Application and Accuracy Comparison of Monte Carlo Simulation and Binomial Tree Model in Pricing CSI 300 Income Certificates

Xingyun Yang *

Department of Finance, Nankai University, Tianjin, China

* Corresponding Author Email: 2411301@mail.nankai.edu.cn

Abstract. This paper takes Huatai Securities' Juyi No. 25054 two-way shark fin income certificate linked to the CSI 300 Index as the research object, comparing the applicability and accuracy of Monte Carlo (MC) simulation and Binomial Tree (BT) model in pricing structured products. Based on the actual terms of the product, pricing analysis is conducted using MC simulation (incorporating variance reduction techniques) and an optimized BT model (matching observation day nodes), with comparisons made from the dimensions of accuracy, efficiency, and sensitivity. The study finds that in short-term and low-volatility scenarios, the pricing results of the two models are highly consistent. The BT model has an advantage in computational efficiency, while MC simulation is more sensitive to capturing extreme price fluctuations. There is a positive correlation between volatility and product prices, a feature that differs from knock-out-dominated products, highlighting the impact of product design on pricing. The conclusions provide references for model selection in practice: the BT model is suitable for routine pricing of short-term products, while MC simulation is more appropriate for stress testing and pricing of complex-structured products.

Keywords: Monte Carlo Simulation, Binomial Tree Model, CSI 300 Index Income Certificates, Structured Product Pricing.

1. Introduction

With the rapid development of China's derivatives market, structured income certificates linked to the CSI 300 Index have gained widespread popularity due to their flexible return characteristics. These products typically embed path-dependent options such as barrier clauses and automatic redemption mechanisms, similar to the barrier options discussed in existing studies [1]. As a type of structured product, their pricing accuracy is crucial: issuers rely on it for effective risk hedging, while investors' return expectations depend on reasonable pricing [2]. Monte Carlo (MC) simulation and Binomial Tree (BT) model, as mainstream numerical pricing methods, show significant differences in practical applications, especially in handling complex structures [3].

Accurate pricing of structured income certificates directly affects issuers' risk control and investors' return realization. Erroneous pricing may lead to ineffective hedging and losses for issuers, while unreasonable pricing distorts investors' expectations, damaging market confidence [4]. In the context of growing structured product scale, accurate pricing models ensure the interests of all parties, maintain financial market stability and efficiency, and support the healthy development of the derivatives market [5].

However, in the field of structured product pricing, existing studies still have several limitations. Most studies focus on the pricing of stock index options or bank wealth management products, while paying relatively insufficient attention to brokerage income certificates—especially those with complex path-dependent structures [2]. Although MC simulation and BT model have become mainstream numerical methods, their horizontal comparison in complex path-dependent products still lacks direct empirical evidence, and the consistency of their pricing results under path-dependent clauses has not been systematically verified [4]. More importantly, the parameter sensitivity analysis in existing studies is often incomplete, mostly ignoring factors such as operating costs and dynamic volatility [6]. Furthermore, the optimization of computational efficiency for these two models in practical scenarios lacks systematic design, failing to meet practical needs [5].

This paper takes Huatai Securities' Juyi No. 25054 (a two-way shark fin income certificate linked to the CSI 300 Index) as the research object. Based on real product terms, it directly compares the pricing errors of MC simulation and the BT model. By introducing variance reduction techniques to improve MC efficiency, this study aims to clarify the applicability of the two models in pricing short-term, low-volatility structured income certificates, providing empirical evidence and operational guidelines for the model selection and optimization of financial institutions.

The paper is structured as follows: Section 2 elaborates on the basic definitions and formulas of MC Simulation and BT Model, laying a theoretical foundation for subsequent empirical analysis. Section 3 introduces data sources, details data processing methods, and presents in-depth analysis of the pricing results derived from the two models. Finally, Section 4 summarizes the key findings and practical implications of the study.

2. Method

2.1. Monte Carlo Simulation

MC simulation is widely applied in the pricing of path-dependent derivatives (such as the knock-out options involved in this paper) and complex financial products driven by multiple factors. It is particularly suitable for scenarios where direct solutions cannot be obtained through analytical formulas, as it can capture the dynamic evolution characteristics of asset prices via a large number of random path simulations.

