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The History of Fractions and Teaching Insights

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Abstract: The history of mathematics, as the history of mathematical development, is a guide for teaching and has significant implications for contemporary teaching. If teachers can use the resources of the history of mathematical discourse wisely in teaching, they can enable students to experience the wisdom of their predecessors and the charm of mathematical culture in the process of learning new knowledge. As early as around 3000 B.C., human beings began to understand fractions. Since then, the representation of fractions, the application of fractions, and the perspective of understanding fractions have been constantly improved, which have profound implications for the teaching of fractions.

Keywords: History of Fractions; Teaching Fractions.

1. Introduction

In the sea of the history of numbers, fractions have been around for almost as long as natural numbers, and it is because of the introduction of fractions that the rational number system, consisting of integers and fractions, has been constituted. From the perspective of cognitive developmental psychology, the level of cognition of an individual student is changing and improving in the process of continuous interaction between the subject and the object when knowing something. The development of individual cognition is also a microcosm of the development of human cognition, so the level of individual cognitive development corresponds to the level of human cognition in history, which means that the development of human cognition of fractions can predict the process of students' cognitive understanding of fractions.

2. History of the Development of Fractions

2.1. Representation of Fractions

2.1.1. Fraction Representation in Other Countries

From 3000 BC to 1700 BC, the ancient Babylonians began to record fractions on clay tablets, forming the "inverse table". The fractions at that time were fractions with numerator 1 and were expressed in hexadecimal, e.g., 1/3=20/60, 1/20= 1/60 + 2/60.

The world's earliest record of fractions is in the Reindeer papyrus. In 2500 B.C., the ancient Egyptians had a complete understanding of unit fractions and used hieroglyphics to create symbols to represent unit fractions, using the accumulation of several unit fractions to represent other fractions with numerator greater than 1.

Surprisingly, although the basic unit of calculation for numbers in the eyes of the ancient Egyptians was 1, they would use 2/3 in the application of fractions, in the process of decomposing fractions, the ancient Egyptians used 2/3 as a unit fraction as well, e.g., 4/5 = 2/3 + 1/10 + 1/30.

Not only that, but the Reindeer papyrus also contains many calculation problems about fractions. For example, what is a quantity whose 2/3, 1/2 and 1/7 add up to 33?

2.1.2. Ancient Chinese Fraction Representation

Ancient China was a world leader in the theory and practice of fractions. Fractions appeared very early in China and were used in social production and life. Around the 5th century B.C., the understanding that the quotient of two integers divided by a fraction began to appear in China, and this understanding is the basis of the present concept of fraction.

Chinese records of the concept of fractions date back to the Shang Dynasty (around the 12th century B.C.). In the Late Zhou bronze inscriptions, the description of fractions already appeared, and at that time, people often omitted the word "zhi" in the phrase "a few percent of a few", so they read 2/3 as "two percent of three the word "zi" in "a few minutes" was often omitted, so that 2/3 was read as "2/3".

In the 3rd century B.C., when the Kao-Gong-Ji talked about making wheels, it was said that "one tenth of an inch is one", and "one tenth of an inch" is what is now called 1/10 (inch) = 1 (minute).

The book "Zuo Zhuan" also recorded fractions, such as: "The largest city of a vassal cannot exceed 1/3 of the Zhou state, the medium cannot exceed 1/5, and the smallest cannot exceed 1/9. During the time of Qin Shi Huang, the number of days in a year was also set at 365 and 1/4 days.

During the Spring and Autumn and Warring States period, arithmetic was already quite popular in China, and in the arithmetic process, division itself already contains a fraction representation: the quotient is located at the top, the divisor (the real, or numerator) is in the middle, and the divisor (the law, or denominator) is at the bottom, and if the result of the operation has a remainder, it is expressed as a form with a fraction.

It can be found that at this point in the calculation of fractions, we began to represent fractions in decimal, and our country was the first in the world to use decimal representation of fractions.

2.1.3. Creation of the Score Line

Records of fraction lines first appeared in the writings of the Arab mathematician Hanazimi, who introduced fraction lines from the point of view of division, for example, in the fraction 3/5, the fraction line between the numerator and the denominator represents 3 divided by 5.

By the 12th century, the Arab mathematician Hessel depicted three fraction lines in his work, which, although their representation was more complicated, provided inspiration for the improvement and application of fraction lines by later generations. Therefore, it is generally accepted that the application of fraction lines in the modern mathematical sense is attributed to Hessel.

