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Parameter Identification Method for Permanent Magnet Synchronous Motors Based on Model Reference Adaptive System

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Abstract: During the actual operation of a permanent magnet synchronous motor (PMSM), parameter mismatch can lead to degraded control performance. To address this issue, this paper proposes an online parameter identification method based on the Model Reference Adaptive System (MRAS). The method constructs reference models and adjustable models for the d-axis and q-axis currents of the PMSM. To resolve the rank deficiency problem in the voltage equations, a stepwise identification strategy is adopted: adaptive laws based on Popov's hyperstability theory are first designed to identify the stator inductance; subsequently, utilizing the identified inductance value, adaptive laws are further designed to simultaneously identify the stator resistance and permanent magnet flux linkage. A simulation model is built in Simulink for verification. The results demonstrate that the proposed MRAS method effectively achieves online identification of motor parameters, with identification errors for stator inductance, resistance, and permanent magnet flux linkage being approximately 2.98%, 2.60%, and 3.10% respectively, and a convergence time of about 0.15 seconds. The waveforms are stable and the accuracy meets the requirements for practical engineering applications, verifying the correctness and effectiveness of the method.

Keywords: Permanent magnet synchronous motor; Control performance; Parameter identification; MRAS.

1. Introduction

With the rapid development of industrial automation, permanent magnet synchronous motor (PMSM) have been widely applied in various fields due to their high efficiency, high power density, and good dynamic performance. However, high-performance control methods for PMSM require precise motor parameters to ensure system stability[1]. Although users can obtain offline identified parameters from the motor nameplate after manufacturing, motor parameters change during actual operation due to factors such as temperature and magnetic saturation[2]. Therefore, online parameter identification has become a key research direction for improving system robustness.

In recent years, numerous scholars domestically and internationally have conducted in-depth research on online parameter identification methods, including the least squares method[3], model reference adaptive method[4], and extended Kalman filter method[5]. While these methods can achieve relatively accurate parameter identification, they also present certain issues. The Kalman filter method requires empirical selection of appropriate covariance matrices, system noise matrices, and measurement noise matrices. The least squares method suffers from data saturation as covariance and gain matrices continuously decrease, leading to reduced identification accuracy. The model reference adaptive method requires iterative trial-and-error tuning of initial system values, proportional coefficients, and integral coefficients.

This paper combines the mathematical model of the PMSM and employs the model reference adaptive method for motor parameter identification. To address the issue of rank deficiency in the voltage equations, a step-by-step identification strategy is adopted. A model is built in Simulink for simulation to verify the correctness and effectiveness of the identification algorithm.

2. Mathematical Model of PMSM

The stator voltage equations of a PMSM in the rotating coordinate system are:

$$\begin{cases} u_d = R_s i_d + \dfrac{d}{dt}\psi_d - \omega_e \psi_q \\ u_q = R_s i_q + \dfrac{d}{dt}\psi_q + \omega_e \psi_d \end{cases} \tag{1}$$

The stator flux linkage equations are:

$$\begin{cases} \psi_d = L_d i_d + \psi_f \\ \psi_q = L_q i_q \end{cases} \tag{2}$$

In Equations (1) and (2): u_d , u_q are the stator voltage components in the $d-q$ axes; R_s is the stator resistance; i_d , i_q are the stator current components in the $d-q$ axes; ω_e is the electrical angular velocity; ψ_d , ψ_q are the stator flux linkage components in the $d-q$ axes; ψ_f is the permanent magnet flux linkage; L_d , L_q are the inductance components in the $d-q$ axes.

For surface-mounted permanent magnet synchronous motor $L_d = L_q = L_s$, Substituting Equation (2) into Equation (1) yields:

$$\begin{cases} u_d = R_s i_d + L_s \dfrac{d}{dt} i_d - \omega_e L_s i_q \\ u_q = R_s i_q + L_s \dfrac{d}{dt} i_q + \omega_e \left(L_s i_d + \psi_f \right) \end{cases} \tag{3}$$

3. MRAS Parameter Identification

3.1. MRAS Theory

The Model Reference Adaptive System (MRAS) is an advanced adaptive control strategy. It can be used not only for sensorless speed and rotor position estimation in PMSMs but also for PMSM parameter identification. The algorithm principle is shown in Figure1. The reference current i output by the reference model is compared with the adjustable current $\hat{i}_1$ output by the adjustable model to obtain the error e_1 This e_1 and the input variables u are fed into a designed adaptive law to obtain the motor parameters to be identified in the adjustable model. The identified parameters are then fed back into the adjustable model, and the process repeats. During this process, the error e_1 gradually decreases. When $e_1 = 0$, the adjustable model is considered equal to the reference model, meaning the identified parameters in the adjustable model match the actual motor parameters.

