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Abstract: Small and medium-sized enterprises often face the risk of uncertain output. We considered a supply chain system composed of a single risk-neutral retailer and a single risk-averse manufacturer. When manufacturer is faced with capital constraints in the production of green products, they can solve it by external financing to third-party financial institutions or internal financing to retailer. At the same time, the retailer can determine the expected order quantity according to the market demand, the wholesale price and the green technology input level of the manufacturer. Based on these, we construct three game models: no financing model, external financing model and internal financing model, and study the financing strategy of manufacturer and the ordering strategy of retailer. The results show that the manufacturer's profit is the largest when producing green products. At this time, both internal financing and external financing are optimal financing decisions for the manufacturer. However, the retailer’s optimal ordering strategy is when only green products are sold and the manufacturer conducts internal financing.

Keywords: Supply Chain Financing; Risk Aversion; Green Technology Input; Capital Constraint.

1. Introduction

Under the background of “double carbon”, Enterprises are required to carry out green transformation. Small and medium-sized enterprises occupy a large scale in the market. They should actively comply with the needs of green development and become the main force of green transformation [1]. However, because of the small scale of small and medium-sized enterprises, it is easy to have the problem of insufficient capital and anti-risk ability. Therefore, when small and micro enterprises choose to continuously improve the level of green technology investment in order to adapt to the environment of green transformation, they are likely to face the problem of financial constraints. In real life, small and medium-sized enterprises often solve the problem of financial constraints through financing. For example, small and medium-sized manufacturers can finance through Jingdong supply chain finance or Ali microfinance [2]. They can also be financed by commercial banks after big data qualification review [3].

In addition to the problem of capital constraints, small and medium-sized manufacturing enterprises will also face the risk of uncertain output in production and operation [4]. In the production process, due to a series of uncontrollable natural factors such as the quality of raw materials and the failure of machinery and equipment, manufacturing enterprises in many industries often cannot produce according to the pre-production plan. Therefore, there is a difference between actual output and expected output [5]. For example, the lack of capacity of battery supplier LG Chemical has led to uncertainty in the output of Volkswagen’s Audi e-tron [6]. The manufacturing process of hydrogen fuel cell involves difficult technologies such as hydrogen energy storage, storage and transportation, and system integration. However, its high cost, high preparation capacity, and high technical standards make the output of hydrogen fuel cell manufacturers uncertain [7]. Relevant research shows that enterprises will show behavioral preferences in the face of uncertain risks, such as risk aversion preference, risk pursuit preference, fairness concern preference and so on [8-10]. These behavioral preferences will have an important impact on corporate decision-making. Therefore, it is of great practical and theoretical significance to study the supply chain financing problem with capital constraints and behavioral preferences.

Most of the existing studies analyze the optimal financing decisions of the supply chain based on the capital constraints of manufacturers or retailers. Among them, the most studied financing methods are external financing and internal financing. On the study of external financing, Kouvelis and Zhao [11] constructed a supply chain of suppliers and capital-constrained retailers, in which capital-constrained retailers need to finance from outside the supply chain (such as banks) to order products. Finally, the equilibrium solution of the order quantity is obtained. Yi and Zhou [12] constructed a two-echelon supply chain with a single manufacturer and a single retailer based on the newsboy model. The retailer applied for mortgage external financing with a product procurement contract signed with the manufacturer. And the analytical formula of financing value ratio is determined. Cao et al. [13] analyzed the financing strategy of risk-averse manufacturers. Capital-constrained manufacturers can choose retailers’ advance payment model and external financing model. They found that advance payment is always optimal for manufacturers. On the research of internal financing, Wang [14] studied the financing strategy of suppliers providing deferred payment to retailers based on capital constraint suppliers, and studied the externalization strategy of internal financing among supply chain members. Yang et al. [15] found that only when the supplier’s risk aversion threshold is medium, trade credit financing can achieve a win-win situation for suppliers and retailers. Yan et al. [16] constructed a dual-channel supply chain, in which a capital-
constrained supplier needs to finance one of the retailers, and the two retailers can make bidirectional free-riding. Based on this, they studied the impact of free riding on internal financing and the optimal pricing and optimal market share.

