A Study on the Application of Multiple Representations in Teaching Fraction Concepts in Elementary Mathematics — Taking “Understanding Fractions” as an Example

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Abstract. The concept of fractions exhibits dual characteristics as both a process and an object, making it difficult for a single representation to fully support students' understanding of its essence. Multiple representations, through diverse forms such as context, manipulation, language, and symbols, effectively reduce cognitive load, align with students' cognitive development stages, and promote the development of mathematical thinking. Taking the elementary mathematics unit “Understanding Fractions” as an example, this paper presents different representations in a progressive manner based on the cognitive sequence of “activity-image-symbol.” It emphasizes the conversion and translation between representation forms, guiding students to transition their understanding of the essence of fractions from intuitive and concrete thinking levels to ultimately achieve a fundamental shift toward abstract thinking, laying a foundation for their subsequent mathematical learning.

Keywords: Multiple representations; fraction concept; teaching strategies; elementary mathematics.

1. Introduction

Mathematical learning is fundamentally an experiential activity, with its subject matter possessing the distinct characteristics of mental objects. Mathematical concepts are not passively received but are progressively constructed through individual practical operations, ultimately taking shape through processes of reflection and abstraction [1]. Consequently, mathematical content can be categorized into two types: “processes” and “concepts.” As Sfard points out, many mathematical concepts possess duality: they manifest as a series of operational processes while also being viewed as static objects or structures. This characteristic determines that mathematical concepts are best understood and taught through multiple representations.

In elementary mathematics, fractions exemplify this “process-object” duality of mathematical concepts. On one hand, they manifest as an operational process: dividing a whole into equal parts to establish the denominator (the fractional unit), then taking one or more of these parts to determine the numerator (the quantity of counting units). On the other hand, they exist as static mathematical objects and structures represented by the symbolic form A/B, embodying a complete, operationally defined mathematical concept [2]. Therefore, the concept of fractions can be represented through multiple forms. This paper uses the sixth unit “Understanding Fractions” from the Beijing Normal University Edition Grade 3 Mathematics textbook as an example to explore the feasibility of teaching mathematical concepts through multiple representations, providing theoretical and practical references for mathematics instruction.

2. The Essence of Mathematical Multiple Representations

The concept of “representation” originated in psychological research. According to psychologist S.E. Palmer, representation typically refers to the re-presentation of an object, concept, or knowledge through some form—whether physical or mental. Thus, a structural correspondence exists between the representational system and its object of representation [3]. In other words, when the cognitive object is not directly present, a representation can be considered any symbol or set of symbols standing in for that object [4]. Mathematical representation is a cognitive approach that uses external or internal forms—such as symbols, graphics, or language—to represent or re-present a mathematical object, establishing a correspondence between the representational form and the represented mathematical object.

Goldin et al. categorize mathematical representations into external and internal representations. External mathematical representations are the external forms reflecting mathematical concepts, propositions, and other mathematical learning objects; internal mathematical representations involve individuals' meaningful attribution and construction of mathematical learning objects, including their mathematical emotional experiences and strategies [5]. Cognitive psychology further categorizes external representations into two major types: narrative representations and descriptive representations. Narrative representations encompass logical representations such as verbal texts, written texts, and mathematical formulas, which are characterized by high levels of generalization and abstraction. Descriptive representations are essentially graphic symbols, including images, drawings, and mathematical models. Multimodal representation broadly refers to diverse external representations, meaning the display of a learning object through multiple forms of narrative and descriptive representations [6]. This paper primarily focuses on research concerning external multimodal representations in mathematics learning. The theoretical origins of mathematical multimodal representation can be traced back to the early 1960s, specifically to the principle of multiple concretizations proposed by British mathematics educator Dienes, particularly its perceptual variation principle. This principle advocates guiding learners to inductively abstract underlying mathematical structures from diverse concrete instances by presenting different physical models or varied physical contexts for a given mathematical concept [7]. Thus, multiple representations primarily function as a learning principle. Tang Jianlan posits that learning mathematics through multiple representations constitutes a vital learning philosophy or strategy [8]. Based on this, the author proposes that mathematical multiple representations primarily refer to a learning principle and teaching strategy. It involves presenting the same mathematical object through multiple external representations, such as narrative representations and descriptive representations, to help learners perceive, abstract, and understand its mathematical structure and essence from different variations. It emphasizes promoting individuals' meaningful construction and deep understanding of mathematical objects through the diversity and interactive transformation of external representations.

