Separation at the Kink: Reassessing the Fisher Model under Imperfect Capital Markets

Yuan Du *

Rutgers Business School, Rutgers, The States University of New Jersey, New Brunswick, NJ, 08901, the United States

* Corresponding Author Email: yd393@scarletmail.rutgers.edu

Abstract. This paper reassesses the Fisher model under imperfect capital markets. And develop a geometric framework with a piecewise Capital Market Line (CML) and an explicit credit‐limit boundary to capture three pervasive frictions: asymmetric borrowing–lending rates, financing constraints, and policy interventions. The framework shows that when the optimal choice lies at a kink or boundary, investment cannot be separated from financing; firm decisions become endogenously tied to the cost and availability of credit. A case study of Boeing in 2020 corroborates the mechanisms: the collapse in global air travel demand (RPK) tightened funding conditions, producing a kinked CML; subsequent policy backstops flattened the effective borrowing segment and shifted the feasible set outward, enabling large-scale bond issuance and term-structure repair. We derive testable implications (H1–H3) linking spreads, issuance windows, liquidity hoarding, and the timing of capital expenditure. The analysis identifies conditions for approximate separation and highlights nonlinear policy multipliers when firms transition from boundary to interior solutions. The results clarify the model's scope of applicability and offer practical guidance for crisis-era credit tools and corporate liquidity management.

Keywords: Fisher model; separation theorem; capital market line (CML); credit constraints; borrowing–lending asymmetry.

1. Introduction

As one of the cornerstones of modern corporate finance, the Fisher Model supposes that under perfect capital market conditions, firm investment decisions and individual consumption decisions can achieve perfect separation. Firms only need to choose the investment scale that maximizes net present value (NPV) within its given production-investment opportunity set, while individuals can base on the capital market line (CML) to borrow and lend across time freely achieve their optimal consumption allocation under specific preferences. Geometrically, this framework is represented by the unique tangency point between the linear CML and the concave transformation curve, which delivers a solution that is both transparent and practically operable.

However, financing friction, information asymmetry, interest rate asymmetry, and policy-induced exogenous shocks generally exist in the real capital market. These types of imperfections will damage the Fisher model systematically when the borrowing rate is higher than the lending rate and there is a credit limit, or individual preferences are non-concave/inflected. Under these circumstances, the CML will degenerate from a single slope straight line to a segmented line, and the optimal point is usually shown on the vertices or boundaries rather than a unique interior point tangent. The distortion of inter-period "price-opportunity" means that investment choices and financing conditions are endogenously coupled, and the separation of investment and consumption is difficult to maintain.

Focusing on the above tension, the research objectives and contributions of this paper are mainly reflected in three points:

A practically compatible geometric framework of "segmented CML + credit limit" is proposed to uniformly explain the impact of financing constraints, preference heterogeneity, and policy intervention on the location of the optimal solution, and to keep it comparable with the Fisher standard plot (linear CML + conversion curve).

Taking Boeing's 2020 crisis as an illustrative case, we lay out the mechanism and the accompanying evidence path that yield a set of testable implications. At the policy level, we show

that when firms are constrained at a boundary or kink, small improvements in price or quantity instruments (e.g., spreads or credit limits) can trigger a nonlinear shift from a boundary to an interior solution, with multipliers far larger than the marginal responses at interior points—providing an analytical benchmark for credit facilities, market-making arrangements, and term-structure management in crisis periods.

2. Theoretical Framework

2.1. The Core Mechanism of the Fisher Model

The Fisher Model is fundamentally built upon the Separation Theorem, where in a perfect competition and non-friction capital market, firms' investment decision strategy is independent from investors' personal preference [1]. Under this ideal circumstance, the only goal of the firm is choosing an investment project that can maximize its net present value (NPV), and investors can borrow and lend freely in the capital market at a uniform interest rate and decide their own inter-temporal consumption allocation [2]. Therefore, investment decisions are determined by the objective market standard, and consumption completely depends on personal preferences, resulting in the separation of investment and consumption entirely [3].