Most of the above studies do not take into account the random output problem. Existing research shows that the output uncertainty in the supply chain is an unavoidable risk in production decision-making, especially for small and medium-sized manufacturing enterprises [17]. In addition, the output uncertainty will affect the manufacturer's optimal decision on production planning, pricing and the retailer's procurement strategy [17-19]. Otherwise, the uncertainty of output will also make decision makers have risk aversion preferences and thus affect the supply chain strategy. However, these studies do not consider the impact of risk aversion preferences caused by output uncertainty on supply chain financing strategies. Therefore, we introduced risk aversion into the supply chain with capital constraints, and construct a two-stage supply chain model composed of capital-constrained and risk-averse manufacturer and capital-sufficient retailer. As the leader of the supply chain, the manufacturer can determine the level of green technology investment, but it will face financial constraints in the production of green products. It can solve financial difficulties through external financing and internal financing. Of course, manufacturer can also choose not to finance, at this time will produce traditional products. Based on the above background, we studied the problem of supply chain financing strategy with risk aversion preference caused by capital constraints and output uncertainty.

2. Problem Description and Assumptions

We considered a supply chain system consisting of a single risk-neutral retailer and a single risk-averse manufacturer. The risk-averse manufacturer is the dominant enterprise, and the retailer proposes the expected order quantity to the manufacturer, and the manufacturer produces the product accordingly. We also considered that the manufacturer's production of green products will face financial constraints. It can consider external financing and internal financing to obtain funds for green product production. In this context, we studied the expected order quantity and green technology investment level of products under the three models of no financing model, external financing model and internal financing model. Finally, we compared the optimal decisions and optimal profits in different situations.

The model construction in this paper is based on the following assumptions:

1. The retail price of the product is affected by the market demand, the level of green technology input and the randomness of output. The prices of traditional products and green products sold by retailer are:

\[ p_1 = a - (q + \varepsilon) \]  
\[ p_2 = a - (q + \varepsilon) + \varepsilon \]

Here, \( p_1 \) is the traditional product sales price, \( p_2 \) is the green product sales price, and \( a \) is the market demand scale. The random variable of output fluctuation \( \varepsilon \) is used to describe the output uncertainty of the manufacturer. If the expected production of the manufacturer is \( q \) (the expected production is also equivalent to the expected order quantity of the retailer to the manufacturer), the actual output is \( q + \varepsilon \). Where, the mean of \( \varepsilon \) is 0, and the variance of \( \varepsilon \) is \( \delta^2 \). At the same time, we assumed that the unit production cost of the product is \( \zeta \), the green level of the product is \( \varepsilon \), the wholesale prices of the manufacturer's traditional products and green products are \( w_1 \) and \( w_2 \). Based on the research of Xu [20], we assumed that the input cost of green technology is \( \varepsilon^2 \).

2. The manufacturer has a risk aversion preference for output uncertainty. Therefore, we used the mean-variance method to measure the decision utility of the manufacturer. The targeted utility function of the manufacturer is \( U(\pi) = E[\pi] - k \sqrt{Var(\pi)} \). Here, \( k \) is used to describe the degree of risk aversion, which reflects the manufacturer's attitude towards risk. The higher the degree of risk aversion, the more conservative the manufacturer's behavior.

3. The manufacturer is the leader of the Stackelberg game, and the retailer is the follower.

4. \( p > w > c \).

All symbols and their meanings involved in this article are shown in Table 1.

