A Comparative Analysis of The Markowitz Model and The Single-Index Model: Mathematical Efficiency Vs. Practical Simplicity

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Abstract. With the increasing interest of emerging economies in the stock market, single asset investment is also shifting towards diversified portfolio strategies, and the trade-off between Markowizt model and index model in terms of evaluation correctness and practical effectiveness is also arising. Based on the historical daily total return data of 21 OEX member companies' stocks over the past 20 years, as well as a S&P 500 stock index and risk-free rate (one month federal funds rate), this article uses a structured data-driven approach to analyze and compare the difficulty of data and computation between the two models, as well as to compare the accuracy of capturing the trade-off between risk and return. Research has found that an increase in the number of assets will bring significant computational pressure to the Markowitz model, and the calculation speed of single indicator models will be faster and more accurate; The Markowitz model framework theoretically captures correlations between assets more accurately, but also faces the phenomenon of high risk and high return, while the single index model tangent investment portfolio has a milder balance of expected risk and return at the cost of slightly lower expected returns.

Keywords: Efficient frontier; Permissible portfolios; Correlation matrix.

1. Introduction

Since the 1980s, there has been growing scholarly and practical interest in stock markets within emerging economies, driven by their potential for high returns alongside significant risks [1]. Events such as the technology bubble in 2000 and the global financial crisis in 2008 have further underscored the importance of robust investment risk management and return optimization [1]. This evolution has also marked a shift from single-asset investments to diversified portfolio strategies [2].

Portfolio theory, which examines how investors allocate capital across various assets to optimize the risk-return trade-off under uncertainty, has become a cornerstone of modern financial research [3]. Introduced by Harry Markowitz in 1952, the mean-variance framework laid the foundation for modern portfolio theory by employing mathematical optimization to balance risk and return [1,4]. While the Markowitz model offers a rigorous approach to constructing efficient portfolios, its reliance on estimating large covariance matrices imposes substantial computational burdens as the number of assets increases. To address these limitations, William Sharpe proposed the single-index model in 1963, which simplifies covariance estimation by using market factor approximations, thereby enhancing practical applicability [3]. Subsequent studies have compared these two models under various market conditions. For instance, a 1989 study by Seler applied both models to the Istanbul Stock Exchange from 1986 to 1987 [2]. Yet, due to contextual differences and evolving financial environments, a consensus regarding their relative effectiveness remains elusive.

This paper examines the trade-off between the mathematical efficiency of the Markowitz model and the operational simplicity of the single-index model. Through mathematical analysis of risk-return relationships, comparative evaluation of covariance-based and index-based estimation methods, and a statistical discussion of estimation errors, this study aims to provide investors and researchers with a clearer framework for selecting the appropriate model based on specific constraints and objectives. It highlights the inherent tensions between theoretical accuracy (favored by the Markowitz approach) and practical scalability (offered by index models).

For scholars, this analysis clarifies theoretical aspects of portfolio construction. For practitioners, it offers guidance on balancing accuracy with implementability. Internationally, prior research has extensively examined the efficiency of Markowitz optimization, often highlighting issues related to estimation errors-some of which may be mitigated through structured covariance estimation methods [5,6]. Recent studies have incorporated multi-factor extensions, machine learning techniques, and ESG (environmental, social, and governance) constraints to enhance traditional models [7,8]. This paper compares the practicality and simplicity of two models with the accuracy of capturing the trade-off between risk and return to determine which model is more suitable in which situation. Also, this paper contributes to this ongoing discourse by systematically comparing both models and outlining context-dependent recommendations.

2. Methdology

2.1. The Basic Model: Markowitz Model & Single-Index Model

2.1.1 The Markowitz Model

The Markowitz mean variance model (1952) provided the foundational framework for modern portfolio theory [1]. Its premise is to assume that investors are risk averse and construct an optimal asset portfolio by balancing risk and return [9]. Balancing risk and return are about maximizing expected return at a given level of risk or minimizing risk at a given level of expected return. At there, the expected return is the average return and risk is measured by the variance (or standard deviation) of portfolio returns [10]. Mathematically, Markowitz optimization requires estimating the expected return of each asset, as well as the complete covariance matrix of asset returns: $n(n-1)/2$ covariances. In addition, portfolio risk will be represented as a quadratic function of asset weights and their covariance.

The expected return theory formula for a portfolio is:

$$E(R_p) = \sum w_i E(R_i) \tag{1}$$

In formula (1), the w_i is the weight of asset i in the portfolio, $E(R_p)$ is the expected return of the portfolio, and $E(R_i)$ is the expected return of asset i.

