

Optimization Research Based on a Multi-Beam Ranging System

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Abstract: This paper studies the deep-water multi-beam swath system, analyzing issues such as the coverage width of swaths and the overlap rate between adjacent swaths, and establishes appropriate mathematical models to address optimization problems in the field of marine depth sounding using multi-beam systems. Firstly, based on the knowledge of multi-beam depth sounding technology, a corresponding mathematical model is established. In the seafloor measurement system, data such as coverage width (W), transducer opening angle (θ), water depth (D), the distance between adjacent survey lines (d), and overlap rate (η) can be obtained. When the seafloor terrain is flat, the calculation formula can be derived using relevant professional knowledge, such as the propagation principle of sound speed in water and knowledge of trigonometric functions in geometry: Since the actual seafloor will have a certain slope, the slope (α) also needs to be taken into account. With the aforementioned data, relevant mathematical equations can be obtained to calculate the results of seawater depth, coverage width, and overlap rate as shown in Table 1. These results help us understand the coverage of the multi-beam depth sounding system at different distances from the center point, and design and plan the survey lines and line spacing reasonably according to the coverage situation.

Keywords: Coverage Width, Overlap Rate, Mathematical Equation.

1. Problem Restatement

Single-beam echosounding systems transmit a single beam towards the seafloor, and by measuring the reflection of the beam, they can determine the depth. Multibeam echosounding systems, on the other hand, can send out multiple beams simultaneously, measuring the depth of different areas of the seafloor. Essentially, both single-beam and multibeam echosounding systems work on the principle of utilizing the propagation characteristics of sound waves underwater to measure water depth. The working principle can be summarized as follows: the transducer of the depth sounder emits a sound wave underwater, and at the same time, the transducer also receives the sound wave reflected from the seabed. By calculating the time difference between the two sound waves and the propagation speed of the sound wave on the seabed, the depth data can be obtained. Since the sound waves emitted by the multibeam system are at a certain angle to the vertical direction, this factor must be taken into account when calculating depth and positional information^[1].

$$\eta = 1 - \frac{d}{W} \quad (1)$$

When the angle of the emitted sound waves from a multibeam system remains constant, the coverage width of the multibeam survey strip varies with the water depth. From the coverage width and the distance between adjacent survey lines, the definition of overlap rate can be derived. If the overlap rate is less than zero, there may be a situation of side leakage; if the overlap rate is too large, there may be excessive overlap in the measurement, leading to redundant data, wasting resources, and affecting measurement resolution, etc^[2]. Therefore, to optimize the multibeam echosounding system, we need to control the overlap rate within a certain range. We need to address the following issues:

(1) Analyze the relationship between seawater depth D and transducer opening angle θ , and slope α .

(2) Analyze the relationship between coverage width W and seawater depth D , and transducer opening angle θ .

2. Problem Analysis

2.1. Analysis of the Problem

First, the coverage width of the multibeam bathymetric swath varies with the transducer beam angle and water depth. If the survey lines are parallel and the seafloor topography is flat, the overlap ratio can be defined as the relationship between the spacing of adjacent survey lines and the swath coverage width. If the overlap ratio is less than zero, it indicates a gap in the survey coverage. To ensure measurement efficiency, and given the actual fluctuations in seafloor topography, a relationship between the coverage width of the multibeam bathymetry and the overlap ratio between adjacent swaths is established. A trigonometric model is built based on these two variables, utilizing the law of sines and the law of cosines. In this trigonometric model, the transducer beam angle and the seafloor slope are fixed, and the distances from the survey line to the central point are traversed to calculate the coverage width and overlap ratio for each scenario compared to the previous survey line.

2.1.1. Model Assumptions

- It is assumed that the seafloor surface is smooth and without undulations.
- The seafloor is considered a flat plane, and errors caused by topographic fluctuations are not considered.
- It is assumed that no other factors affecting measurement quality exist in the surveyed area.
- There are no debris or other obstructions in the surveyed area that could interfere with the survey lines, ensuring measurement quality is unaffected.
- It is assumed that the survey vessel can move along the designated direction without deviating from the set course.
- The movement direction of the survey vessel is not affected by external forces, so the effects of water currents or air flow causing the vessel to deviate are not considered.

2.1.2. Explanation of Symbols

Symbol	Description
d	The distance between two adjacent survey lines
θ	The opening angle of the multibeam transducer
D	The depth of seawater at the center point of the sea area
η	The overlap rate between adjacent strips/with the previous survey line
W	The coverage width of the multibeam sounding strip
α	Slope
β	The angle between the survey line direction and the normal projection on the horizontal plane of the seabed slope
O	The intersection point of the survey line and the seabed slope
h	Seawater depth
H	The distance from the survey line to the center point

2.2. Establishment and Resolution of the Model

2.2.1. Model Establishment

Based on the assumption that the survey line direction is perpendicular to the horizontal plane, a right-angle coordinate system is established with the intersection point of the survey line and the seabed slope as the origin. The relationship between the seawater depth (h) and the depth of seawater at the center point of the sea area (D) has been identified, as well as the relationship between the coverage width (W) and the slope (α), and the relationship between the overlap rate (η) and the coverage width (W). A schematic diagram is made to illustrate these relationships.