In theoretical research on mathematical multiple representations, two representative classification approaches exist. Lesh et al. categorized external representation systems based on their cognitive functions into five types: concrete situations, manipulative models, graphs or charts, spoken language, and written symbols (as shown in Figure 1), emphasizing that translation and conversion between these representations are crucial for mathematical understanding. On the other hand, Xu Binyan, grounded in practical mathematical teaching needs, categorizes mathematical representations into four major types: formal representations, pictorial representations, action-based representations, and linguistic representations [9]. From the perspective of mathematical concept learning, this paper further subdivides mathematical representations into verbal representations, symbolic representations, pictorial representations, contextual representations, and operational representations.

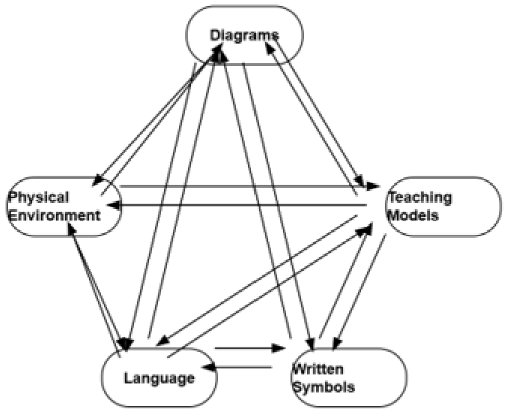

Figure 1. Lesh's Five External Representations.

3. The Necessity of Multiple Representations in Fraction Instruction

3.1. Multiple Representations: The Key to Alleviating Cognitive Load in Fraction Learning

Cognitive load theory, originally proposed by Australian psychologist John Sweller, centers on the finite capacity of human working memory. Cognitive load is defined as the total amount of intellectual activity simultaneously activated and requiring processing within working memory—that is, the aggregate information an individual must process within a specific timeframe [10]. Cognitive load can be categorized into three types: intrinsic cognitive load, extrinsic cognitive load, and effective cognitive load. Intrinsic cognitive load is determined by the characteristics of the learning task itself and its interaction with the learner's existing knowledge structure. Extrinsic cognitive load often stems from deficiencies in instructional resources or activity organization. Effective cognitive load manifests as meaningful cognitive activities and their outcomes [9]. Since both extrinsic and related cognitive loads can be regulated through instructional interventions, educators should strive to minimize both intrinsic and extrinsic cognitive loads in teaching practice while enhancing beneficial effective cognitive load.

The dual nature of fractions and the interactivity between teaching agents make it difficult to effectively reduce cognitive load by relying solely on visual representations or verbal explanations. Consider teaching the concept of “one-half.” If a teacher only uses verbal explanations—such as “When you divide one cake equally into two parts, one part is one-half of the whole”—students must simultaneously process multiple abstract terms like “equal division,” “part,” and “whole” along with their logical relationships in working memory, resulting in high intrapersonal cognitive load. Conversely, presenting only a static image of a cake already divided into two halves may lead students to perceive merely a “semicircle,” failing to grasp the dynamic process and critical role of “equal division.” Irrelevant visual details could even introduce additional external cognitive load. Single representations have limited effectiveness in reducing cognitive load when helping students understand fraction concepts. To effectively enhance students' comprehension of fraction meaning and alleviate their internal cognitive load, diverse representation methods must be employed.

3.2. Multiple Representations: Fraction Instruction Aligned with Children's Cognitive Development

Research on mathematical representations both domestically and internationally initially focused on students. In China, studies on mathematical representations first explored students' representations of mathematical problems from a cognitive psychology perspective [11]. Shenyang et al. categorized research based on grade levels and mathematical content, finding that researchers most frequently focused on elementary-level numbers and operations, as well as junior and senior high-level equations and functions—particularly the representation of fraction concepts and operations in elementary education. Piaget divided children's cognitive development into four successive stages: the sensorimotor stage (0–2 years), the preoperational stage (2–7 years), the concrete operational stage (7–12 years), and the formal operational stage (12–15 years). Elementary students typically reside in the Concrete Operational stage, characterized by: understanding the essential properties of concepts through manipulating concrete objects; developing an initial capacity for conservation of quantity; and performing basic logical operations like classification and ordering. However, cognitive activities during this period remain reliant on physical objects or contextual support, lacking fully formed, systematic thought structures and exhibiting a degree of fragmentation and partiality. In summary, Piaget's theory of cognitive development explains “why” multiple representations should be employed in elementary mathematics instruction. These representations are grounded in students' cognitive patterns and scientifically leverage these patterns to organize and guide mathematical learning. By utilizing multiple representational modes—such as physical objects, diagrams, and symbols—students gain cognitive anchors. They consciously establish connections and autonomously switch between representations, thereby integrating fragmented experiences into coherent cognitive structures.