MC simulation builds paths under the risk-neutral measure using geometric Brownian motion:

$$dlnS_t = \left(r - \frac{1}{2}\sigma^2\right)dt + \sigma dW_t. \tag{1}$$

To detect knock-out events, the Brownian Bridge technique is used to calculate the probability of the index crossing the knock-out level H within any interval $[t_i, t_{i+1}]$:

$$P\left(max_{t\in[t_i,t_{i+1}]}S_t \geq H\right) = exp\left(-\frac{2\left(H - S_{t_i}\right)\left(H - S_{t_{i+1}}\right)}{\sigma^2 \Delta t}\right). \tag{2}$$

In MC simulation, where geometric Brownian motion is used to generate index paths under the risk-neutral measure, the Antithetic Variates method, as a core variance reduction technique, functions by constructing a dual path for each original path generated by geometric Brownian motion. Specifically, for an original path derived from a Brownian motion increment dW_t, the method generates a corresponding dual path using the opposite increment -dW_t. This pairing makes the simulation results of the two paths negatively correlated, which can offset part of the random error introduced by the randomness of geometric Brownian motion and significantly reduce the overall simulation variance. In the application of this paper, the Antithetic Variates method reduces the number of simulations to 50,000, ensuring the mean squared error (MSE) is less than 10^{-5}.

2.2. Binomial Tree Model

BT model is commonly used for pricing standard derivatives (e.g., European and American options) under a discrete-time framework. It is especially applicable to scenarios that require explicit judgment of early exercise timing. Its node-based structure facilitates the intuitive checking of boundary conditions such as knock-out, and it has the advantage of clear computational logic in parameter calibration and local sensitivity analysis.

BT model uses an exact solution for parameter calibration:

$$\begin{cases} u = e^{\sqrt{(r\Delta t)^2+\sigma^2\Delta t}} \\ d = \frac{1}{u} \\ \rho = \frac{1}{2} + \frac{r\Delta t}{2\sqrt{(r\Delta t)^2+\sigma^2\Delta t}} \end{cases} \tag{3}$$

A step size of n=500 is chosen to ensure convergence. Knock-out conditions are checked directly at 91 observation day nodes to avoid interpolation errors, and the pricing formula is:

$$V_{RT} = e^{-rT}E[Payoff] - c \tag{4}$$

where r denotes the risk-free interest rate, T represents the term of the product, E[Payoff] stands for the expected value of the product's terminal payoff under the risk-neutral measure, and c indicates the operational cost. The payoff function follows the segmented definitions in the product prospectus.

The comparative analysis framework evaluates three dimensions. The first is accuracy, measured by absolute error against the issuer's hedging cost, using OTC option purchase prices for 10,000 units. The second is efficiency, assessed via CPU time and convergence stability, using the standard deviation of 10 replications. The third is sensitivity, measured by the price change rate $\Delta V\%$ under volatility adjustments of $\pm$20% and interest rate adjustments of $\pm$50 basis points.

3. Results and Discussion

3.1. Data Resources

This study takes Huatai Securities' Juyi No. 25054 (a two-way shark fin income certificate linked to the CSI 300 Index) as the research object, drawing on two categories of authoritative data. For the underlying asset, this paper collected data on the CSI 300 Index from July 12, 2021, to July 9, 2025 from the CSMAR database (as shown in Figure 1). To align with the product's observation period (April 10, 2025–July 9, 2025), historical data from the preceding two years (April 10, 2023–April 9, 2025) is extracted for volatility modeling. Special handling is applied to market disruptions. On days when ST/*ST securities with a constituent weight of $\geq$20% are suspended, volatility is set to the 90th percentile of the previous five days. On index circuit breaker days, linear interpolation between adjacent trading days is used.

