

Exploring the "Teaching and Learning" of Calculus Course from the Perspective of Teaching Connection

Maojun Zhou, Feng Xu, Xiaowei Cai

School of Statistics and Applied Mathematics, Anhui University of Finance and Economics, Bengbu, 233030, China

Abstract: In the current Mathematics education system, there is an obvious disconnection between college mathematics (taking calculus as an example) and middle school mathematics. If the connection is not smooth, it will directly affect the teaching effect of college mathematics. This article analyzes the causes of poor connection from the perspectives of differences in teaching objectives and methods, disconnection and overlap in teaching content, differences in learning methods, external environmental influences, and individual differences in student sources. We have proposed bridging strategies from the perspectives of teachers' "teaching" and students' "learning", providing teaching references for improving the effectiveness of university mathematics teaching and cultivating high-quality mathematics talents.

Keywords: Teaching Connection; Disjointed; Middle School Mathematics; Calculus.

1. Introduction

For many students who have gone through the college entrance examination and just entered the university gate, this stage is a leap, an improvement, and also faces new tasks and challenges. Adapting to a new lifestyle, improving professional knowledge and skills, and planning future development directions have come one after another, with the primary task being to learn the professional courses offered during the university stage. Calculus is the first mathematical course offered by many first-year students in higher education institutions, and it is an important fundamental theoretical course. It is a continuation of mathematical knowledge in high school, as well as a foundation for subsequent mathematical courses in university and a practical tool for professional courses, playing a role in connecting the past and the future. The content of this course is also a significant part of the mathematics subjects that students take in graduate exams. Learning Calculus well plays a crucial role in cultivating high-quality talents and promoting students' further development.

Due to various factors, some new students may not be able to adapt to the learning style of this course for a long period of time at the beginning of the new semester, believing that the learning difficulty is high and the learning effect is poor, which has affected their confidence. There are both objective and subjective reasons for this. There are significant differences in the classroom environment and teaching methods between Calculus and high school mathematics, as students cannot "move in" according to their previous learning. The poor connection caused by the differences between the two stages is an important factor in the occurrence of this phenomenon. There are many factors such as the repetition and disconnection of textbook content, changes in teaching methods, students' ability and self-awareness to learn independently, individual differences in students, and regional differences in student sources. If students cannot keep up with the pace of the teacher at the beginning, their later learning will become more difficult, and in the long run, they will gradually lose confidence in learning mathematics well, and even affect their self-confidence and mental health.

In response to the current situation of high school mathematics curriculum standards in terms of curriculum objectives, curriculum content, students' learning methods, and teachers' teaching methods, the teaching of university mathematics must be reformed and adjusted accordingly. Therefore, the effective teaching connection between college mathematics and middle school mathematics is a practical problem that cannot be ignored, and it is a subject worthy of in-depth study by Mathematics education in colleges and universities.

2. Review of the Literature

The research on the issue of educational connection between universities and secondary schools abroad mainly focuses on the adaptation of college freshmen to enrollment, teaching methods of teachers, curriculum design for the first year of university, and measures to strengthen the connection in the academic system. The International Mathematics Education Committee (ICMI) discussed the transition from middle school to university, the relationship between middle school and university, and the improvement of training methods for university mathematics teachers at a meeting held in the United Kingdom from November 21 to 24, 1997 [1]. At the international seminar held in Singapore from December 8th to 12th, 1998, there was a thematic discussion on the transition from high school to university [2]. In 1999, Russia issued the "Minimum Contents Must Be Learned in Middle School Mathematics education", the most prominent of which is the "Preliminary Mathematical analysis", including sequence limit, continuity of function, derivative, Newton Leibniz formula, optimal solution and other contents of Further Mathematics [3].