For example, when teaching "understanding one-half," the teacher asks students to take circular paper pieces representing mooncakes and fold them in half, requiring that "both sides align perfectly" and emphasizing "equal division" (as shown in Figure 2). This provides students with the essential "concrete support and assistance" needed for thinking. Students understand the concepts of 'division' and "equality" through multiple senses rather than abstract imagination. Next, unfold the paper, trace the crease with a finger, shade one half, and describe: "We divided one mooncake equally into two parts. One of these parts is its half." This process presents both visual and verbal representations, helping students extract the essential concept of "part-whole relationships." Next, the teacher writes $\frac{1}{2}$on the shaded portion and explains: "The 2 indicates the total number of equal parts, the 1 indicates the number of parts taken, and this short line represents the division." This introduces symbolic representation, transforming it from an unfamiliar abstract code for students. Finally, the teacher asks: "Can you represent $\frac{1}{2}$using rectangular paper?" " What does your representation of $\frac{1}{2}$ mean?" This deliberate 'connection' and "translation" teaching actively helps students systematically integrate scattered activities and experiences—folding paper, coloring, speaking, writing symbols—into a structured, holistic understanding of $\frac{1}{2}$.

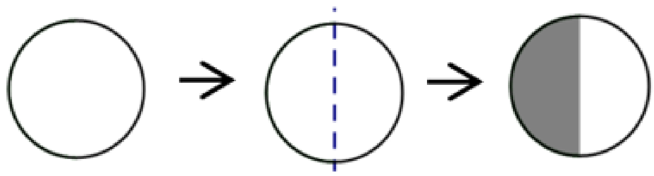

Figure 2. Process of folding a circular paper strip.

3.3. Multiple Representations: Promoting Children's Mathematical Thinking Development

In mathematical learning, knowledge representations can be categorized into two types: external representations and internal representations, which exhibit significant interaction and synergy. Appropriate external representation forms facilitate the establishment and refinement of an individual's internal cognitive structures. Simultaneously, the quality and organizational patterns of internal representations directly influence the selection and presentation effectiveness of external representations [12]. Mathematical multi-representation involves both the internalization of external mathematical knowledge and the externalization of internal mathematical thinking. This process requires mutual conversion between external and internal representations [13]. Through multi-representation, students' mathematical thinking not only becomes richer but also deepens.

For example, when teaching "Understanding Fractions," the teacher guided students to represent fractions using their preferred method to express "one-half." As a result, students employed diverse representations (as shown in Figure 3). Under the teacher's guidance, Student 1 used a situational representation: one small rectangular cake and two children.

Favorite way to represent "one-half." Students produced diverse representations (as shown in Figure 3).

Under the teacher's guidance, Student 1 adopted a contextual representation: Given one rectangular cake and two children, to ensure each child receives an equal share, the cake must be divided into two equal parts. Each child takes one of these parts, meaning each receives "one-half." Student 2 used an operational representation: folding a rectangular piece of paper in half, cutting it apart, and dividing the paper equally into two parts. One of these parts is its half. Student 3 used a graphical representation: folding a rectangular piece of paper in half, cutting it to divide it equally into two parts, and coloring one part black. The shaded portion represented one-half of the entire shape. Student 4 used a symbolic representation: writing the fraction $\frac{1}{2}$ on paper and identifying the line in the middle as the fraction bar. The part below the bar was the denominator, and the part above was the numerator. Student 5 used verbal representation, while the teacher recorded the description on the board: "When one whole is divided equally into two parts, one of those parts is one-half."

Multiple representations not only reflect the diversity and richness of students' mathematical thinking but also foster a comprehensive understanding and deep insight into mathematical concepts.