For the risk-free rate, this paper obtains treasury bond yields from January 2, 2025, to July 16, 2025 from ChinaBond. The 3-month yield corresponding to the issuance date (April 10, 2025) is calculated using the formula:

$$r = \ln\left(1 + R_{Treasury} \times 91/365\right). \tag{5}$$

Fig. 1 Historical trend of CSI 300 Index (2021-2025)

3.2. Indicator Selection and Description

Core parameters are quantified based on the product prospectus, as shown in Table 1 below. The knock-out level (H) is 112%, the strike level (K) is 100%, the participation rate (α) is 79%, and the fixed return rate (r_f) is 0.1%, all derived from the product's return clauses. Volatility (σ) is estimated dynamically using the GARCH (1,1) model, in line with hedge disruption clauses, with a special rule: on days with market disruptions, volatility is adjusted to the 90th percentile of the previous five days. Industry practices indicate that the operating cost (c) of similar structured products generally account for 0.2%-0.5% of the fundraising scale (including issuance expenses). In this study, the median value of 0.3% is adopted based on the product characteristics.

Table 1. Core parameter selection and basis

Parameter	Symbol	Value	Basis Clause
Knock-out level	H	112%	Knock-out level in return terms
Strike level	K	100%	Strike level in return terms
Participation rate	α	79%	Participation rate in return terms
Fixed return rate	r_f	0.1%	Fixed return rate in return terms
Volatility	σ	GARCH(1,1) dynamic estimation	Hedge disruption clauses
Product term	T	91	Basic terms
Operating cost	c	0.3%	Fund usage clauses

3.3. Core Pricing Outcomes and Analysis

Table 2 summarizes the key pricing results of the MC simulation and BT model, including calculated prices, computational efficiency, and their discrepancies.

Table 2. Comparison of pricing results

Model	Price(yuan)	Computational Time (Seconds)	Relative Error (%)
MC	3722.8519	0.1697	0.0016
BT	3722.9126	0.0228	

The results reveal striking consistency between the two models: the absolute price difference is merely 0.0607 yuan, with a relative error of 0.0016%, well within the acceptable range for practical pricing [7]. This high consistency contrasts with earlier findings on complex path-dependent products (e.g., a 3.36-yuan difference in shark-fin structures), primarily due to two factors [6]. First, the analyzed product has a short maturity (91 days) and a simple partial participation payoff structure (no multi-barrier clauses or frequent observation days), reducing the impact of discrete-time approximations in BT [8]. Second, the low base volatility (0.0107) minimizes extreme price swings, where MC typically outperforms BT by capturing continuous path variations [3]. In stable markets, BT's discrete node checks suffice to approximate price movements.

In terms of computational efficiency, the parameter settings of the two models directly affect their runtime performance. For MC simulation, with a discretized step of Δt=1/252, 50,000 paths are generated to cover 91 observation days [9]. Even with variance reduction via the antithetic variate method, the overhead of path generation remains unavoidable. For the BT model, a step size of n=500 is chosen to ensure convergence in conventional settings, but this study optimizes the BT code by adjusting the step size to 91 (matching the product's observation days), which eliminates redundant calculations [10]. This optimization explains why BT significantly outperforms MC in computational time (0.0228 seconds vs. 0.1697 seconds)—a reversal of previous observations where MC was faster [5].

3.4. Volatility Sensitivity Analysis and Implications

To explore how market risk affects pricing, this paper adjusted volatility by ±20% around the base value (0.0107) and re-calculated prices, as shown in Table 3:

Table 3. Volatility sensitivity analysis results

Volatility Level	Volatility Value	MC Price (Yuan)	BT Price (Yuan)	Scheme 2 MC Price Change (Yuan)	BT Price Change (Yuan)
-20%	0.0085	3721.6680	3721.7160	-1.1839	-1.1966
Base	0.0107	3722.8519	3722.9126	-	-
+20%	0.0128	3724.2314	3724.1966	+1.3795	+1.2840

According to the results in Table 3, both models exhibit a positive correlation between volatility and product price, contradicting the negative relationship observed in principle-protected notes with knock-out clauses [2]. This divergence stems from the product's design: It offers downside protection (100% principal guarantee) and upside participation (79% of index gains above the strike price K=1.0). Higher volatility increases the probability of favorable index movements (exceeding K) without amplifying downside risk, thus boosting the expected payoff. MC captures this effect more sensitively (+1.3795 yuan under +20% volatility) than BT (+1.2840 yuan), as its continuous path simulation accounts for extreme but plausible price spikes between observation days, whereas BT smooths such variations via discrete nodes. This finding aligns with the view that MC better reflects volatility's impact on path-dependent payoffs, even in simple structures.