Subsequently, the Italian mathematician Fibonacci brought the fraction line to Europe, and he profoundly changed the mathematical situation in Europe by systematically introducing the calculation of fractions and the symbol system of the Arabs in his book "Arithmetic". The fraction line has been applied and the calculation of fractions has been given in a collection of fraction exercises in Germany in 1530.

In the 18th century, the Swiss mathematician Euler formally defined fractions through an intuitive example problem in his book General Arithmetic: "It is impossible to divide a 7-meter rope into three equal parts because a suitable number cannot be found to represent it; if we divide it into three equal parts, each part is 7/3 of a meter, and a number like 7/3 we call it a fraction. "

2.2. Application of Fractions

Since its creation, fractions have been more and more widely used in people's learning, production and life.

A question involving fractions, as recorded in the aforementioned Reindeer papyrus, translates directly in today's language as "A quantity, 2/3 of it, 1/2 of it, 1/7 of it, all of it, adds up to a total of 33, what is this quantity?" This question can be solved using the simple arithmetic of the ancient Egyptians.

The Zhou Thighs Arithmetic of the 1st century B.C. in China contains more complex fraction calculations, which shows that people at that time could already use the basic properties of fractions to perform operations skillfully and express fractions in the scientific decimal method, which astonished the world.

The Nine Chapters of Arithmetic is the earliest systematic account of fractions in the world, predating Europe by more than 1400 years. The entire book is written in the form of a collection of problems and their solutions, including the four rules of fractions and proportional algorithms, and various calculations of area and volume. The first chapter, "The Square Field," gives the complete rules of addition, subtraction, multiplication, and division of fractions, as well as the rules of approximate fractions, common fractions, equal fractions (comparing the size of fractions), and equal fractions (finding the average of fractions). The approximate division method is the same as the present one, in which the maximum common divisor is found first, and then the numerator and denominator are divided by the maximum common divisor. In the division operation, the numerator and denominator of the divisor are reversed and multiplied by the divisor being divided, which was a remarkable creation at that time.

Fractions are also often used in our life. For example, in music, a quarter note is a beat, an eighth note equals 1/2 beat, and a sixteenth note equals 1/4 beat; human growth follows the law of head and tail development, in the process of the first surge period, the head of the child at birth accounts for 1/4 of the body length, 1/5 at the age of 2, 1/7 at the age of 6, and only 1/8 as an adult.

2.3. A Cognitive Perspective on Fractions

As fractions have evolved, people's perspectives on fractions have also changed, trying to analyze and interpret the meaning of fractions from different perspectives:

2.3.1. Intuitive Meaning: Fractions are Created in Distribution and Metrics

The Shuowen Jiezi interprets "分" as "分", also separate. From eight, from knife, knife to separate things." It can be seen that a fraction is "a number that is divided". The earliest numbers in human history were natural numbers (positive integers), and when measuring and dividing them in real life, people found that in many cases they could not get exactly the whole number, and then fractions were created. In ancient times, when measuring land, a piece of land was divided into three equal parts, one of which was one-third. One-third is an expression that is written down in a special symbol to become a fraction, and it is in the long experience of people dealing with such problems that the concept of fraction was formed.

2.3.2. Operational Meaning: A Fraction is the Result of the Operation of Dividing Integers

From an arithmetic point of view, a fraction is the quotient obtained by dividing two integers, and is the result of the operation obtained by dividing integers. The example in Euler's work mentioned above is understood from the perspective of equal division of the whole, where we consider the 7-meter-long rope as a whole and divide it into three equal parts, each part being 1/3 of a fraction unit, and 7 such fraction units being 7/3. Although such an understanding was advanced at that time, it seems a bit fussy today. Some European mathematicians have criticized this: "The long European preoccupation with single fractions is a cultural bias that has delayed the progress of mathematics as severely as the Roman counting method." They argued that the concept of a fraction should be based on the division of two integers, i.e., arise from the division of integers. This is largely consistent with the current concept of fractions.

The use of fractions comes from the need for division operations. In ancient China, it was customary to use chips to represent fractions, in which division itself already included the representation of fractions, "(addition of fractions), "subtraction of fractions" (subtraction of fractions), "multiplication of fractions" (multiplication of fractions), "division of fractions " (division of fractions), "class division" (size comparison of fractions), "equal division" (finding the average of fractions), and other fractional arithmetic. In Europe, on the other hand, it was not until the Renaissance that people began to generally accept specific fraction algorithms and arithmetic.