Since the mathematical model in Equation (1) is a two-dimensional equation system, simultaneously identifying three parameters leads to rank deficiency in the voltage equations. Therefore, a step-by-step identification approach is adopted. First, the stator resistance and permanent magnet flux linkage are treated as constants and input into Adaptive Law1 in Figure1 to identify the stator inductance. Then, based on the identified stator inductance, Adaptive Law2 is used to identify the stator resistance and permanent magnet flux linkage.

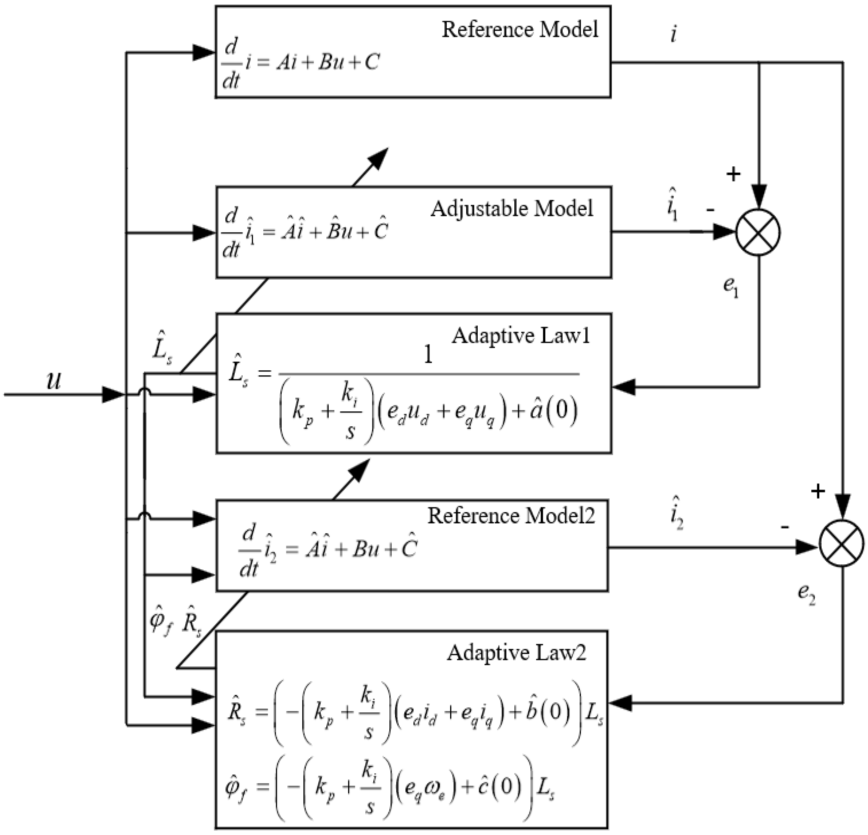

Figure 1. MRAS Structure Diagram

In the MRAS algorithm, the appropriateness of the adaptive law design is crucial for identification performance. Adaptive laws are typically designed using either Popov hyperstability theory or Lyapunov stability theory. Popov hyperstability can directly handle nonlinear time-varying systems, and its adaptive law design is simpler, whereas Lyapunov requires constructing complex energy functions. Comparatively, Popov is more suitable for rapid tracking of motor parameters, balancing stability and real-time performance. Therefore, this paper employs Popov hyperstability theory to design the adaptive laws.

3.2. Stator Inductance Adaptive Law Design

Equation (3) is taken as the reference model and simplified as:

$$\frac{d}{dt}i = Ai + Bu + C \tag{4}$$

where $i = \begin{bmatrix} i_d & i_q \end{bmatrix}^T$ $u = \begin{bmatrix} u_d & u_q \end{bmatrix}^T$. Let $a = 1/L_s$ then the coefficient matrix A, B, C are:

$$A = \begin{bmatrix} -aR_s & \omega_e \\ -\omega_e & -aR_s \end{bmatrix} B = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} C = \begin{bmatrix} 0 \\ -a\psi_f \omega_e \end{bmatrix}$$