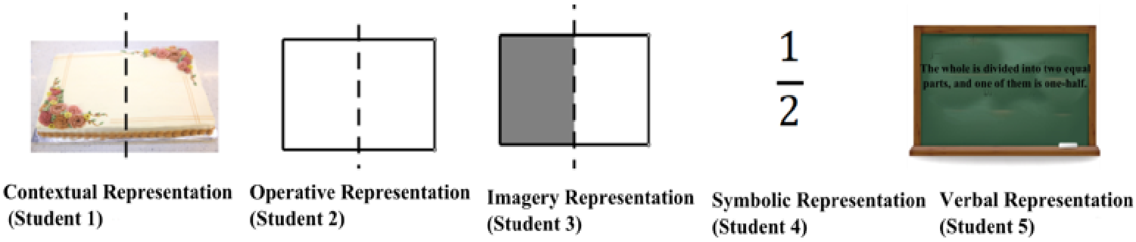

Figure 3. Different External Representations of $\frac{1}{2}$.

Xi Aiyong categorizes students' thinking levels into "performative representation," "pictorial representation," and "symbolic representation" based on the ascending complexity of mathematical representations [13]. In actual teaching practice, educators should guide students to progressively transition from an initial representation stage reliant on actions and physical demonstrations to a deeper thinking stage capable of expressing concepts through abstract symbols. This facilitates the development of mathematical cognition from superficial understanding to grasping underlying principles. Simultaneously, this approach reflects how intrinsic mathematical representations are interconnected and externalizable[14]. Therefore, teachers must promote the conversion and translation between diverse representations while emphasizing the multifaceted externalization of mathematical concepts. In the following three diagrams (as shown in Figure 4), first, students divide apples into different portions and describe how they divided them and what each portion represents. Next, they are encouraged to draw their own diagrams to represent $\frac{2}{3}$. Finally, use mathematical symbols and formulas to abstractly represent mathematical concepts. Guide students from their diagrams to writing the fraction $\frac{2}{3}$, and have them verbally describe their symbolic representations, such as "the number of red apples is two-thirds of the total apples." Simultaneously, encourage students to use alternative verbal representations like "2 portions of red apples out of 3 total portions" or "6 red apples out of 9 total apples represent two-thirds." Throughout this process, students' thinking progresses from performative representation to symbolic representation, deepening from superficial to profound [15].

Additionally, facilitating the translation between diverse representations further promotes the deepening of students' thinking. In Figure 4, regardless of the total number of apples or the quantity of red apples, the proportion of red apples to the total can be expressed as the fraction $\frac{2}{3}$. This requires students to perform a conversion within the graphical representation system: viewing the total number of apples as unit 1, dividing it equally into 3 parts, where 2 parts constitute $\frac{2}{3}$ of the total. This process also involves translating the graphical representation into verbal and symbolic representations of $\frac{2}{3}$. Taking verbal representation as an example, when faced with the third image in Figure 4, some students translate it into $\frac{2}{3}$ from the perspective of parts, while others translate it into $\frac{4}{12}$ from the perspective of quantities.

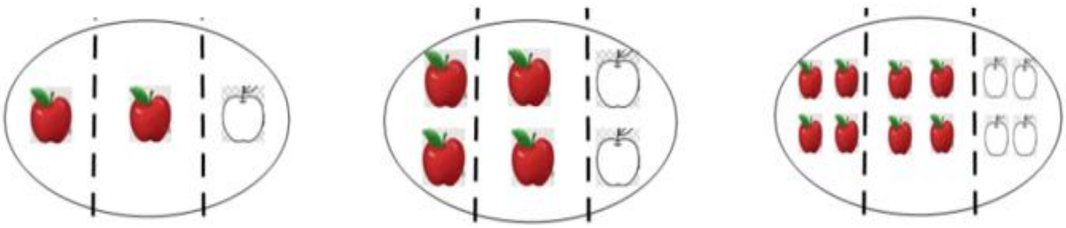

Figure 4. Graphical representation comparing 2/3 under different total quantity conditions.

In summary, multiple representations not only effectively reduce students' cognitive load, making fraction concepts easier to grasp, but also align with the cognitive development stage of elementary students, who primarily engage in concrete operations. By utilizing tangible objects and visual graphics, these representations help students build a bridge from concrete to abstract thinking. More importantly, multiple representations facilitate a two-way process of externalizing and internalizing mathematical thinking, guiding students from superficial to deeper understanding.