Closely related to the separation theorem is the construction of the capital market line (CML). In the standard Fisher model, the transformation curve shows that when the market allows free-risk lending, the curve will be extended into a linear CML [1]. When the slope of CML is equal to the Sharpe Ratio, it represents the best risk-return trade-off available on the market [4]. Graphically, the point of tangency between the CML and the efficient frontier corresponds to the market portfolio. This portfolio under equilibrium conditions should be held by all rational investors. Differentiation of each investor's options only reflects location on CML: Investors with a strong risk appetite leverage operations through borrowed funds, while risk-averse investors prefer reducing the risk by lending funds [5].

In summary, the combination of both separation theorem and CML constructs the core logic of the Fisher Model: maximizing return on investment for firms on an objective level, while allocation of consumption and risk based on investors' preferences on a subjective level. This mechanism laid the foundation for modern corporate finance theory and portfolio theory. However, as the following chapters proved, this framework highly relies on the premise of perfect capital market, which is often difficult to establish in real cases [2].

2.2. Three Major Situations Where Reality Deviates

2.2.1 Investor Behavioral Biases

One of the key assumptions implicit in the Fisher model is that investors are completely rational and risk aversion leading to strictly concave functions. This represents, in the C0-C1 graph, that the indifference curve of investors is smooth and convex to the origin, that creates the only tangent point between CML and the indifference curve, reflecting the separation of investment and consumption. However, in the real market, behavioral finance research shows that investor preferences often deviate from this assumption [6]. Over-confidence usually will cause the investor to overestimate information advantage, leading to excessive risk-taking. Loss aversion makes investors more inclined to increase risk exposure when facing losses to "recover their investment". A status quo bias may cause them to cling to inefficient portfolios. All these irrational behaviors will make the utility function have convex intervals, inflection points, or even discontinuities that there might be several potential intersections, so it destroys the logic of the unique solution of the Fisher model.

2.2.2 Financing Constraints (Imperfect Capital Markets)

Investors being able to borrow and lend freely in the capital market at a constant interest rate, is another core condition for the separation theorem to hold [1]. However, in the reality, it's common that financing constraints and credit asymmetry. Borrowing rate is usually higher than the saving rate,

especially during a financial crisis or credit crunch. Simulation studies by Gu show that when the borrowing rate is 20% and saving rate is 5%, the capital market line will no longer be a straight line[1]. The new CML curve will become kinked, recoupling investment and consumption decisions [1]. Under this condition, even when firms face high-return projects, they might be forced to abandon or postpone investment due to high-cost of financing. As result, the investment decision will be directly restricted by the financing conditions [7]. This behavior indicates that imperfections in the capital market will make "Financing Irrelevance" emphasizing in the Fisher Model, become difficult to implement in practice.

2.2.3 External Shocks and Market Intervention

In the Fisher Model, it assumes that the operation of capital markets is independent of the external environment, and the interest rate is depending on market demands. Nevertheless, the monetary policy from the central bank (like quantitative easing and tight interest rate policies) and the government's fiscal stimulus or credit support will directly change market financing conditions. For example, firms can significant corporate financing costs and changes in investors' optimal consumption-investment choices, because the central bank will lower the benchmark interest rate and implementing a large-scale asset purchase program [8]. At this point, CML is no longer determined solely by market mechanisms, experiencing horizontal shifts or slope changes because of the policy interference. The outcome of the separation of investment and consumption is therefore directly influenced by external policies.

In summary, the combined effects of investor behavioral biases, financing constraints, and policy intervention make the real market deviates significantly from the idealized setting of the Fisher Mdel. These deviations not only destroy CML's linear structure but also weaken the effectiveness of the separation theorem. As a result, the application of Fisher Model in the real case will inevitably face limitations which provides theoretical background for subsequent case analysis in part 3.