The relevant domestic research is mainly concentrated in the 1990s, when the curriculum reform in the field of basic education in China, which has changed from Teaching to the test to quality education, began to be implemented, and the connection between college mathematics and middle school Mathematics education teaching gradually emerged. Especially since the beginning of the 21st century, with the introduction of new curriculum standards for middle school mathematics, it has become particularly necessary to study the issue of two-level teaching connection. Domestic experts

and scholars have achieved many argumentative research results on the differences and connections between university mathematics and middle school mathematics in teaching concepts, teaching content, teaching methods, learning methods, and cohesive response measures. Hui-Lan Zhang [4] (2001) discussed the differences between university mathematics and middle school mathematics in teaching content, teaching methods, and student learning methods, and explored them from four aspects: the connection of teaching content, teaching mode, student learning methods, and student thinking ability. Yong-Hong Xiao [5] (2009) investigated and analyzed the differences between Further Mathematics and middle school mathematics teaching, and proposed countermeasures and methods for teaching convergence. Liu-Ping Shen [6] (2014) summarized the bottleneck problem in the connection of college mathematics and middle school Mathematics education from three aspects: students' psychological conflict, students' Role conflict, and teachers' professional quality, and gave corresponding operational countermeasures. Hui-Jing Zhan [7] (2019) proposed the connection of the teaching method of "practice first, spiral teaching method" in mathematics between universities and middle schools. Shu-Xia Feng [8-9] (2020) explored the fundamental differences between university mathematics and middle school mathematics in terms of computational and structural aspects of mathematics, and their impact on learning and teaching.

Some researchers choose specific content from mathematical textbooks as the research object to study the problem of connection. Jin-Bo He [10] (2007) suggests using intuitive description, visual expression, rough presentation, precise definition, and geometric analysis to teach the limit. Meng-Ri Xu [11] (2007) proposed several suggestions on the connection between the "preliminary algorithm" part of high school mathematics and university teaching. Kuang Wang [12] (2021) conducted a specific comparative analysis of the derivative teaching case of functions in calculus.

In recent years, with the rise of online courses, some university teacher teams have created relevant video courses to provide a more comprehensive learning platform and pathway for transitional students. For example, Advanced Further Mathematics prepared by the teaching team of Southwestern University of Finance and Economics, and Calculus Enlightenment recorded by Beihang University.

To sum up, many Mathematics education workers have attached great importance to the connection between middle school mathematics and college mathematics, and have also achieved fruitful research results. However, the depth of research is not detailed enough, the results are often repeated, lack of innovation, and the recommendations put forward by some scholars are not operational. Based on practical experience in the teaching process, this article conducts a study on the necessity, feasibility, and countermeasures of the connection between calculus course teaching and middle school mathematics, in order to improve the teaching effect of calculus course and make some meaningful explorations for enriching the reform of university mathematics teaching.

3. Analysis of the Causes of Poor Connection between College Mathematics and Middle School Mathematics

3.1. Differences in Teaching Objectives and Methods

Although the new curriculum standards emphasize putting people first and advocating active exploration of learning methods, in the middle school mathematics teaching stage, due to the enormous pressure of entering the college entrance examination, the main teaching philosophy is still based on the college entrance examination outline as the reference standard, and the focus of teaching is on students' problem-solving ability. Often, the teaching of basic mathematical knowledge and the cultivation of basic computing ability are strengthened through problem-solving tactics, more emphasis is placed on the repeated practice of knowledge points that are easy to examine for the purpose of scores, lacking systematic knowledge and the exploration of internal connections. At the university stage, professional talents need to be cultivated, with more emphasis on the cultivation of students' preciseness, scientificity and innovation. Students should have a deep understanding of Formula, master the principles, be able to derive specifically, and be able to generalize and extend to draw inferences from one instance and understand by analogy. It is necessary to not only master mathematical knowledge, but also learn mathematical thinking methods and be able to flexibly apply them. The course "Calculus" has a complete knowledge system, and the study of calculus is the key to cultivating students' mathematical literacy. It cultivates students' thinking ability in induction and abstraction, from concrete to general association ability, hands-on ability in reasoning and verification, as well as independent learning ability in questioning, criticizing, and correct cognition. From the perspective of teaching objectives, there are higher requirements for students' knowledge reserves and learning abilities, resulting in some high school graduates not meeting the basic requirements for learning the course "Calculus".