4. Teaching Strategies for Multiple Representations in “Understanding Fractions”

Given the crucial role of multiple representations in conceptual understanding and problem-solving in elementary mathematics, and considering current shortcomings in teaching “Understanding Fractions”—such as limited representational forms, unclear progressive levels, and insufficient conversion between different representations—these issues not only hinder students' deep grasp of fraction concepts but may also complicate subsequent learning in “Properties and Significance of Fractions.” Therefore, this paper emphasizes multidirectional expression, progressive levels, and autonomous conversion of representations. Guided by children's cognitive development patterns, it integrates multiple representational forms—including contextual, manipulative, linguistic, and symbolic representations—to guide students' understanding of the essence of fractions[16]. This process facilitates a gradual transition from intuitive, concrete thinking to the fundamental shift toward abstract thinking.

4.1. Uncovering the Essence of Fractions Through Progressive Levels of Multiple Representations.

Bruner categorized mathematical representations into three types based on cognitive development: activity-based, pictorial, and symbolic representations, which emerge and develop sequentially. Grounded in students' psychological characteristics and the elementary mathematics content “Understanding Fractions,” this paper selects four representational forms—contextual, manipulative, verbal, and symbolic—and presents them in this order during classroom instruction.

4.1.1. Contextual Representation—Concrete Cognition of Fractions

Placing fraction concepts within authentic, engaging scenarios enables students to form concrete, intuitive perceptions of fractions. This phase primarily aims to spark learning interest, introduce fraction concepts, and reinforce the fundamental principle of “equal distribution,” thereby facilitating a smooth transition from integer cognition to fraction cognition.

At the start of class, the teacher introduces a scenario where children are having a picnic. How should 4 bananas be divided among 2 children? Students respond in unison: “Two bananas each.” The teacher follows up: “Why two bananas each? Couldn't one child get three bananas?” Students respond: “That's unfair!” This reveals that children prioritize ‘fairness’ when sharing food—the mathematical concept of “equal division.” This naturally introduces the foundational premise of fractions: equal division. The teacher then asks: “How should we divide one large cake among these two children?” Almost all students answer: “One half each.”

However, students soon realize that none of the whole numbers they've learned can represent “half.” This allows the teacher to introduce the fraction $\frac{1}{2}$ and describe its meaning: dividing one cake equally into two parts, each part being $\frac{1}{2}$. This gives students an initial contextual understanding of $\frac{1}{2}$ and helps them gradually grasp the concept of fractions.

4.1.2. Manipulative Representation—The Transition to Abstracting Fractions

For elementary students, fractions are inherently highly abstract concepts. By manipulating paper pieces of various shapes, students connect abstract mathematical ideas with concrete visuals, making abstract relationships like “whole,” “equal division,” and “part” tangible and understandable. This facilitates a smoother transition from concrete to abstract thinking.

After introducing the concept of $\frac{1}{2}$, the teacher asked students to take out differently shaped paper pieces representing cakes. As shown as Figure 5,through hands-on activities, they represented $\frac{1}{2}$ (half) and colored the shaded portion. The teacher displayed student work and asked them to explain the meaning of the shaded area:

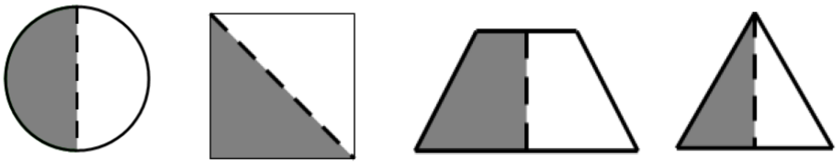

Figure 5. Examples of Representing 1/2 After Equidivision of Various Plane Figures.

Student response: Treating the square (circle, trapezoid, or triangle) paper as one cake, dividing it equally into two parts, where the colored section represents half of the cake, or $\frac{1}{2}$.

Teacher prompt: When dividing the cake, you emphasized "equal division." Why must it be "equal division"?

Student response: If not divided equally, each portion would be different sizes, not truly half or $\frac{1}{2}$.

Teacher adds: "Equal division" is the foundation and prerequisite for learning fractions. The essence of fractions is representing parts of a whole after equal division. To express "quantity" with fractions, equal division must come first. Though the shapes, positions, and sizes of the shaded areas vary across different figures, why can they all represent $\frac{1}{2}$?

Guide students to understand: Regardless of the paper's shape, whenever a whole is divided equally into two parts, each part represents $\frac{1}{2}$of the whole.

