approx 104 > 865 \times 0.10 = 101$, and the batch of spare parts is rejected directly. Therefore, when the defective product is greater than 101, no subsequent testing will be done, regardless of whether 865 sample values are tested or not.

(2) The condition H_1 for the condition to hold is $865 \times 0.08 \approx 69 < 865 \times 0.10 = 101$, i.e., there must not be more than 101 nonconforming products.

When the sample size is 865, different defective rates have certain effects on rejection and acceptance, so we carry out a sensitivity analysis to make the defective rate change and observe the change of its rejection critical value and acceptance critical value. We change the acceptance and rejection thresholds according to the trend of changing different defective rates as shown in Figure 1 below.

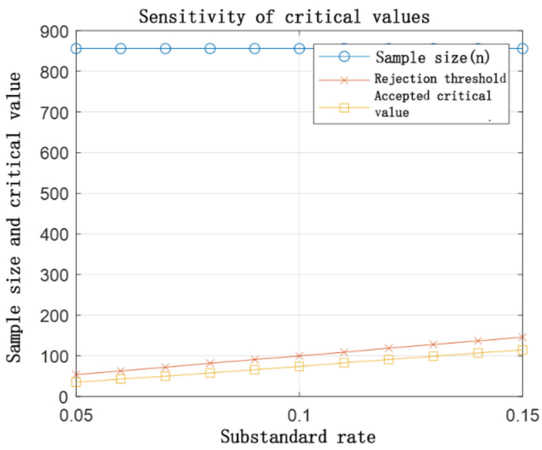

Figure 1. Changes in acceptance and rejection thresholds under changes in defect rates

From the above figure, we can conclude that the rejection threshold and acceptance threshold are constantly adjusted with the change of substandard rate. Enterprises can adjust the critical value of rejected and accepted spare parts according to the difference of their substandard rate.

3. A production decision model based on minimum objective planning

In this chapter, we discuss the firm's decision-making problem in the manufacturing process for the production of spare parts, semi-finished products, and finished products. We have collected data on the production process. The firm produces a best-selling electronic product and needs to purchase spare parts separately and assemble them into finished products at the firm. In the production process, the enterprise needs to make decisions on each link, the specific decisions include whether to quality check the parts, whether to quality check the finished product, whether to disassemble the finished product when it is unqualified, whether to disassemble the returned unqualified product again and handle the operation, etc.

Let us assume that there are n part 1 and n part 2 and that the finished product consists of one part 1 and one part 2. Based on the test of spare parts, decide whether to continue production; test the finished product, decide whether to sell; and then decide whether to dismantle the substandard finished product dismantled spare parts to re-enter the spare parts inspection.

If the defective rate of part 1 is p_1, then the number of parts 1 available after testing is:

$$N_1 = N \cdot (1-p_1) \tag{13}$$

If Part 1 is tested, the detected defective parts are directly discarded, then the number of remaining usable Parts 1 is:

$$N_1 = x_1 \cdot N \cdot (1-p_1) + (1-x_1) \cdot N \tag{14}$$

Similarly, if the defective rate of spare part 2 is p_2, then the number of remaining usable spare parts 2 is:

$$N_2 = x_2 \cdot N \cdot (1-p_2) + (1-x_2) \cdot N \tag{15}$$

Then, from the above, it can be seen that the quantity of the finished product is determined by the lesser quantity of spare part 1 and spare part 2:

$$N_{\text{product}} = \min(N_1, N_2) \tag{16}$$

The production decision of the product contains four parts: spare parts inspection, finished product inspection, unsold disassembly of nonconforming products, sold disassembly of nonconforming products, etc. The costs incurred in these four processes are; spare parts inspection costs, finished product inspection costs and nonconforming products replacement losses, assembly costs and disassembly costs.

Parts testing costs:

(1) for the detection cost of spare parts 1:

$$\mathrm{C}_{d1-total}{}^{(k)} = x_1^{(k)} \cdot N^{(k)} \cdot (C_{d1} + C_{b1}) + (1-x_1^{(k)}) \cdot N^{(k)} \cdot p_1 \cdot C_{b1} \tag{17}$$

(2) For the cost of inspection of spare parts 2:

$$\mathrm{C}_{d2-total}{}^{(k)} = x_2^{(k)} \cdot N^{(k)} \cdot (C_{d2} + C_{b2}) + (1-x_2^{(k)}) \cdot N^{(k)} \cdot p_2 \cdot C_{b2} \tag{18}$$

where $\mathrm{C}^{(k)}_{d1-total}$ and $\mathrm{C}^{(k)}_{d2-total}$ are the cost of the first round k of testing for Part 1 and Part 2, respectively, $x_1^{(k)} \in \{0,1\}$ indicate whether or not to test Part 1, $x_2^{(k)} \in \{0,1\}$ indicate whether or not to test Part 2, $N^{(k)}$ are the total number of pieces produced, C_{d1} indicate the cost of testing a Part 1, C_{d2} indicate the cost of testing a Part 2, C_{b1} indicate the cost of purchasing a Part 1, C_{b2} indicate the cost of purchasing a Part 2, p_1 indicate the defective rate of a Part 1, p_2 indicate the defective rate of a Part 2.