<table>
<thead>
<tr>
<th>Symbols</th>
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<tbody>
<tr>
<td>( q_i )</td>
<td>Expected order quantity under model ( i )</td>
<td>( p_j )</td>
<td>Retail price of ( j )</td>
</tr>
<tr>
<td>( \varepsilon_i )</td>
<td>Green technology input level under model ( i )</td>
<td>( w_j )</td>
<td>Wholesale price of ( j )</td>
</tr>
<tr>
<td>( \pi_M^i )</td>
<td>Manufacturer’s profit under model ( i )</td>
<td>( c )</td>
<td>Production cost of traditional products</td>
</tr>
<tr>
<td>( \pi_R^i )</td>
<td>Retailer’s profit under model ( i )</td>
<td>( k )</td>
<td>Risk aversion degree</td>
</tr>
<tr>
<td>( E[\pi_R^i] )</td>
<td>Retailer’s expected profit under model ( i )</td>
<td>( a )</td>
<td>Market size</td>
</tr>
<tr>
<td>( U(\pi_R^i) )</td>
<td>Manufacturer’s target utility function under model ( i )</td>
<td>( r )</td>
<td>Financing rate</td>
</tr>
<tr>
<td>( j \in {1,2} )</td>
<td>{traditional products, green products}</td>
<td>( i = A )</td>
<td>No financing model</td>
</tr>
<tr>
<td>( i = B )</td>
<td>External financing</td>
<td>( i = C )</td>
<td>Internal Financing</td>
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3. Model Construction

3.1. No Financing Model (Model A)

When manufacturer does not have financing channels, it sells traditional products by using its own funds. In decentralized decision-making, the decision-making principle of manufacturer and suppliers is to maximize their own interests. The manufacturer decides the wholesale price \( w_i \) in advance, the retailer decides the expected order quantity \( q \) according to the wholesale price, and the manufacturer organizes production activities accordingly. The sales price is determined by the market demand.

According to the above, the retailer’s expected profit is:
\[ E[p^*_A] = (a - q - w_i)q - \delta^2 \]  

(3)

Based on the mean-variance method, the target utility function of the risk-averse manufacturer is:

\[
U(p^*_M) = E[p^*_M] - k\sqrt{\text{Var}(p^*_M)} = (w_i - c)q - k(w_i - c)\delta
\]  

(4)

Because the manufacturer is the leader of the Stackelberg game, the reverse induction is used to solve the problem. Because the second derivative of \(E[p^*_M]\) with respect to \(q\) is -2, let the first derivation of \(E[p^*_M]\) with respect to \(q\) be 0. The retailer’s optimal expected order quantity expression is \(q^* = \frac{1}{2}(a - w_i)\). Substituting \(q^*\) into \(U(p^*_M)\), the optimal wholesale price of the manufacturer \(w^*_M\) can be obtained. At this time, the Nash equilibrium solution of the supply chain is:

\[
w^*_M = \frac{1}{2}(a + c - 2k\delta)
\]  

(5)

\[
q^* = \frac{1}{4}(a - c + 2k\delta)
\]  

The optimal expected utility of the manufacturer and the optimal profit of the retailer are:

\[
E[p^*_M] = \frac{1}{16}(a^2 + c^2 - 4ck\delta - 4(4 - k^2)\delta^2 - 2a(c - 2k\delta))
\]  

(6)

\[
U(p^*_M) = \frac{1}{8}(c - a + 2k\delta)^2
\]  

3.2. External Financing Model (Model B)

When the manufacturer produces green products, it needs to bear the processing cost of green products \(c^2\), and the manufacturer will be underfunded. At this time, it can choose to finance loans through banks. The decision order of the manufacturer will be underfunded. At this time, it can choose to finance loans through banks. The decision order of the manufacturer will be underfunded. At this time, it can choose to finance loans through banks. The decision order of the manufacturer will be underfunded. At this time, it can choose to finance loans through banks. The decision order of the manufacturer will be underfunded. At this time, it can choose to finance loans through banks. The decision order of the manufacturer will be underfunded. At this time, it can choose to finance loans through banks.