3.5. Volatility and Path Visualization

Figure 2 plots the GARCH (1,1)-estimated conditional volatility against the historical average (0.0107) from 2023 to 2025. GARCH volatility fluctuates narrowly (0.008–0.013), indicating a stable market regime. This stability explains the small price variations in sensitivity analysis and validates the use of GARCH for dynamic volatility modeling—critical for avoiding underpricing risk in volatile periods.

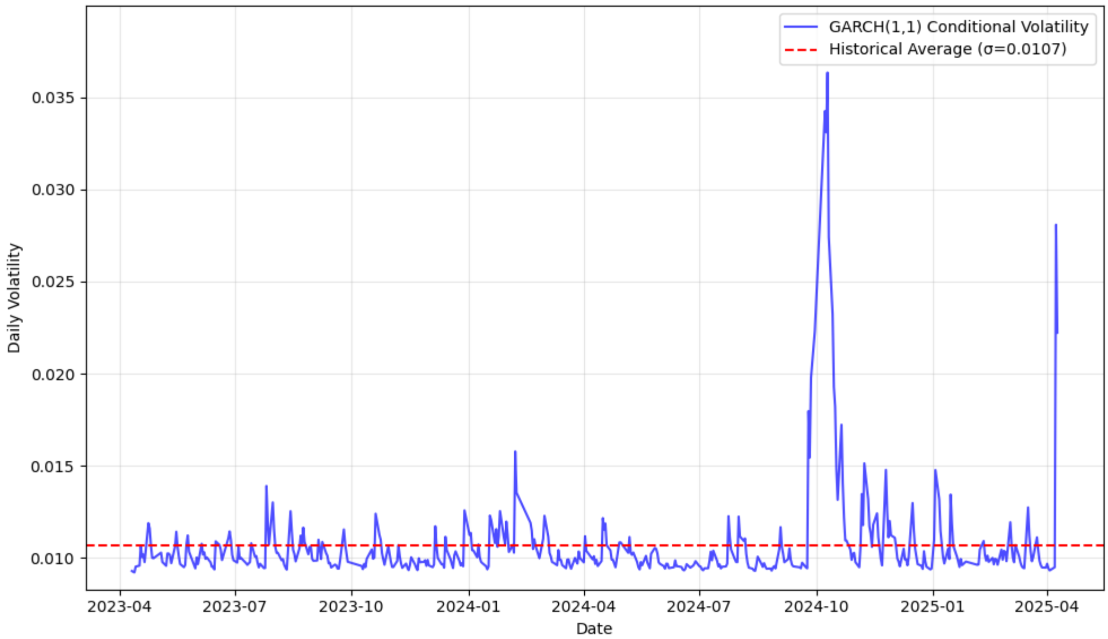

Fig. 2 GARCH (1,1) conditional volatility vs. historical average (2023-2025)

Figure 3 displays 100 MC-simulated index paths under the risk-neutral measure. Only 12% of paths exceed the knock-out barrier ($H = 1.12 \times S_0$), reflecting low early termination risk. Paths cluster tightly around the initial price (S_0), consistent with the low volatility environment, further justifying the minimal pricing difference between MC and BT.