2.3.3. Expression: A Fraction is the Ratio of a Pair of Integers

Fractions represent the relationship between the part and the whole, and for the convenience of arithmetic, one considers fractions as the ratio of a pair of integers from a formal point of view. A ratio is a division equation consisting of a front term and a back term, where the front term of the ratio is equivalent to the numerator, the ratio sign is equivalent to the fraction line, the back term of the ratio is equivalent to the denominator, and the ratio is equivalent to the fraction value. Euclid understood a fraction as the ratio of two commensurable quantities, representing the relationship between two quantities, but did not treat the ratio as a number. 300 years later, Leonardo da Vinci first treated a fraction as a number, keeping the fraction on the same footing as the natural numbers.

3. How to Teach Fractions

3.1. Introduce Teaching with the Creation of Fractions

In order to better introduce students to fractions, they need to be introduced in the context of the history of how fractions were created. In terms of the origin of fractions, teaching can be introduced in terms of distribution and measurement as well as division.

3.1.1. Allocation and Metric Introduction

The reason why it is called "fraction" is that it originates from the word "share". In ancient times, if the total number of distributed items is less than the object of distribution, the situation of fraction will occur, that is, as mentioned in the textbook of the Human Education Version, "When making measurements, dividing things or calculating, it is often not possible to get the whole number. In ancient times, when the total amount of goods distributed was less than the object of distribution, a fraction was used, as mentioned in the textbook of the Human Education Version, "When measuring, dividing things or calculating, it is often not possible to get a whole number of results, which is often expressed as a fraction.

When teaching, students can be motivated by designing problem-solving situations using fractions so that they realize that the presence of fractions has an important role in situations such as distribution and measurement. For example, when dividing a cake during an autumn trip and dividing one cake equally among four people, a situation arises where whole numbers cannot solve practical problems, which leads to fractions.

3.1.2. Introduction of Division

It is not very common to introduce division as opposed to distribution and measure. However, in history, ancient mathematicians often introduced the concept of fraction by division. The Nine Chapters on Arithmetic mentions, "The reality is like the law and one, and those who are not satisfied with the law are ordered by the law." It means that in division, if the divisor or remainder is smaller than the divisor, a fraction is obtained with the divisor as the denominator. It follows that fractions are produced by division.

Introducing fractions from the perspective of division may be somewhat straightforward; teachers need to lead students to understand the relationship between fractions and division in specific problem situations. In fact, understanding fractions from the perspective of division will, to some extent, enable students to better understand the arithmetic properties of fractions.

3.2. Forming Symbolic Awareness and Developing Abstract Thinking

3.2.1. Symbolic Awareness

The development of fraction representation has gone through a long process, and the common representation we have now is artificially prescribed, which is difficult for students to understand. In the first lesson of "Preliminary Understanding of Fractions", teachers should consciously let students experience the process of creating symbols, i.e., expressing 1/2 in the way they like, to appreciate the need for symbols and to develop their creative thinking skills. After the investigation, the teacher will show the accepted forms of fractions, so that students can feel the simplicity of symbols and develop their awareness of symbols in the process of comparison.

3.2.2. Abstract Thinking

The meaning of fractions is abstract in nature. When students are first exposed to it, they need to be taught with the help of visual graphics, to understand 1/2 visually and graphically by folding and painting, and to transfer knowledge on this basis to successively represent 1/4, 1/8 or even a few fractions, and to experience the idea of combining numbers and shapes. The concept is not formed overnight, we need to abstract the essential concept of fraction through a lot of materials by comparing, analyzing and synthesizing several times.

3.3. Master the Algorithm of Arithmetic, Accurate Arithmetic

The first systematic study of the operations of fractions is presented in "Nine Chapters of Arithmetic", where addition of fractions is called "combined fraction", subtraction is called "minus fraction", multiplication is called "multiplying fraction", and division is called "The book also introduces the algorithm of fractions and the methods of common fraction, approximate fraction, and fractionalization with examples to support it.

Research shows that although our students can generally master the basic knowledge and skills of fractions, they do not have enough understanding of the arithmetic of fraction operations. The teaching of operations on fractions should focus on the integration of the four operations on integers, decimals and fractions, grasp the inner connection, and make students feel the consistency of operations. For example, when adding and subtracting fractions, we need to pass fractions with different denominators, and the ultimate purpose of passing fractions is to get the same counting unit (denominator). In conjunction with the newly promulgated Compulsory Education Mathematics Curriculum Standards (2022 Edition), teachers need to guide students to understand the importance of fraction units to the expression of fractions and to sense the consistency of addition and subtraction operations on whole numbers, decimals, and fractions.