The variance (risk) theory formula for a portfolio is:

$$\sigma_p^2 = \sum\sum w_i w_j \sigma_{ij} \tag{2}$$

In formula (2), the w_i is the weight of asset i in the portfolio, the w_j is the weight of asset j in the portfolio, σ_{ij} is the covariance between returns of assets i and j, σ_p^2 is the variance of the portfolio.

$$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j \tag{3}$$

In formula (3), ρ_{ij} is the correlation between asset i and j. σ_i and σ_j are standard deviation of assets i and j.

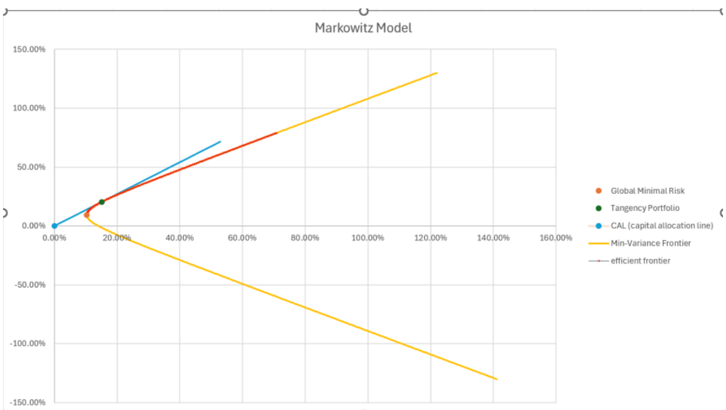

Fig.1 Important elements of Markowitz Model for portfolio investment

In Fig.1, the Minimum - Variance Portfolio is the portfolio of risky assets with the lowest possible risk among all feasible portfolios [9]. It is the leftmost point of the parabola-shaped feasible set. After that, the set of all portfolios that provide the maximum expected return for a given level of risk is called efficient frontier [9]. Graphically, it is a curve that lies above the global minimum-variance portfolio. The capital allocation line (CAL) is a straight line that represents all possible risk-return combinations available to an investor by combining a risk-free asset with a risky portfolio [9]. Moreover, the intersection point between the Efficient Frontier and CAL is called tangency portfolio, which represents the optimal mix of risky assets when combined with a risk-free asset [9].

2.1.2 The Single-Index Model

The single index model simplifies portfolio construction by simplifying the estimation of covariance matrix problems, decomposing risk into systematic and firm-specific components, and assuming that the common motion between individual securities is driven by their linear relationship with the common market index [11]. The single index model uses market index to replace common factor and variance covariance matrix can be decomposed into systematic risk (beta coefficient) and firm-specific risk (residual variance) [11].

The single-index model can be written as the following regresssion equation:

$$Ri = \alpha i + \beta i\ Rm + \epsilon i \tag{4}$$

In formula (4), the αi is the intercept, which represents the security's expected excess returns when market excess return is zero. βi refers to the amount by which the return on a security increase or decreases for every 1% increase or decrease in the index return [11]. ϵi is the firm-specific surprise, also called residual.

The expected return formula of the single-index model is:

$$E(Ri) = \alpha i + \beta i\ E(Rm) \tag{5}$$

In formula (5), the αi is the non-market risk premium and E(Rm) is the market risk premium.

The variance of an asset's return is:

$$Var\ (Ri) = \beta i\ \text{^}2\ \sigma\text{^}2m + \sigma\text{^}\ \epsilon i \tag{6}$$

In formula (6), σ^2m is the variance of the market index return and σ^ ϵi is the variance of the residual term.

2.2. Methods for Making Comparison

The comparative analysis between the Markowitz model and the single-index model in this paper will be conducted through a structured data-driven approach. This study selected the historical daily total return data of 21 OEX member companies' stocks over the past 20 years, as well as one (S&P 500 stock index) equity index and risk-free interest rate (one-month federal funds rate). OEX refers S&P 100 Index, which is a stock market tracks 100 major, large-cap U.S. companies [12].

Some researchers believe that small samples cannot capture diverse traces, and a small sample size can bias the results towards firm-specific risk rather than systematic risk [11]. This can result in the efficient frontier being too loose when generating visualizations. However, an excessive number of stocks can lead to difficulties in covariance calculation and comparison due to excessive numerical values. Based on the above considerations, this study selected 21 stocks and one stock index (S&P 500 index) as the data.

These 21 stocks belong to 5 different industry sectors, which are communications, cyclical consumer, non-cyclical consumer, technology and financial respectively. In Table 1, there are the ticker symbols of the 22 risky-assets.