2) The retailer decides its expected order quantity \(q\) according to the manufacturer’s decision. From the above, the manufacturer’s target utility function and the retailer’s optimal expected order quantity expression can be obtained. At this time, the Nash equilibrium solution of the supply chain is:

\[ w^*_M = \frac{4a(1+r) + c(3 + 4r) - 8k(1+r)\delta}{7 + 8r} \]  

(12)

\[ e^* = \frac{a - c - 2k\delta}{7 + 8r} \]  

(13)

4. Comparing the Decision Variables under Three Models

4.1. Comparing the Decision Variables under Three Models

By comparing and analyzing the decisions of manufacturer and retailer under the above three models, we can get:

Proposition 1

\[ w^*_M - w^*_i = \frac{a - c - 2k\delta}{14 + 16r} > 0 \]  

\[ q^* - q^* = \frac{a - c - 2k\delta}{28 + 32r} > 0 \]  

\[ e^* = \frac{a - c - 2k\delta}{7 + 8r} > 0 \]  

3.3. Internal Financing Model (Model C)

When the manufacturer produces green products, the manufacturer needs to bear the processing cost of green products, and the manufacturer faces financial constraints. In addition to using external financing to obtain funds (Model B) for green product production \(c^2\), manufacturer can also finance within the supply chain. This paper assumes that the interest rate of manufacturer’s internal financing is equal to the interest rate of external financing. The decision order of the supply chain is: 1) the manufacturer decides the wholesale price \(w_2\) and the green technology input level \(e\) in advance. 2) The retailer decides the expected order quantity \(q\) according to the manufacturer’s decision. The retailer’s expected profit, the manufacturer’s target utility function:

\[ E[p^*_M] = (a - q + e - w_2)q - \delta^2 + re^2 \]  

(10)

\[ U(p^*_M) = E[p^*_M] - k\sqrt{\text{Var}(p^*_M)} = (w_2 - c)q - k(w_2 - c)\delta - (1 + r)e^2 \]  

(11)

From the equation (10), the second derivative of \(E[p^*_M]\) with respect to \(q\) can be obtained and the result is less than 0, so the expected profit of the retailer’s optimal expected order quantity can be obtained as \(q^* = \frac{1}{2}(a + e - w_2)\). Then substitute \(q^*\) into \(U(p^*_M)\), and the optimal wholesale price and green technology input level of the manufacturer can be obtained by jointly solving \(\frac{\partial U(p^*_M)}{\partial w_2} = 0\) and \(\frac{\partial U(p^*_M)}{\partial e} = 0\). At this time, the Nash equilibrium solutions of the supply chain are:

\[ w^*_M = \frac{4a(1+r) + c(3 + 4r) - 8k(1+r)\delta}{7 + 8r} \]  

(12)

\[ e^* = \frac{a - c - 2k\delta}{7 + 8r} \]  

(13)
It is proved that \( a-c-2k\delta >0 \) can be obtained from
\[
w_k^* - w_i^* = a - c - 2k\delta > 0.
\]
Therefore, \( w_k^* > w_i^* \), \( q^* = q^e > q^a \).

Proposition 1 explains:
When the manufacturer is constrained by the funds for producing green products, whether it chooses external financing or internal financing, the input cost of the manufacturer is fixed. Therefore, the wholesale price decided by the manufacturer is the same under the external financing model and the internal financing model. The retailer makes decisions based on the wholesale price given by the manufacturer, so the expected order quantity decided by the retailer is equal under the two models. From the above, the manufacturer has the same interest rate for financing under the two models, and the cost of green production is the same, so the level of green technology investment is the same under the two models. However, under the no-financing model, the input cost of traditional products is lower than that of green products, so the wholesale price of traditional products is lower than that of green products. In real life, China’s green consumers surge, and the concept of green consumption prevails. Retailer will expect to order more green products to meet the needs of the market in order to maximize their own profits. Therefore, the retailer’s expected order quantity under the external financing model and the internal financing model is greater than that under the no-financing model, and lowercase letters.