Finished Product Inspection and Exchange Losses:

Finished Product Inspection Cost:

$$C_{dc}{}^{(k)} = y^{(k)} \cdot N^{(k)} \cdot C_{dc} \tag{19}$$

Loss on exchange:

$$C_{\text{exchange loss}}{}^{(k)} = (1-y^{(k)}) \cdot N^{(k)}_{\text{product}} \cdot p_c \cdot L \tag{20}$$

where C_{dc} denotes the cost of finished product testing, $C_{\text{Exchange losses}}$ denotes the exchange loss; when $y^{(k)} = 1$ denotes that the finished product is tested, when $y^{(k)} = 0$ denotes that it is not tested; C_{dc} denotes the cost of finished product testing; $p_c = 1-(1-p_1)(1-p_2)$ denotes the defective rate of the finished product; and L denotes the exchange loss caused by the defective product after entering the market.

Assembly Losses:

The assembly cost per bearing flat is a fixed cost, i.e.:

$$\mathrm{C}_{a-total}{}^{(k)} = N^{(k)}_{\text{product}} \cdot C_a \tag{21}$$

where $\mathrm{C}^{(k)}_{a-total}$ is the cost of the first round of assembly, C_a expressed as the cost of assembling a finished product.

Disassembly cost:

If a non-conforming finished product is disassembled, the corresponding cost is:

$$\mathrm{C}_{r-total}{}^{(k)} = z^{(k)} \cdot N^{(k)}_{\text{product}} \cdot p_c \cdot C_r \tag{22}$$

where $\mathrm{C}_{r-total}$ represents the total dismantling cost, when $z = 1$ indicates the choice of dismantling, dismantling the unqualified finished products; $z = 0$ denotes the direct scrapping of unqualified products without dismantling; C_r denotes the dismantling cost of a finished product.

Summarize the above can be obtained from the total cost of the objective function:

$$C_{total} = \sum_{k=1}^{K} [C_{d1-total}{}^{(k)} + C_{d2-total}{}^{(k)} + C_{dc}{}^{(k)} + C_{\text{exchange loss}}{}^{(k)} + C_{r-total}{}^{(k)} + C_{a-total}{}^{(k)}] \quad (23)$$

We consider minimizing the total cost, i.e:

$$\min C_{total} \quad (24)$$

Since the decision variables $x_1, x_2, y, z \in \{0,1\}$ are binary decision variables, then they can be solved by linear programming, which we have collected for six scenarios of the firm's production, as shown in table 1:

And it is assumed that there will be a maximum of three rounds of the same spare parts returned with unsatisfactory assembly. Based on the assumptions and the minimum cost formula obtained above, the minimum cost for six scenarios can be derived:

Case 1:

From the above table 2, it can be seen that when all three rounds are tested and the nonconforming product is directly discarded, the minimum cost required under the conditions of case 1, i.e., the minimum cost is ¥28,600.

Case 2:

From the above table 3, it can be seen that when all the three rounds are tested and the nonconforming products are directly discarded, the minimum cost required under the condition of case 2, i.e., the minimum cost is ¥29,200.

Case 3:

From the above table 4, it can be seen that the minimum cost required under the conditions of case 3, i.e., the minimum cost is ¥31,000 when the inspection is performed only at the leveling step in all the three rounds and the nonconforming product is discarded directly.

Case 4:

From the above table 5, it can be seen that the minimum cost required under the conditions of Case 4, i.e., the minimum cost is ¥29,400, when spare part 1 is tested in the first round and the finished product is tested in all three rounds as well as when the rejected product is simply discarded.

Case 5:

From the above table 6, it can be seen that the minimum cost required under the conditions of Case 5, i.e., the minimum cost of ¥28,600, is required when only Spare Part 2 is tested in the three rounds as well as when the nonconforming product is simply discarded.