There are also differences in the teaching methods of teachers in the two stages. The content of middle school mathematics is relatively simple and intuitive, and teachers pay more attention to the training of skills and methods. Each knowledge point is explained accurately and in detail, and the same question type is repeatedly practiced. Through homework, exams, and other feedback, until most students can master it. Students have also adapted to passive indoctrination learning, relying excessively on the learning methods, problem-solving ideas, and knowledge summarization provided by teachers, lacking the spirit and ability of independent thinking and autonomous learning. College mathematics has rich content, strong theoretical content, abstract concepts, and more emphasis on the completeness of knowledge structure and the rationality of logical reasoning, which exercises students' mathematical thinking methods and ability to solve practical problems. This results in a large amount of information and knowledge points in the classroom content of college mathematics, and the same question type will not be practiced repeatedly. Homework, tests, stage exams, etc. are not as frequent as in middle school. Most university mathematics teachers play the role of guides, requiring students to learn independently, think independently, and summarize after class. The transformation of teaching

methods has made it difficult for some students who are accustomed to the teaching methods of high school mathematics to adapt quickly.

3.2. Disjointed Teaching Content

Primary school, junior high school, and high school belong to the same education system, and the connection between textbooks is very good, with a hierarchical relationship. However, university mathematics places more emphasis on subject education, and the textbook system cannot be well compatible with the primary, junior, and senior stages. This also prevents freshmen from transferring knowledge from their past learning experiences. After the implementation of the new curriculum reform, there have been significant adjustments in the content setting of middle school mathematics, adding some learning content with characteristics of the times, such as statistics and probability, estimation, approximate calculation, and computer applications in middle school. In high school, content such as derivatives, vectors, and preliminary algorithms have been added, as well as mathematical modeling activities and mathematical exploration activities to cultivate mathematical abilities. Part of the content is ready to connect with college mathematics, but it also cuts down and weakens some basic knowledge necessary for college mathematics, such as cotangent function, secant function, cosecant function, sum difference product sum difference formula, Inverse trigonometric functions, Polar coordinate system, Parametric equation, complex number, etc. Knowledge points in liberal arts mathematics such as permutation and combination, Mathematical induction, Binomial theorem, etc. are not involved. Although high school mathematics already involves functions, limits, derivatives, integrals, and other contents, it is a different construction process from university mathematics textbooks. Without strict limit formalization and precision theory as the foundation, middle school students' understanding of limits, derivatives, and other contents is not thorough and unclear. However, after entering university, they face the same concepts from a more abstract and systematic perspective, which is a leap for students. The textbook setting and teaching reform in universities are separate from those in middle schools, without clear and reasonable allocation and boundaries, lacking communication and exchange, resulting in a lot of repetition and disconnection in content, resulting in waste of some teaching resources and gaps in the knowledge system. Some college teachers have long been committed to the research of certain branches of Further Mathematics, and pay little attention to the curriculum content after the curriculum reform of middle school mathematics. For some disjointed parts, they can't have too much time to "remedy" after being found in the classroom. Generally, they can only list the formulas for students to learn independently after class. If students can't learn by themselves after class, it will affect the follow-up learning. Contradictions in classroom teaching can arise and affect the effectiveness of classroom teaching.

3.3. Differences in Student Learning Methods and the Impact of External Environment

In the middle school learning stage, many students have adapted to a passive learning method of knowledge instillation, using "memorization" and "imitation" learning methods, rarely connecting and comparing the content before and after, relying on teachers to systematically teach and summarize knowledge, and providing feedback on students'

learning outcomes in the classroom, lacking the awareness and ability of autonomous learning. At the university stage, students are required to summarize and test themselves, think and explore independently, and teachers are mainly guided in the classroom. Each lesson covers a wide range of knowledge points, and the teaching progress is fast. If students lack thinking, Rote learning, and are not good at summarizing, they cannot form a change from passive indoctrination to active learning, and will not achieve good learning results.