3. Case Study: Boeing's Financing and Investment Decisions During the 2020 Crisis

3.1. Case Background: Demand Collapse and Credit Market Freeze (2020Q1–Q2)

To further test the applicability of the Fisher Model, Section 3 examines Boeing's financing and investment choices during the 2020 COVID-19 crisis as an empirical case. In the first quarter of 2020, the demand for global civil aviation travel experienced a steep drop due to the COVID-19 pandemic. IATA monthly data shows that global RPK fell by -52.9% and -94.3% year-on-year in March and April 2020, respectively shows in Figure 1 [9] [10]. The speed and depth of demand contraction are both rare in history. IATA's annual review estimates a 66% decrease in RPKs for the full year 2020 [11]. Such external shocks, combined with Boeing's own uncertainties (such as the tail effect of the 737 MAX project), quickly led to cash flow and refinancing pressures. Delayed order deliveries, prepayments, and operating cash flow from bills were squeezed, while the commercial paper and corporate bond markets showed signs of drying up liquidity and soaring spreads in mid-March (Figure 2 shows the key time points in the timeline).

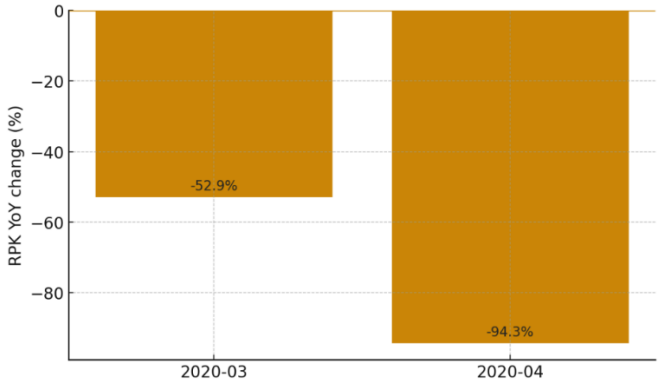

Fig. 1 Global RPK YoY in 2020 (IATA) [9] [10]

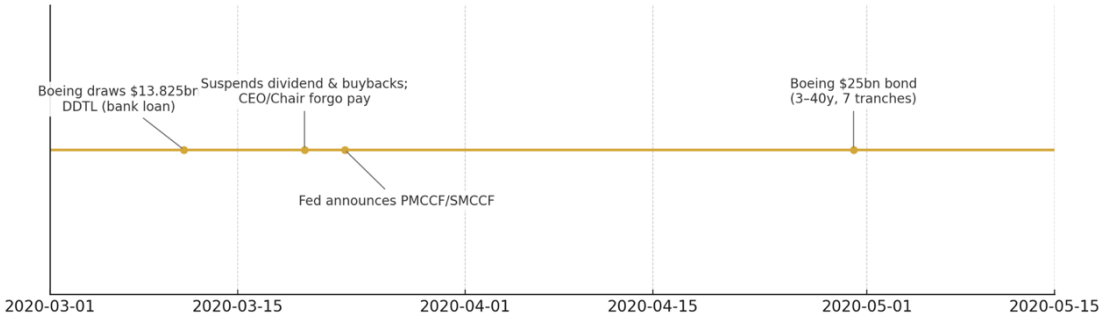

Fig. 2 2020 crisis timeline: Boeing Actions & Fed Facilities [11]

Under this background, Boeing activated emergency terms of the 'Delayed Draw Term Loan (DDTL) on March 11, 2020, a one-time drawdown of a two-year bank loan of US$13.825 billion, to quickly replenish cash positions and hedge against the uncertainty of refinancing in the capital market [12][13]. Subsequently, on March 20, the company suspended dividends and share buybacks, and management waived their full-year salary, clearly conveying its "liquidity first" approach [14] [15]. From a macro perspective, on March 23, the Federal Reserve announced the establishment of the PMCCF/SMCCF to provide liquidity support for the primary and secondary markets of corporate bonds [16] [17]. Policy signals stabilize the issuance business window. After the policy "bottoming out" came into being, Boeing completed excess bond financing of US$25 billion and seven tranches (3-40 years) on April 30 and stated that it no longer needed to rely on direct government bailouts [18] [19]. Figure 2 visualizes the above key nodes along the timeline: the causal chain is clearly visible in the time series, showing how the demand shock led to liquidity defense, which in turn triggered policy support and eventually enabled market refinancing.

3.2. Ideal Model Compare with Realistic Choice: The Failure of Investment - Consumption Separation

According to the Fisher Model, under frictionless, symmetric interest rate conditions, firms only need to make investment decisions that maximize NPV; individuals can freely borrow and lend in the capital market along the linear CML to complete intertemporal consumption allocation, completely separating the two. However, in this case, three types of real-world frictions jointly drive deviations.