4.1.3. Verbal Representation—The Leap Toward Abstracting the Concept of Fractions

Building upon the concrete understanding of abstract concepts achieved through contextual and operational representations, verbal representation further helps students integrate their previously fragmented and vague understanding of fractions. This enables them to articulate the meaning of fractions and their formation process clearly using more rigorous and standardized mathematical language. Simultaneously, by analyzing, comparing, and abstracting different representational forms, students gain a deeper grasp of the mathematical essence underlying diverse representations. Therefore, this process emphasizes guiding students to describe "one-part-of-whole" and encourages them to cite real-life examples to deepen their understanding of the nature of fractions.

The teacher asks students to explain the meaning of a fraction based on its operational process and to provide real-life examples of $1\frac{1}{2}$. Building on their understanding of the fraction $\frac{1}{2}$, students draw $\frac{1}{3}$ and $\frac{1}{4}$ on paper.

Student summary: When one whole is divided equally into two parts, one of those parts is $\frac{1}{2}$. For example, dividing one apple equally into two parts makes each part $\frac{1}{2}$ of the whole; dividing one mooncake equally into two parts makes each part $\frac{1}{2}$ of the whole, and so on. The same applies to $\frac{1}{3}$ or $\frac{1}{4}$: dividing one whole object equally into three or four parts, taking one part represents $\frac{1}{3}$ or $\frac{1}{4}$(as shown below):

Teacher Follow-up: Based on your understanding of fractions $\frac{1}{2}$, $\frac{1}{3}$, and$\frac{1}{4}$, can you summarize their common characteristics?

Student Response: When expressing fractions, we first divide something equally. Whether it's an object or a shape, dividing one whole into equal parts gives us the denominator, and the number of parts taken becomes the numerator. Taking just one part represents "how many parts" of the whole.

Through comparing and generalizing fractions like $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, students gradually abstract the common property of fractions like "one-half," one-third, and one-fourth. They begin to construct a foundational model for fractions. With this model as support, they can further understand more complex fractions, indicating they have grasped the essence of fractions.

4.1.4. Symbolic Representation—The Fundamental Transformation in Abstracting the Concept of Fractions

Symbolic awareness constitutes a vital component of core mathematical literacy. In fraction learning, verbal representations establish an initial cognitive framework for students, while symbolic representations further refine this foundation. They present the concept of fractions in a highly concise form, laying a solid groundwork for students to engage in complex mathematical operations and reasoning. As the renowned British mathematician Whitehead observed: "The characteristic of mathematics is conciseness—expressing the most complex things in the simplest and clearest terms." Symbols constitute the language of mathematics, and expressing the world through this language is one of the core competencies emphasized in the 2022 Edition of the Primary Mathematics Curriculum Standards for Compulsory Education.

Teacher asks: We have expressed the fraction $\frac{1}{2}$ through a diagram and described it verbally. Do you know how to write it? Please write the fraction $\frac{1}{2}$ on your paper and name each part.

Student response: In the fraction $\frac{1}{2}$, the line in the middle is called the fraction line. The number 1 above the line is the numerator, and the number 2 below the line is the denominator.

Teacher supplement: In the fraction $\frac{1}{2}$, the fraction line represents division into equal parts. The numerator 1 indicates the number of parts taken, and the denominator 2 indicates the total number of parts.

Student supplement: In the fraction$\frac{1}{3}$, the denominator is 3 and the numerator is 1. It represents dividing the whole into 3 equal parts, with each part being $\frac{1}{3}$.

At this point, students can apply the established fraction model to similar situations, achieving a fundamental shift from intuitive understanding to abstract generalization of the fraction concept.

4.2. Deepening Understanding of Fractions Through Conscious Conversion Among Multiple Representations.

While Lesh et al. did not emphasize a fixed developmental sequence within representation systems, they argued that conversion and translation between these forms play a crucial role in students' comprehension of mathematical concepts.

Task 1: Comparing Sizes of Like Fractions: Compare the sizes of $\frac{1}{2}$ and $\frac{1}{4}$.

Some students concluded that $\frac{1}{4}$ is larger than $\frac{1}{2}$ because 4 is a larger denominator than 2. Others argued that $\frac{1}{2}$ is larger, reasoning that when a circular paper is divided equally into 2 parts versus 4 parts, taking one part from each division shows that one 2-part is larger than one 4-part, thus proving $\frac{1}{2} > \frac{1}{4}$ (as illustrated Figure 6 below).