Based on the reference model, the adjustable model is designed as:

$$\frac{d}{dt}\hat{i} = \hat{A}\hat{i} + \hat{B}u + \hat{C} \tag{5}$$

where $\hat{i} = \begin{bmatrix} \hat{i}_d & \hat{i}_q \end{bmatrix}^T$ $u = \begin{bmatrix} u_d & u_q \end{bmatrix}^T$. Let $\hat{a} = 1/\hat{L}_s$, $\hat{L}_s$ is the identified stator inductance value. The coefficient matrix $\hat{A}, \hat{B}, \hat{C}$ are:

$$\hat{A} = \begin{bmatrix} -\hat{a}R_s & \omega_e \\ -\omega_e & -\hat{a}R_s \end{bmatrix} \hat{B} = \begin{bmatrix} \hat{a} & 0 \\ 0 & \hat{a} \end{bmatrix} \hat{C} = \begin{bmatrix} 0 \\ -\hat{a}\psi_f \omega_e \end{bmatrix}$$

Define the error $e = \begin{bmatrix} i_d - \hat{i}_d & i_q - \hat{i}_q \end{bmatrix}^T$. Subtracting Equation (4) from Equation (5) gives:

$$\begin{aligned} \frac{d}{dt}e &= Ai + Bu + C - (\hat{A}\hat{i} + \hat{B}u + \hat{C}) \\ &= Ae + (\Delta A\hat{i} + \Delta Bu + \Delta C) \\ &= Ae - M \end{aligned} \tag{6}$$

Let $M = -(\Delta A\hat{i} + \Delta Bu + \Delta C)$. Based on Equation (6), the state equation for the nonlinear feedback channel can be constructed as:

$$\begin{cases} \frac{d}{dt}e = Ae - M \\ V = De \end{cases} \tag{7}$$

In the equation, setting $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the MRAS structure is shown in Figure 2. The necessary and sufficient conditions for system stability are: 1) The transfer function $H(s)$ of the linear time-invariant forward path is strictly positive real , and 2) The nonlinear time-varying feedback path satisfies the Popov integral inequality.

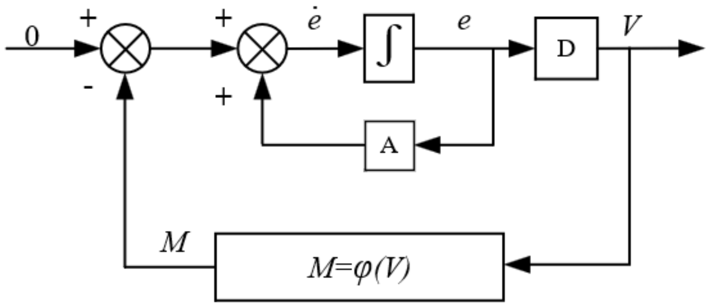

Figure 2. Nonlinear Feedback System

According to the positive real lemma[6], the transfer function $H(s)$ is strictly positive real if and only if there exist symmetric positive definite matrices P and Q satisfying:

$$\begin{cases} PA + A^T P = -Q \\ PI = D \end{cases} \tag{8}$$

Let $p = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$. Substituting into Equation (8) shows that when $k > 0$, the transfer function is strictly positive real. Since the compensator $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, set $k = 1$.

After proving the forward path transfer function is strictly positive real, it must also be proven that the feedback path satisfies the Popov integral inequality:

$$\eta(0,t) = \int_0^t V^T M dt \geq -\gamma^2, \forall (t > 0) \tag{9}$$

Substituting V and M into Equation (9) and splitting the integral inequality yields:

$$\begin{cases} \eta_1(0,t) = \int_0^t (a - \hat{a})(R_s e_d \hat{i}_d + R_s e_q \hat{i}_q) dt \geq -\gamma_1^2 \\ \eta_2(0,t) = \int_0^t (\hat{a} - a)(e_d u_d + e_q u_d) dt \geq -\gamma_3^2 \\ \eta_3(0,t) = \int_0^t (a - \hat{a}) e_q \psi_f \omega_e dt \geq -\gamma_3^2 \end{cases} \tag{10}$$