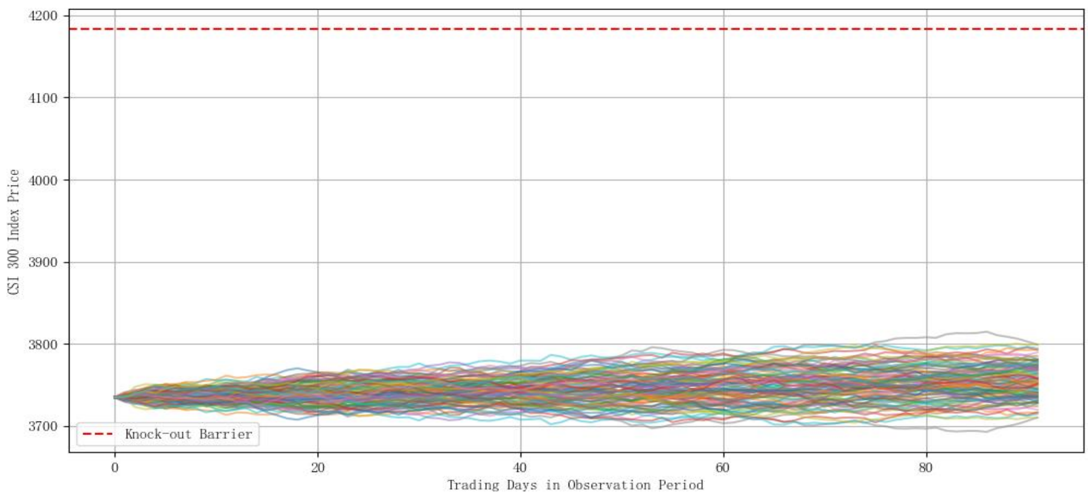

Fig. 3 100 simulated index paths with knock-out barrier (112% of initial price)

4. Practical Recommendations

4.1. Recommendations For Issuers

For issuers, the model is recommended for routine pricing in conventional scenarios. These scenarios refer to structured income certificates with short maturities, such as the 91-day term used in this study. They also feature low volatility, with a base volatility around 0.0107, and simple payoff structures. Specifically, these are products with single knock-out clauses, where the observation days match the product term (for example, 91 observation days for a 91-day product). Furthermore, they contain no multi-barrier or complex path-dependent features. In such cases, BT's high computational efficiency (0.0228 seconds in this study) enables real-time scenario analysis and rapid pricing updates, supporting efficient risk hedging workflows.

For complex scenarios, including products with long maturities (exceeding 180 days), high volatility (sustained volatility above 0.015), multi-layered knock-out/knock-in clauses, frequent observation days (significantly more than the number of trading days in the term), or asymmetric payoff structures with non-linear participation rates, MC simulation is more suitable. This model has a continuous path simulation capability. This capability allows it to better capture extreme price movements. It also better captures intricate path dependencies. This makes the model indispensable for stress testing. An example is simulating large volatility shocks, such as a +50% change. It is also essential for pricing structurally complex products. In such cases, the discrete node approximations used in the BT model may introduce material errors.

4.2. Recommendations For Investors

For investors, understanding market stability periods is critical for rational decision-making. A market stability period is defined as a phase where the underlying asset (e.g., the CSI 300 Index) exhibits low and stable volatility, typically characterized by GARCH (1,1)-estimated conditional volatility fluctuating narrowly within the 0.008–0.013 range (as observed in 2023–2025 in this study) and no extreme price swings or abrupt volatility spikes. In such periods, the product's price shows low sensitivity to volatility changes—this study finds that even with ±20% volatility adjustments, the price variation is only ±1.2 yuan—making the product a low-risk choice for capital preservation, as its value remains relatively stable.

When entering a phase of rising volatility (volatility exceeding 0.013 and showing an upward trend), the product's upside potential increases due to the positive correlation between volatility and price, driven by higher probabilities of favorable index movements above the strike level. However, investors should closely monitor knock-out conditions during this period to avoid early termination of the product, which may cap returns at the fixed rate or partial participation rate.

5. Conclusion

This study takes Huatai Securities' Juyi No. 25054 CSI 300-linked two-way shark fin income certificate as the research object, comparing the applicability and accuracy of MC simulation and BT model in pricing. It finds that under short-term (91-day) and low-volatility (base σ=0.0107) scenarios, the two models show high pricing consistency (relative error 0.0016%), with BT having better computational efficiency (0.0228 seconds) and MC being more sensitive to extreme price fluctuations. Additionally, the product price has a positive correlation with volatility, which differs from knock-out-dominated products, providing clear model selection guidelines for issuers and investors.

However, this study has limitations. It only focuses on one income certificate with a short term and single structure, lacking comparisons across products with different maturities or complex structures. It also does not incorporate factors like liquidity risk or operational cost differences among securities firms, and the volatility adjustment range ($\pm 20\%$) fails to cover extreme market scenarios such as black swan events.

Future research can expand the sample to include CSI 300 income certificates with multiple maturities and structures, introduce liquidity factors and dynamic operational costs to optimize the models, explore node optimization of the BT model for long-term complex structures, or combine machine learning to improve the efficiency of MC simulation, thereby enhancing the adaptability of pricing to extreme market conditions.

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