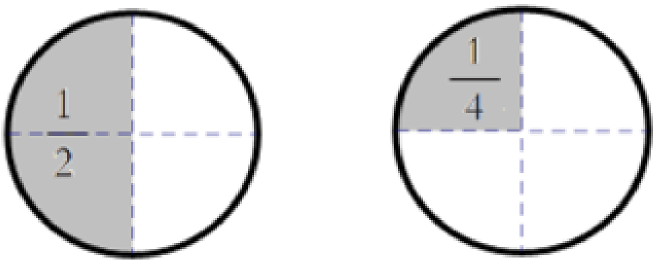

Figure 6. Comparing the sizes of 1/2 and 1/4 using a circular division diagram

Teacher follows up: Similarly, what about $\frac{1}{4}$ and $\frac{1}{6}$?

Student responds: $\frac{1}{4}$ is larger than $\frac{1}{6}$. If we divide the same circle into 4 equal parts and then take one of those 4 parts, it is larger than one of the 6 parts. Thus, $\frac{1}{4}$ is larger than 1/6 (as shown in the Figure 7):

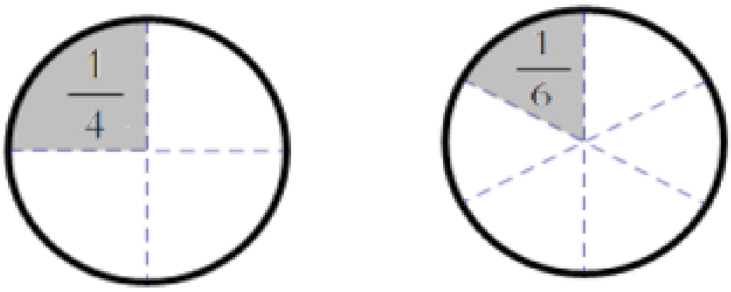

Figure 7. Comparing the sizes of 1/4 and 1/6 using a circular division diagram.

Through observation, comparison, and reasoning, the teacher guides students to conclude: For circles of equal size, the more equal parts they are divided into, the smaller each part becomes; the fewer the parts, the larger each part becomes. That is, for two fractions with the same numerator, the larger the denominator, the smaller the fraction.

Task 2: Comparing Fractions with the Same Denominator: Compare the sizes of $\frac{1}{4}$ and $\frac{2}{4}$.

Having learned to compare fractions with the same numerator, students can now mentally construct the corresponding fraction diagrams for $\frac{1}{4}$ and $\frac{2}{4}$ without relying on visuals and immediately state that $\frac{2}{4}$ is larger than $\frac{1}{4}$. To facilitate observation and summarization, the teacher displays the corresponding diagrams:

Students summarize: When dividing a whole into equal parts, the larger the number of parts represented, the larger the fraction; the fewer parts represented, the smaller the fraction. Thus, among fractions with the same denominator, the one with the larger numerator is greater.

This process—transitioning from "number" to "shape" and then back from 'shape' to "number"—demonstrates the mutual conversion and translation between different representations. During learning, students first actively transform abstract symbolic representations into intuitive graphical representations, using the visual nature of shapes to understand the relationship between fraction sizes. Subsequently, they consciously return to symbolic representations to accurately judge and express the magnitude of fractions.

5. Conclusion

The concept of fractions possesses a dual nature as both a process and an object, making it difficult for a single representation method to support students' understanding of its essence. Diverse representations—through contexts, operations, graphics, language, and symbols—effectively reduce cognitive load, align with cognitive developmental stages, and enhance students' mathematical thinking. In instructional strategy, following the cognitive development sequence of "activity-image-symbol," different representations should be introduced progressively. Contextualization establishes perception, manipulation facilitates transition, verbal expression drives systematic articulation, and ultimately, symbolic abstraction achieves fundamental transformation. Concurrently, the importance of conversion and translation between representations must not be overlooked, guiding students to achieve deep conceptualization of fraction meaning through the integration of numbers and shapes.

In summary, multiple representations serve as an effective tool for teaching mathematical concepts and a crucial pathway for advancing thinking from concrete to abstract. Educators should emphasize the diversity and hierarchical nature of representations, encouraging students to autonomously shift between them to lay a solid foundation for future learning.

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