The adaptive law is derived using the second equation in Equation (10). To maintain system adjustment when the error approaches zero, a PI form is adopted for the adaptive law:

$$\hat{a} = \int_0^t f_1(\tau) d\tau + f_2(t) + \hat{a}(0) \tag{11}$$

Substituting Equation (11) into the second equation of Equation (10) and splitting gives:

$$\begin{cases} \int_0^t \left(\int_0^t f(\tau) d\tau + \hat{a}(0) - a \right) \left(e_d u_d + e_q u_q \right) dt \geq -\gamma_{11}^2 \\ \int_0^t f_2(t) \left(e_d u_d + e_q u_q \right) dt \geq -\gamma_{12}^2 \end{cases} \tag{12}$$

To ensure Equation (12) holds, set:

$$\begin{cases} \frac{d}{dt} F(t) = e_d u_d + e_q u_q \\ kF(t) = \int_0^t f_1(\tau) d\tau + \hat{a}(0) - a \end{cases} \tag{13}$$

Substituting Equation (13) into Equation (12), the first part of Equation (12) satisfies the following inequality:

$$\frac{k}{2}\left[f^2(t) - f^2(0) \right] \geq -\frac{k}{2} f^2(0) \tag{14}$$

Equation (14) confirms the first equation of (12) holds. Next, differentiating both sides of the second equation in Equation (13) gives:

$$f_1(t) = k(e_d u_d + e_q u_q) \tag{15}$$

Substituting Equation (15) into the second equation of Equation (12) satisfies the inequality. Therefore, the adaptive law for the parameter to be identified $\hat{a}$ is:

$$\hat{a} = \frac{1}{L_s} = \left(k_p + \frac{k_i}{s} \right) \left(e_d u_d + e_q u_q \right) + \hat{a}(0) \tag{16}$$

3.3. Stator Resistance and Permanent Magnet Flux Linkage Adaptive Law Design

After identifying the stator inductance, it is treated as a known quantity to design the adaptive laws for stator resistance and permanent magnet flux linkage. The steps are identical to those for the stator inductance. Therefore, the adaptive laws for stator resistance and permanent magnet flux linkage are presented directly without detailed derivation.

$$\frac{\hat{R}_s}{\hat{L}_s} = -\left(k_p + \frac{k_i}{s}\right)\left(u_d e_d + u_q e_q\right) + \hat{b}(0) \tag{17}$$

$$\frac{\hat{\varphi}_f}{\hat{L}_s} = -\left(k_p + \frac{k_i}{s}\right)\left(\omega_e e_q\right) + \hat{c}(0) \tag{18}$$

4. MRAS Parameter Identification Simulation Analysis

To verify the accuracy and effectiveness of the MRAS parameter identification, a simulation model of PMSM vector control with MRAS parameter identification was built in Simulink. The structure is shown in Fig. 3, and the motor parameters are listed in Table 1.

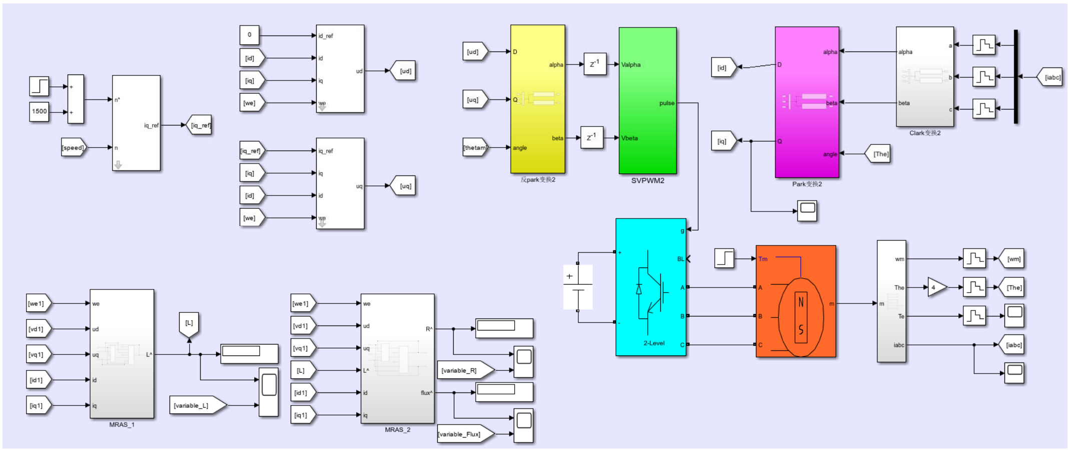

Figure 3. PMSM Vector Control and MRAS Identification Model

Table 1. PMSM Parameter Table

Parameter Name	Parameter Value
Stator Resistance	0.89Ω
Stator Inductance	0.00062H
Permanent Magnet Flux Linkage	0.00838Wb
Pole Pairs	4