Case 6:

From the above table 7, it can be seen that the minimum cost required under the conditions of case 6, i.e., the minimum cost is $28,500, when only testing of spare part 2 is performed in all three rounds as well as when the nonconforming product is directly discarded. In the above tables 2 to 7 where 0 means no testing and 1 means testing, we can get the specific decision making scenarios for different cases.

Table.1 Six cases of costs, fees and unit prices in different situations

Case	Parts 1			Parts 2			Product				Unqualified products	
	Defective rate	Prices	Testing costs	Defective rate	Prices	Testing Costs	Defective rate	Assembly cost	Testing Costs	Selling prices	Exchange losses	Dismantling costs
1	10%	4	2	10%	18	3	10%	6	3	56	6	5
2	20%	4	2	20%	18	3	20%	6	3	56	6	5
3	10%	4	2	10%	18	3	10%	6	3	56	30	5
4	20%	4	1	20%	18	1	20%	6	2	56	30	5
5	20%	4	8	20%	18	1	10%	6	2	56	10	5
6	5%	4	2	5%	18	3	5%	6	3	56	10	40

Table.2 Decision-making and minimum total cost under case 1 conditions

Number of detection rounds	Parts 1	Parts 2	Product	Dismante	Minimum total cost
1	0	0	0	0	28600
2	0	0	0	0	
3	0	0	0	0	

Table.3 Decision-making and minimum total cost under case 2 conditions

Number of detection rounds	Parts 1	Parts 2	Product	Dismante	Minimum total cost
1	0	0	0	0	29200
2	0	0	0	0	
3	0	0	0	0	

Table.4 Decision-making and minimum total cost under case 3 conditions

Number of detection rounds	Parts 1	Parts 2	Product	Dismante	Minimum total cost
1	0	0	1	0	31000
2	0	0	1	0	
3	0	0	1	0	

Table.5 Decision-making and minimum total cost under case 4 conditions

Number of detection rounds	Parts 1	Parts 2	Product	Dismante	Minimum total cost
1	1	0	1	0	29400
2	0	0	1	0	
3	0	0	1	0	

Table.6 Decision-making and minimum total cost under case 5 conditions

Number of detection rounds	Parts 1	Parts 2	Product	Dismante	Minimum total cost
1	0	1	0	0	28600
2	0	1	0	0	
3	0	1	0	0	

Table.7 Decision-making and minimum total cost under case 6 conditions

Number of detection rounds	Parts 1	Parts 2	Product	Dismante	Minimum total cost
1	0	1	0	0	28500
2	0	1	0	0	
3	0	1	0	0	

4. Conclusions

This paper conducts a systematic research on the decision-making problems of an enterprise in the production process, especially the sampling and testing of spare parts and production cost optimization. The paper firstly simplifies the sampling and testing model of spare parts based on normal distribution, and effectively applies the sequential inspection technique to reduce the number of sampling times, thus optimizing the sampling and testing costs. Subsequently, this paper constructs a minimum objective planning model to model the production process of spare parts and finished products and their costs in detail. The optimal production cost is solved for six production scenarios faced by enterprises, and the results show that the proposed model can effectively reduce production cost and improve decision-making efficiency. The results not only provide a scientific decision support tool for enterprises, but also provide a new research perspective in the field of production management. Through the combined application of sequential test and minimum objective planning method, this paper effectively solves the decision-making problem in the production process and realizes the dual objectives of cost control and efficiency improvement. These findings are of great significance for enterprises to maintain their competitiveness in the fierce market competition, and also provide valuable references and inspirations for subsequent related research.

References

[1] ZHANG Xuejian, CHEN Chunyan, ZHAO Na. Application of counting sampling test procedure in quality management of inspection and testing organizationsi J]. China Standardization, 2024, (10): 163-166.

[2] Zhou Q. Approximation of normal distribution for binomial distribution[J]. Shanghai Middle School Mathematics,2024, (Z1):1-3.

[3] Zhu Cui. Detailed description of normal distribution [J]. New Century Intelligence,2024, (Z9):37-40.

[4] Fang Maoda. Research on truncated-tailed sequential optimality test of exponential distribution based on dynamic programming method [D]. Guizhou University, 2023.

[5] Lu Jiaxing. Optimization of Warehouse Picking Paths Based on Minimizing Cargo Loss Cost [D]. Linyi University,2024.

[6] Zhao X. How to do a good job of project cost management in assembly construction enterprises [J]. China Business,2024, (02):106-108.

[7] Yu Yang. Optimization research on production management of assembly line of company H based on lean production[D]. North Central University, 2023.