3.2.1 Financing Constraints Cause a "Breaking Point" in CML

In March, the credit market stress led to an increase in borrowing rates and a temporary closure of the issuance window, but the marginal return of "lending-depositing" did not rise at the same rate, so the CML split into two segments with different slopes (the slope of the borrowing segment is $-1-r_{borrow}$ and the slope of the lending segment is $-1-r_{lend}$, and $r_{borrow} > r_{lend}$). This undermines the prerequisite of "being able to make inter-period exchanges at a uniform price at any investment scale". Enterprises cannot choose the investment scale based solely on NPV but must include "financing at an acceptable cost" as a constraint. Boeing's full use of bank credit ($13.825 billion) can

be seen as a liquidity defense of "trading time for space"[12] [13]. As shown in Figure 3, under the asymmetric borrowing/lending interest rates, the CML has a kink point, which cannot guarantee the unique tangent point with the conversion curve, thereby destroying the "separation" of the Fisher model (which is consistent with the market stress and policy support scenario in Figure 1) [16] [17].

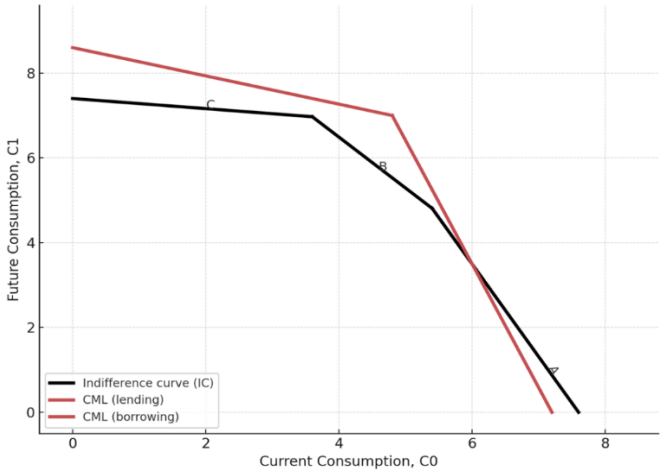

Fig 3. Non-linear CML under borrowing-lending rate asymmetry

3.2.2 Cash Conservatism and Project Delays

Under uncertainty and contractual restrictions (debt covenants, rating thresholds), companies tend to "preserve cash first and discuss investment later." After the policy stabilized, the company chose to conduct long-term debt financing ($25 billion) under a bottoming environment, giving priority to repairing cash reserves and maturity structure rather than immediately expanding capital expenditures; at the same time, it suspended dividends/buybacks to reduce external financing pressure and the "rigid expenditure" of shareholder distributions. These choices are significantly different from the conclusion of the Fisher model that "invest immediately as long as there is a positive NPV project"[16].

3.2.3 Policy Intervention Changes the Feasible Set

The announcement of the PMCCF/SMCCF exogenously improved financing availability and expanded the feasible opportunity set by reducing credit spreads and resuming market making and subscription. This not only changed the "slope" (price) but also moved the "boundary" (opportunity set) outward, making points that were originally inaccessible due to financing restrictions accessible (Figure 4) [17]. This is structurally different from the premise of the Fisher model that "the market is self-sufficient and coordinated only by price signals."

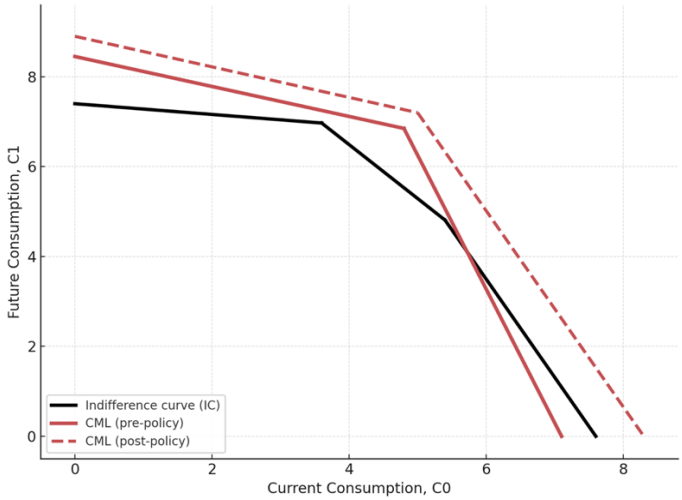

Fig 4. Policy backstop: Outward Shift & Flatter Effective CML

3.3. Mechanism Mapping and Testable Implications

To match the theoretical deviation in Part 2 with the implementation of this case, three testable implications are proposed:

H1 (financing constraints): During the crisis, the borrowing-lending spread widened, and the issuance of corporate bonds plummeted, which coincided with the increase in corporate cash positions - Boeing fully withdrew DDTL ($13.825 billion) on March 11, 2020, and completed the placement of long-term bonds worth $25 billion on April 30, 2020, after policy support.

H2 (interaction between preferences and policies): After the announcement of PMCCF/SMCCF on March 23, the financing spread fell and the issuance window reopened, and companies shifted from "short-term cash defense" to "maturity structure reconstruction" (replacing short-term bank credit with long-term bonds).

H3 (Investment Timeline): When demand has not yet recovered (RPK is still deeply negative), companies prioritize capital expenditure and capacity expansion after “cash and credit repair” (Figure 2 is demand-side evidence; used in conjunction with Figure 1 on key financing events).

4. Theoretical revision and expansion

4.1. The “Segmented CML” Framework under Financing Constraints

Question: When the capital market is imperfect and credit rationing exists, the borrowing rate > lending rate, and the CML is no longer a straight line with a single slope. At the same time, there is a maximum credit limit, and the CML stops extending after a certain point. This means that the separation of investment and consumption is no longer automatically true, and the optimal solution may appear at the inflection point or boundary, rather than the traditional "single tangent point of the linear CML and the conversion curve." See the figure (the borrowing slope $-1-\mathrm{r}_{\mathrm{borrow}}$ is steeper, the lending slope $-1-\mathrm{r}_{\mathrm{lend}}$ is flatter; $\mathrm{r}_{\mathrm{borrow}} > \mathrm{r}_{\mathrm{lend}}$; there is a credit ceiling).

Modification 1: Piecewise CML.

Represent the CML as two or more linear segments:

$$\text{Lending segment: } \mathrm{C}_1 = \mathrm{C}_1^{\mathrm{M}} - (1+\mathrm{r}_{\mathrm{lend}})(\mathrm{C}_0 - \mathrm{C}_0^{\mathrm{M}}) \quad (1)$$

$$\text{Borrowing segment: } \mathrm{C}_1 = \mathrm{C}_1^{\mathrm{M}} - (1+\mathrm{r}_{\mathrm{borrow}})(\mathrm{C}_0 - \mathrm{C}_0^{\mathrm{M}}) \quad (2)$$

and impose a credit limit L of $\mathrm{C}_0 \le \mathrm{C}_0^{\mathrm{M}} + \mathrm{L}$.

The decision rule is changed to: If the investor's optimal consumption point lies within the lending segment, the firm's investment can still be approximately determined by maximizing NPV; if the optimal point falls at the breakpoint/borrowing segment/cap boundary, the investment scale and financing conditions are intrinsically coupled, and the separation no longer holds (echoing the "failure" in 3.2). Furthermore, as the borrowing scale increases (or lending scale increases), $\mathrm{r}_{\mathrm{borrow}}$ increases and $\mathrm{r}_{\mathrm{lend}}$ decreases, and the "average slope" of the effective CML decreases. A piecewise or smooth function can be used.

4.2. Preference Heterogeneity and the Condition of “Approximate Separation”

In the presence of non-concave preferences or behavioral biases (such as loss aversion, status quo bias, and overconfidence), even with minimal market friction, the unique tangent point may be destroyed due to convex segments or kinks in the indifference curve. To restore a certain degree of separation at a practical level, three conditions must be satisfied. First, the consumption optimum must lie within the lending segment, meaning that it does not trigger the credit limit or enter the borrowing segment. Second, the slope of the local indifference curve should remain monotonically decreasing and tangent to the slope of the lending segment. Finally, the investment opportunity set must be strictly concave within this neighborhood. If any of these conditions fail (e.g., preferences shift to "cash conservatism" during a crisis), the situation degenerates into the "segment-boundary solution" described in 4.1 (see 3.2 for empirical evidence and mechanistic discussion).