After entering university, without strict supervision from teachers and parents, coupled with the completion of the college entrance examination, some students lack learning motivation and are in a state of relaxation and laziness psychologically. They take it for granted that the pressure and difficulty of studying in university are low, and they have not given enough attention or clear learning goals. The education management model of universities is relatively loose, and a large amount of time is at the students' own disposal. Students with poor self-discipline will lower their demands on themselves. Due to the popularity of electronic products and the richness and openness of online information, some students have become overly addicted to online games, dramas, and other entertainment content, unable to arrange their study time reasonably, and their enthusiasm for learning has weakened. Some students prefer to search online for the exercise questions assigned by the teacher, lacking independent thinking and the ability to identify right and wrong. Their learning attitude is perfunctory, which can directly affect the learning effect.

3.4. Individual Differences among Students

Due to the setting of enrollment conditions, most majors in economic and management colleges have both arts and science, and students have obvious differences in arts and science and regional differences. After entering the same professional class, they are all on the same starting line, which is difficult to achieve differentiated treatment of teaching, which also brings learning difficulties to some students with weak Foundations of mathematics.

4. Ways to Solve the Connection between "Teaching and Learning"

The two teaching systems of college mathematics and middle school mathematics are not isolated and independent, but must be closely connected and scientifically coordinated with each other. To solve the problem of connection well, it must be a two-way effort between teachers and students, and a joint effect of "teaching" and "learning". Teachers play a leading role, being guides and guides. Students are the main body, the key core, the beneficiaries of education, and also the testers of teaching effectiveness. Only through unified interaction between the two can teaching effectiveness be truly improved. The following respectively elaborate on the ways to solve the problem of cohesion from the perspectives of "teaching" by university mathematics teachers and "learning" by college students.

4.1. The Connection of Teacher's "Teaching"

(1) Strengthen enrollment education and ideological awareness

To enhance students' awareness of the importance of studying college mathematics, enhance their sense of urgency, eliminate the slack mood after the college entrance

examination, and adjust for the negative impact of psychological changes after enrollment, face pressure and competition correctly, and find their own positioning and goals. Teachers should also emphasize the characteristics and difficulties of university mathematics learning, so that students have a psychological preparation to face and overcome difficulties in advance. They should adjust their learning methods from "dependency based" to independent learning and thinking. We can invite some outstanding senior students to exchange learning experiences with new students to promote their rapid adaptation to university mathematics learning.

(2) Make good connections in teaching content

From a long-term perspective, in order to effectively connect teaching content, it is necessary for middle school teachers to take a step forward and university teachers to take a step back. Middle school mathematics teachers should be familiar with the connection between Further Mathematics and Elementary mathematics, be aware of it, and properly stand at the height of Further Mathematics to rethink and analyze the problems of Elementary mathematics. On the other hand, they should guide students to pay attention to the improvement of mathematical thinking methods, and do a good job in the transfer of knowledge and the integration of thinking methods for entering university learning. College mathematics teachers should be prepared to take over, first of all, to make students consciously transition from "obedience" to "autonomy" in learning methods, ideological understanding, and other aspects; It is necessary to consciously guide and inspire students to use the theories and methods of higher mathematics to analyze topics related to elementary mathematics, so that students can feel some content in middle school mathematics that cannot be "thoroughly explained" at that time. How to provide corresponding explanations through higher mathematics, and give a comprehensive and systematic understanding of the original mathematical content. It is necessary to find the consistency and integration between higher mathematics and elementary mathematics as much as possible from within the textbook.