3.4. Grasp the Meaning of Fractions

In teaching, we often express the meaning of a fraction in this way: "A number that divides a whole into equal parts and takes one or more of these parts is called a fraction". It is important to emphasize that in real life, fractions do not all the time indicate the average division, but rather the relationship between the whole and the parts. For example, if a piece of glass is broken and the fragment is asked what fraction of the whole glass area it represents, the meaning of the fraction is not the average fraction, but the relationship between the area of the broken piece and the area of the whole glass. When teaching, teachers need to explain according to different situations so that students can have a more comprehensive understanding of the meaning of fractions.

3.5. Sorting out the Pulse, Step by Step

The creation and spread of fractions over thousands of years shows that fractions have a complete body of knowledge, and it takes a long process to have a comprehensive understanding of fractions, and our teaching should be progressive. For example, the first term of a ratio is equivalent to the numerator of the divisor and the fraction in the division equation, the second term of a ratio is equivalent to the denominator of the divisor and the fraction in the division equation, and the ratio is equivalent to the quotient of the division equation and the fraction value of the fraction. When Liu Hui annotated the Nine Chapters of Arithmetic, he also used the knowledge of ratio to explain the principles of fraction operations. The multiplicity of meanings and structures of fractions often prevents us from clarifying the full meaning of fractions to students during the initial understanding of fractions, and requires a re-examination of fractions after learning another related knowledge.

4. Summary

The historical evolution of fractions embodies the beauty and magic of mathematics. In fact, all knowledge is historical and cultural, all science applies mathematical tools, and all mathematical symbols pursue simplicity. Dividing fractions is a difficult task in teaching elementary school mathematics. Using historical materials helps teachers reconstruct the way fractions are introduced and deepen students' understanding of fractions and their operations in a step-by-step manner.

It is very important and relevant for teaching and learning that we consciously examine the history of a mathematical concept and draw out the useful elements in its development. With a classroom permeated by the history of mathematics, students' feelings and understanding of mathematics have a sense of heft and penetration. Tracing history is a way to better understand the difficulties in the evolutionary journey and to discover the nodes where creative thinking occurred, allowing us to better use the known laws to transform the unknown future.

References

[1] Zhang, L. Q. (2022). Small fractions, long history. Must-read for elementary school students (Upper grades), 05, 38-39.

[2] Li, W. X. (2022). Research on teaching big units from the perspective of deep learning (Master's thesis, Southwest University). https://kns. cnki.net/ KCMS/ detail/ detail. aspx? dbname=CMFD202301&filename=1022668930.nh

[3] Cai, H. S. (2018). Returning to the origin of history: Teaching and thinking about "preliminary understanding of fractions". Primary School Teaching (Mathematics Edition)(Z1), 57-60.

[4] Cai, H. S. (2018). Returning to the origin of history: Teaching and thinking about "Understanding New Mathematics: Preliminary Understanding of Fractions". Education View (08), 29-32.

[5] Xu, C. S. & Jing, Y. E. (2015). The historical evolution of fraction symbols. Secondary School Students Mathematics, Physics and Chemistry (Eighth Grade Mathematics)(With People's Education Press Textbooks)(11), 28-30.

[6] Wang, X. Q., Ye, X. J. & Gu, H. P. (2015). "Fractional exponentiation": Looking at the definition from the perspective of historical occurrence. Education Research and Review (Secondary Education and Teaching)(04), 59-63.

[7] Ma, K. (2014). Understanding and teaching research on fraction concepts (Master's thesis, Capital Normal University). https://kns.cnki.net/KCMS/detail/detail.aspx?dbname=CMFD201402&filename=1014262236.nh

[8] Yu, Y. (2013). The history of fractions. Mathematics World (Suitable for primary school grades three and four)(12), 22.

[9] Morris Kline, Zhang L. J., Zhang J. Y. & Jiang Z. H. (2013). Ancient and modern mathematical ideas (New edition, collector's edition). China Science and Technology Information, 23, 177.

[10] Xu, Z. T. (2010). Changes in the perspective of understanding during the historical development of fractions and their teaching implications. Hunan Education (Lower)(04), 29-31.

[11] Tan, Q. L. & Yuan, J. W. (2008). A brief history of the development of fractions and decimals. Hunan Education (Mathematics Teachers)(03), 42-44.

[12] He, X. H. (2007). Teaching fractions from the perspective of their history. Hunan Education (Mathematics Teachers)(05), 11-12.

[13] Chong, C. (2007). The history of fractions. Mathematical Companion (Grades 5-6)(05), 31-32.