4.1. Three-Phase Current and Torque Waveform Analysis

Figure 4 and Figure 5 show the three-phase current and torque waveforms, respectively, under the simulation environment defined in Table 1. Figure 4 shows that the starting phase requires a large current, reaching the limit value of 4A, and the current waveform is distorted. After approximately 0.02 seconds, due to the action of the PI regulator, the distorted three-phase current waveform returns to normal, stabilizing at an amplitude of 2.5A under no-load conditions. Due to current overshoot, the torque during startup reaches the rated torque maximum of 0.2 Nm (Fig. 5), then quickly recovers to the normal no-load state value of 0.125 N · m. The torque oscillation amplitude is small, indicating good control performance.

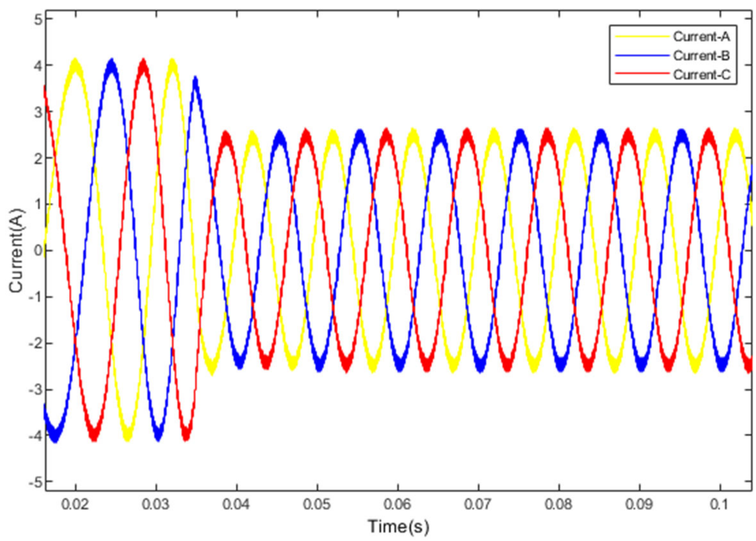

Figure 4. Three-phase current waveform

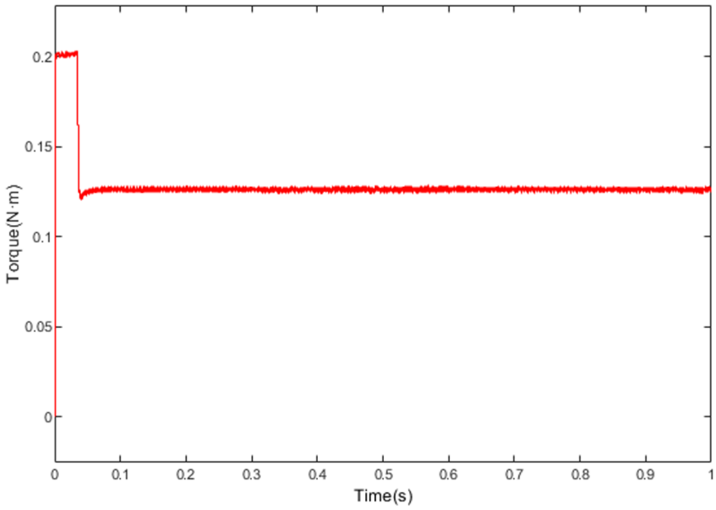

Figure 5. torque waveform

4.2. Online Parameter Identification Waveform Analysis

Figures 6-8 show the simulation waveforms for the online identification of stator inductance, stator resistance, and permanent magnet flux linkage, respectively. The convergence times of the identified values show a similar trend, converging in approximately 0.15 seconds. The identified stator inductance is 0.0006015 H (Fig. 6), with an identification error of about 2.98%. The identified stator resistance is 0.867 Ω (Fig. 7), with an error of about 2.60%. The identified permanent magnet flux linkage is 0.008121 Wb (Fig. 8), with an error of about 3.10%. The identification accuracy is high, waveform oscillation is small, and stability is good, meeting the requirements for practical engineering applications.