4.3. The “boundary shift + slope change” effect of policy intervention

Policy tools (such as PMCCF/SMCCF) are modeled as joint perturbations to the feasible set and prices:

Policy intervention simultaneously expands the feasible set—by restoring market making, subscription, and reducing credit spreads so that previously inaccessible consumption bundles such as (C_0, C_1) become attainable—and flattens the effective Capital Market Line, as the borrowing rate decreases while the lending rate remains stable.

Geometrically, Figure 4 can be directly reused: The overall post-policy boundary shifts outward, and the "effective CML (post)" is flatter than the "pre" one, shifting firms from "short-term cash defense" to "reshaping the term structure." Therefore, policy not only changes "prices" but also "reachable domain," which is the key bridging mechanism for Boeing's transition from fully withdrawn DDTL to excess long-term debt issuance in 3.1–3.3.

4.4. Proposition and Applicable Boundaries

Proposition 1 (Necessary Conditions for Separation): If (a) the consumption optimum lies within the lending segment, and (b) the firm's chosen investment scale does not trigger the credit limit, then the Fisher-style "investment-consumption separation" approximately holds. Otherwise, investment choices significantly depend on the financing function $r_{\text{borrow}}(x)$, , and the upper limit L, and the separation fails (see the inflection point and upper limit in Figure 3).

Proposition 2 (Nonlinearity of Policy Effectiveness): If the firm is constrained by the frontier/inflection point, a small policy improvement ($\Delta r_{\text{borrow}} < 0,\ \Delta L > 0$, or a shift outside the feasible frontier) will produce a nonlinear jump in investment response (from the frontier to the interior solution), with a multiplier effect greater than the marginal response at the interior point (see Figure 4).

Proposition 3 (Case Consistency): When demand is still experiencing deep negative growth, the timing of firms' cash withdrawals followed by investment can be explained by the "segmented CML + ceiling + policy shift" model: first, defensive withdrawals are taken under the ceiling to maintain cash (3/11), then, after policy shift, they turn to long-term bond issuance to reshape the maturity structure (4/30), and then capital expenditures are resumed at an appropriate time (consistent with H1–H3 in 3.3).

5. Conclusion

Starting from the Fisher separation theorem, this paper identifies and systematically characterizes the destructive effects of financing constraints, interest rate asymmetry, credit limits, preference heterogeneity, and exogenous policy shocks on the standard optimal solution of "linear CML + unique tangent point" in real capital markets. Using a geometric framework of piecewise CML + credit limits, we demonstrate that when the optimal solution falls at an inflection point or boundary, firms' investment choices are inevitably endogenously coupled with financing conditions, making the investment-consumption separation mechanistically untenable.

The Boeing case study shows that under the combined influence of a deep contraction in demand and credit market stress, businesses rationally prioritized liquidity over investment, navigating the most intense phase of uncertainty through defensive withdrawals and restructuring of the maturity structure. Policy-based credit instruments, however, not only operated through price channels (slope changes) but also shifted the reachable domain by shifting the opportunity frontier outward, thereby triggering a transitional response from a boundary solution to an internal one. This approach provides a clear analytical framework for understanding the financing-investment linkage of businesses during periods of uncertainty and how policies produce nonlinear effects under boundary conditions.

At the research level, this article uses a more intuitive graphical framework to connect the traditional Fisher model with real-world corporate decision-making for comparative analysis. At the policy level, this framework reminds policymakers that in crisis management, while relying solely on

"price tools" such as interest rates and interest rate spreads, they must also employ "quantity tools" such as credit, market making, and quotas. These two tools, when combined, often manifest at key locations such as "boundaries/inflection points." Future research could proceed along two avenues: first, incorporating firm heterogeneity (asset-liability structure, contractual constraints, and rating trajectories) to identify entities more prone to borderline states; second, employing an "event window + cross-section" identification design to estimate transition probabilities and corresponding investment elasticities from the internal solution of the frontier regression, and to compare the relative effectiveness of different policy tools.

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