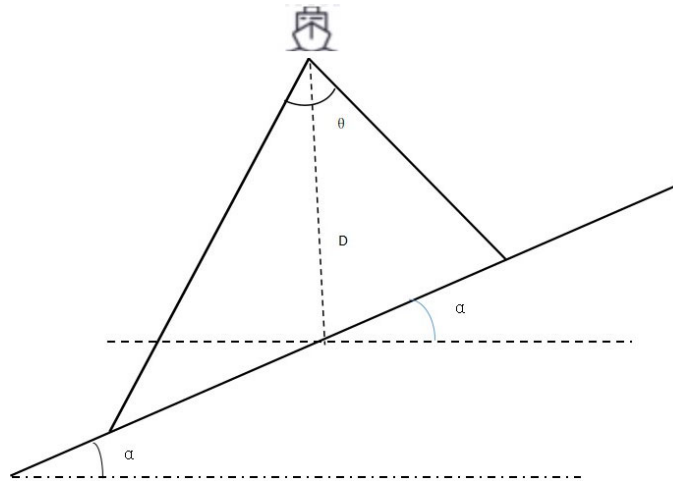


Figure 1. Model of the Problem

2.2.2. Solution of the Model

According to the formula

$$h = D - H \times \tan 1.5^\circ \quad (2)$$

$$W = 2 \times D \times \tan \frac{\theta}{2} \quad (3)$$

$$\eta = 1 - \frac{d}{W} \quad (4)$$

Given known quantities h , D , α , d , and W , the seawater depth can be calculated using the propagation speed

of sound in seawater and the propagation time. Due to the influence of the seabed slope, the seawater depth is not a fixed indicator value. However, the coverage width of the sounding strip has a certain range, which means that the distance between adjacent survey lines does not need to be too small, but it also cannot be too large. Otherwise, in areas affected by seabed slopes, there may be no coverage, leading to a coverage rate less than zero. This situation represents a missed measurement and does not meet the accuracy requirements of our values. This schematic diagram represents a special case of the problem. Considering the unknowns from a planar perspective, the calculations are based on the formulas derived from the model. Record all the results obtained, write the code, and output the results as the solution to the model given in this paper.

Table 1. Calculation Results

The distance from the survey line to the center point/m	-800	-600	-400	-200	0	200	400	600	800
Water depth/m	90.94874	85.71155	80.47437	75.23718	70	64.76282	59.52563	54.28845	49.05126
Coverage width/m	314.9477	296.8118	278.6759	260.5399	242.404	224.2681	206.1322	187.9963	169.8603
Overlap rate with the previous survey line/%	—	0.326172	0.28232	0.232363	0.174931	0.10821	0.029749	-0.06385	-0.17744

3. Advantages of the Model

a) **Rigorous Mathematical Validation:** The model has undergone strict mathematical proofing, with the use of the Cosine and Sine Theorems to verify the accuracy of the mathematical model and the data. The direct calculation of seawater depth based on distance and angle from the survey line makes the process more convenient.

b) **High Field Measurement Efficiency:** The model is highly efficient in field measurements as it employs area-based measurement, providing full coverage with minimal omissions^[3].

4. Extension of the Model

The paper abstracts the survey vessel as a geometric point, which simplifies calculations and facilitates the construction of the model. This model can be extended to positioning technology, measurement technology, and other related fields. The multibeam sonar system serves as a powerful tool for marine scientific research, resource surveys and development, engineering construction, and military activities. It can accurately obtain seabed topographic and geomorphological information within the area of interest, providing fundamental data and support for corresponding needs^[4].

5. Conclusion

Aiming at the application of multi-beam sonar system in deep water environment, this paper systematically analyzes the key factors such as coverage width, overlap rate and seabed slope by establishing a mathematical modeling method based on trigonometry. The results show that the multi-beam sonar technology can significantly improve the

mapping efficiency and effectively reduce the measurement omissions, so as to achieve more accurate seabed feature recognition in deep water environment.

Model verification shows that the mathematical framework has high rigor and practical value, and can provide theoretical basis for the optimal design of multi-beam sonar system. At the same time, this study reveals the important influence of the seabed slope on the measurement accuracy, and provides a new idea for the development of related technologies.

In the future, with the wide application of multi-beam sonar technology in Marine monitoring and military fields, the mathematical model proposed in this paper can be further extended to complex water environment and other practical scenarios, laying a foundation for improving measurement efficiency and accuracy.

References

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