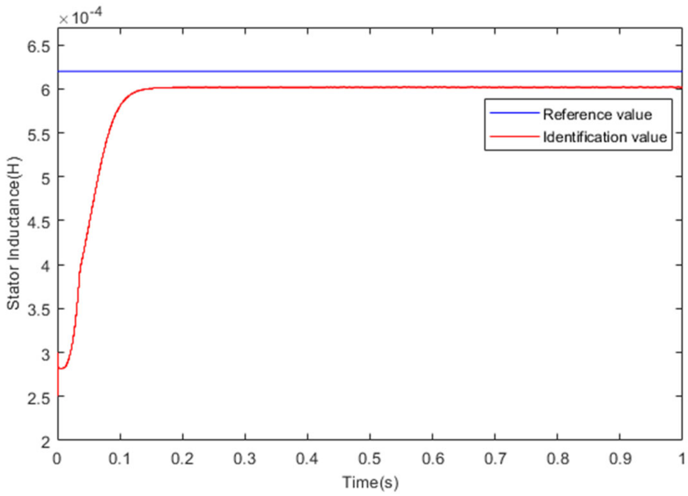

Figure 6. Stator inductance identification waveform

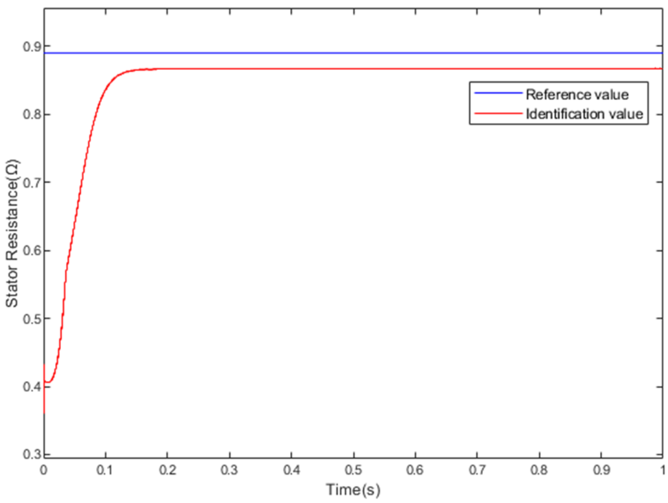

Figure 7. Stator resistance identification waveform

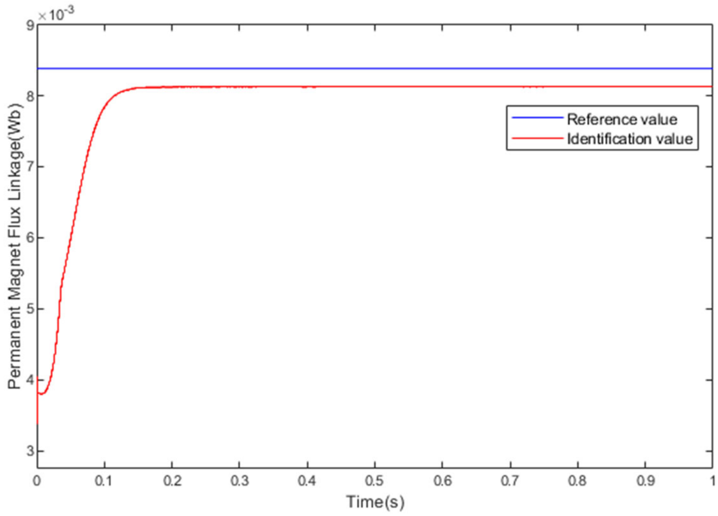

Figure 8 Permanent magnet flux linkage identification waveform

5. Conclusion

This paper proposes an online parameter identification method for permanent magnet synchronous motors (PMSM) based on a Model Reference Adaptive System (MRAS), addressing control performance degradation caused by parameter mismatch during operation. By constructing reference and adjustable models for d/q-axis currents, a stepwise identification strategy resolves the rank deficiency issue in voltage equations: first, adaptive laws designed using Popov hyperstability theory identify stator inductance, followed by identification of stator resistance and permanent magnet flux linkage using the known inductance. Simulink simulations demonstrate that this method achieves online parameter identification, with identification errors for stator inductance, resistance, and permanent magnet flux linkage approximately at 2.98%, 2.60%, and 3.10% respectively. The convergence time is about 0.15 seconds, featuring stable waveforms and accuracy meeting engineering application requirements, thereby validating the correctness and effectiveness of the proposed approach.

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