**Proposition 2**

\[
U(\pi_{a^+}) - U(\pi_{a^-}) = U(\pi_{a^+}) - U(\pi_{a^0}) = \frac{(a-c-2k\delta)^2}{8(7+8r)} > 0
\]

\[
E[\pi_{a^+}^*] - E[\pi_{a^0}^*] = \frac{r(a-c-2k\delta)^2}{(7+8r)^2} > 0,
\]

\[
E[\pi_{a^-}^*] - E[\pi_{a^0}^*] = \frac{(a-c-2k\delta)(r(a-c-2k\delta)+2k(13+16r)\delta)}{16(7+8r)^2} > 0
\]

\[
E[\pi_{a^+}^*] > E[\pi_{a^0}^*] > E[\pi_{a^-}^*]
\]

**Proposition 2** shows:
From proposition 1, it can be seen that the optimal wholesale price of the manufacturer’s decision and the optimal expected order quantity of the retailer’s decision are equal whether the manufacturer carries out internal financing or external financing. Moreover, the cost of external financing and internal financing of the manufacturer is \( r e^- \), so the optimal profit of the manufacturer under the external financing model is equal to the optimal profit under the internal financing model. According to proposition 1, under model B and model C, the optimal green technology investment level of the manufacturer’s decision is also the same, so the sales price of the green product \( p_r = a - (q + c) + e \) is equal under the two models, and then it is easy to obtain that the retailer’s optimal profit under the external financing model is equal to the optimal profit under the internal financing model. However, because the wholesale price of traditional products is less than the optimal wholesale price of green products, and the retailer’s optimal expected order quantity for green products is greater than the optimal expected order quantity of traditional products. Therefore, compared with the non-financing model (that is, manufacturer only produces traditional products), the optimal profits of manufacturer and retailer are higher under the external financing model and the internal financing model (both models produce green products).

### 4.2. Analyzing the Influencing Factors of Game Participants’ Optimal Profit, Product Wholesale Price, Product Green Technology Input Level and Expected Order Quantity

**Corollary 1**

\[
\frac{\partial w_k^*}{\partial k} = -\delta < 0, \quad \frac{\partial q^*}{\partial k} = \frac{\delta}{2} > 0,
\]

\[
\frac{\partial U(\pi_{a^+}^*)}{\partial k} = -\frac{1}{2}\delta(a-c-2k\delta) < 0,
\]

\[
\frac{\partial E[\pi_{a^+}^*]}{\partial k} = \frac{1}{4}\delta(a-c+2k\delta) > 0
\]

When the manufacturer only produces traditional products, the manufacturer does not need to finance. After calculating the first-order derivative of each decision variable \( k \) and the optimal profit solution of the manufacturer and the retailer on the risk aversion degree of the manufacturer, it is concluded that \( w_i^* \) and \( U(\pi_{a^+}^*) \) decrease with the increase of the risk aversion degree of the manufacturer \( k \). However, \( d^* \) and \( E[\pi_{a^+}^*] \) increase with the increase of the manufacturer’s risk aversion.

Corollary 1 shows the impact of the manufacturer’s risk aversion on the decision variables and the revenue functions of the retailer and the manufacturer. As the manufacturer’s risk aversion increases, the degree of risk it can bear will be reduced accordingly. Manufacturer tends to adopt a low-price strategy, while manufacturer adopts a high-price strategy. At the same time, retailer is more inclined to cooperate with manufacturer with high risk aversion and make more profits, while manufacturer’s risk aversion preference will have a negative impact on their returns.

**Corollary 2**

\[
\frac{\partial w_k^*}{\partial r} = \frac{8(1+r)\delta}{7+8r} > 0, \quad \frac{\partial w_k^*}{\partial c} = \frac{4(a-c-2k\delta)}{(7+8r)^2} < 0,
\]

\[
\frac{\partial e^*}{\partial r} = \frac{2\delta}{7+8r} < 0, \quad \frac{\partial e^*}{\partial c} = \frac{8(a-c-2k\delta)}{(7+8r)^2} < 0,
\]

\[
\frac{\partial q^*}{\partial r} = \frac{(3+4r)\delta}{7+8r} > 0, \quad \frac{\partial q^*}{\partial c} = \frac{2(a-c-2k\delta)}{(7+8r)^2} < 0,
\]