For freshmen who have entered the university stage of learning, the realization of convergence in teaching content mainly depends on college mathematics teachers, which is determined by the purpose of Mathematics education in middle school, which aims at college entrance examination scores. College mathematics teachers should first of all have a full understanding of the content of middle school mathematics teaching, and the actual mathematical learning situation of enrolled students, so as to have a targeted view. For the disconnection caused by the course content setting, it is necessary to achieve natural connection and smooth transition. For some of the knowledge points in middle school mathematics that are optional or only suitable for college mathematics that must be mastered, such as Inverse trigonometric functions, secant and cosecant in Trigonometric functions, sum difference product, product sum difference formula, polar coordinates, etc., corresponding supplements must be given instead of assuming or defaulting that students have mastered them, or there will be a fault in the knowledge structure that will affect the learning effect of students. The specific implementation methods can be flexibly adjusted according to the teaching situation, such as classroom explanations, online teaching, student self-study, etc., which can be supplemented in a centralized manner or dispersed in

different chapters. For the obviously repeated part, it can be appropriately deleted, such as using derivatives to determine the monotonicity of Simple function, finding the extreme value of function, etc. Emphasize and value the content that needs to be deepened and expanded. Familiarize oneself with high school mathematics textbooks, grasp the depth and breadth of university mathematics teaching based on the content of the textbooks, both solidify the foundation and improve it.

(3) Connection in teaching methods

In high school mathematics, only one theorem or method is usually taught in one class, followed by repeated practice, while in college mathematics, there are usually two classes at a time, with several pages of lecture notes to be taught, which is like "pouring rain". Therefore, in the first few classes, the progress should be slowed down as much as possible, reminding students to develop the habit of taking notes before class, and summarizing in a timely manner after class. The key and difficult points in the class should be listed as outlines for students to review. After class, some questions should be left for students to think about, such as summarizing the internal connections of relevant content or checking the corresponding knowledge points, gradually cultivating students' ability to learn independently and think independently. When selecting example questions, it is best to consider practical applications related to the major, which can not only enhance students' interest in learning but also broaden their knowledge. High school mathematics has also added mathematical modeling and mathematical exploration sections after the new curriculum standards, but due to the baton effect of the college entrance examination, these contents cannot truly enter the classroom. University mathematics teachers can choose appropriate "mathematical modeling" cases based on students' existing knowledge system, allowing them to experience the fun of mathematics and exercise their ability to use mathematical tools to solve practical problems. We should follow the laws of students' learning and growth, the process of developing and changing their thinking and cognitive levels, and gradually optimize the teaching mode in teaching practice to achieve a smooth transition.

4.2. The Connection of Students' "Learning"

(1) Emphasize the improvement of thinking ability and self-learning ability

Start from yourself, try to change your own learning concept and learning methods. On the basis of following the good learning habits of preview, listening to lessons and review developed at the middle school stage, pay attention to the summary of methods, pay attention to the training and cultivation of your own thinking, learn to use mathematical logic thinking to connect mathematical concepts, Formula and other knowledge points, pay attention to the conditions for the establishment of formulas, theorems, laws, etc., and grasp the essence of the problem. Be able to draw inferences and strive to build one's own mathematical knowledge system. Master some commonly used problem-solving methods, such as induction, analogy, analysis, synthesis, variable replacement, identity transformation, and so on, so as to adapt to college Mathematics education faster and better.

(2) Correct learning attitude

Being able to cross the threshold of the college entrance examination and enter university indicates that most students have excellent academic performance during middle school,

but changes in the environment or lax thinking have led to difficulties for some students in learning college mathematics. Students should establish confidence and take practical actions, establish learning goals, not be lost or lazy, actively strive, and live up to the precious youth and struggle time during their university years.

5. Conclusion

The process of mathematics teaching is a continuous system engineering, and each subsystem needs to be coordinated and organically connected to produce good teaching results. In the actual teaching process, due to differences in teaching content, teaching methods, learning methods, and learning perspectives, there has been a disconnect between college mathematics and middle school mathematics, which has affected the teaching effectiveness of college mathematics. In order to effectively connect the transition period of junior college students and solve this boundary bottleneck problem, it is necessary to work together from various aspects such as university mathematics teachers, middle school mathematics teachers, students, and school managers to promote a smooth transition in this stage, and make practical contributions to improving the quality of university mathematics education and cultivating innovative talents.

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