\[
\frac{\partial U(\pi_{a^+}^*)}{\partial r} = \frac{4(a-c-2k\delta)}{7+8r} < 0
\]

\[
\frac{\partial U(\pi_{a^+}^*)}{\partial c} = \frac{4(a-c-2k\delta)}{(7+8r)^2} < 0
\]

\[
\frac{\partial E[\pi_{a^+}^*]}{\partial r} = \frac{2(3+4r)\delta(2(a-c)(1+r)+k(3+4r)\delta)}{(7+8r)^2} > 0,
\]

\[
\frac{\partial E[\pi_{a^+}^*]}{\partial c} = \frac{4(a-c-2k\delta)(2(a-c)(1+r)+k(3+4r)\delta)}{(7+8r)^2} < 0
\]

Corollary 2 shows:
Under the external financing model: 1) \( w_k^e, e^e \) and \( U(\pi_{a^e}^e) \) decrease with the increase of \( k \), \( q^e^e \) and \( E[\pi_{a^e}^e] \) increase with the increase of \( k \). 2) \( w_k^i, e^i, U(\pi_{a^i}^i) \), \( q^i^i \) and \( E[\pi_{a^i}^i] \).
all decrease with the increase of \( r \).

Wholesale price, product green level, and manufacturer's expected utility are inversely proportional to the manufacturer's risk aversion. The retailer's expected order quantity and its expected utility are proportional to the manufacturer's risk aversion. When the manufacturer's risk aversion is higher, manufacturer and retailer tends to adopt a low-cost strategy, while reducing the level of product green technology investment to prevent risks. At this time, the expected utility of the manufacturer will also decrease. Although the retailer receives internal financing interest, the increase in interest rates leads to a greater impact of wholesale price height on the retailer's expected utility. Therefore, retailer will reduce cooperation with manufacturer, and a medium interest rate level is more favorable for retailer.

5. Conclusion

We focused on a two-stage supply chain consisting of a single manufacturer and a single retailer, where the manufacturer can choose not to finance the production of traditional products, and can also choose to produce green products through external financing or internal financing. Based on the above, considering the coexistence of decision makers' risk attitude and output uncertainty, we studied the manufacturer's financing strategy selection and the retailer's ordering strategy. Through the above analysis, the following conclusions can be drawn:

(1) When manufacturer produces green products, they carry out external financing or internal financing. At this time, the wholesale price of manufacturer is the same, and it is greater than that of traditional products without financing. Moreover, when manufacturer produces green products, external financing and internal financing will not affect their green technology investment level. This is because, regardless of its choice of external financing or internal financing, the manufacturer's input costs are fixed, so the wholesale price and the level of green technology investment are the same in both cases. The input cost of traditional products is lower than that of green products, so the wholesale price of traditional products is less than that of green products.

(2) The retailer’s expected order quantity is equal when the manufacturer produces green products and conducts external or internal financing, and is greater than the non-financing situation when producing traditional products. Because the green technology investment level and wholesale price in the financing situation are the same, the retailer’s expected order quantity is naturally the same, but in real life, the retailer will expect to order more green products to meet the needs of the market in order to maximize its own profits. Therefore, the retailer’s expected order quantity under the external financing model and the internal financing model is greater than that under the non-financing model.

(3) In the three cases, when the manufacturer's risk aversion is large, the risk-averse manufacturer often adopts a low-cost strategy and reduces the level of green technology investment. At this time, the retailer is more inclined to cooperate with the manufacturer to make more profits. When the financing interest rate is high, the cost of green product production will increase. Manufacturer tends to adopt a high-price strategy and reduce the level of green technology investment to prevent risks. At this time, the expected utility of the manufacturer will also decline, and the retailer will also reduce its cooperation with the manufacturer due to the higher wholesale price.

For manufacturer, the optimal decision is to produce green products through financing. Whether it is external financing or internal financing, the manufacturer's expected profit is constant. For retailer, the optimal decision is that manufacturer produces green products and carry out internal financing, so retailer should actively promote the financing of manufacturer.
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References