Table 1. Ticker symbols of 21 stocks and one (S&P 500 stock index) equity index

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
SPX	CMCSA	DIS	CSCO	T	VZ	MCD	SBUX	TGT	WMT	GILD	CL	PG	MDLZ	CVS	JNJ	LLY	MRK	PFE	MSFT	ORCL	BLK

In order to reduce non-Gaussian effects, this study summarizes daily stock data as monthly observations. From the weekly sequence, this article will calculate the average return of each stock. Based on monthly observations of these stocks, the first step will be to calculate the return estimates and risk estimates under Markowitz model and single-index model. After that, the investment portfolio will be constructed and optimized in Excel, and the visualizations of models will be generated. In order to present the precise numerical form of the allowed investment portfolio area, this study will use the following constraints when running the solvertable: minimum risk or variance boundary (curve): investment portfolio return range: -130% to 130%, step size of 0.5%; effective boundary (curve), investment portfolio standard deviation range: 8% to 79%, step size of 0.5%; when run the solver table, use GRG Nonlinear. Moreover, when use the solver and run the solvertable in excel, each of these optimization problems MM and IM, this study uses the optimization constraint: $\sum|wi| <= 2$, which is based on the regulation of the US FINRA, account holders are allowed to purchase stocks with up to 50% margin - that is, people can hold positions up to twice their principal, simulating the leverage or short selling restrictions imposed by securities firms.

2.3. Evalution Criteria

Practical simplicity is one criterion for comparing two models in this study: the optimization method of this model and the difficulty level of constructing the investment portfolio in implementation, as well as the requirements for data processing and calculation. Secondly, the accuracy of capturing the trade-off between risk and return is another criterion for comparison in this study. In the case of the same sample, the magnitude of the impact of estimation errors on the two models will be presented by comparing the composition of the investment portfolio and the effective frontier.

3. Results

3.1. Interpretation the Comparison Results

3.1.1 Data Processing and Computational Difficulty of Each Model

Firstly, and foremost, the data processing and computational difficulty of the two models during the optimization process will be compared. Since the Markowitz model requires the use of the complete correlation (or covariance) matrix of all assets to calculate risk, it requires n (n-1)/2 correlations for standard deviation (or n (n-1)/2 covariance estimates for n assets). The total estimates are approximately equal to (n^2 + n)/2, which grows quadratically with n.

Table 2. Correlation matrix of the 22 risky assets in Markowitz model

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
		SPX	**CMCSA**	**DIS**	**CSCO**	**T**	**VZ**	**MCD**	**SBUX**	**TGT**	**WMT**	**GILD**	**CL**	**PG**	**MDLZ**	**CVS**	**JNJ**	**LLY**	**MRK**	**PFE**	**MSFT**	**ORCL**	**BLK**
0	**SPX**	100%	63%	71%	64%	46%	42%	53%	55%	53%	35%	31%	49%	45%	52%	48%	58%	34%	38%	49%	65%	66%	75%
1	**CMCSA**	63%	100%	55%	42%	43%	43%	32%	39%	31%	22%	21%	33%	35%	42%	33%	41%	12%	17%	32%	31%	43%	46%
2	**DIS**	71%	55%	100%	41%	34%	32%	40%	53%	45%	20%	12%	33%	28%	41%	37%	37%	15%	23%	23%	40%	44%	50%
3	**CSCO**	64%	42%	41%	100%	28%	24%	26%	44%	34%	24%	27%	28%	33%	36%	37%	34%	21%	22%	33%	45%	53%	50%
4	**T**	46%	43%	34%	28%	100%	65%	36%	25%	22%	19%	26%	36%	37%	31%	28%	32%	16%	32%	29%	26%	33%	37%
5	**VZ**	42%	43%	32%	24%	65%	100%	35%	25%	18%	26%	16%	34%	37%	28%	30%	37%	19%	35%	31%	35%	28%	35%
6	**MCD**	53%	32%	40%	26%	36%	35%	100%	35%	27%	26%	14%	46%	47%	49%	22%	49%	16%	32%	33%	38%	34%	40%
7	**SBUX**	55%	39%	53%	44%	25%	25%	35%	100%	40%	15%	15%	29%	25%	31%	27%	30%	14%	22%	24%	34%	36%	44%
8	**TGT**	53%	31%	45%	34%	22%	18%	27%	40%	100%	40%	7%	18%	22%	24%	36%	25%	11%	19%	21%	29%	33%	38%
9	**WMT**	35%	22%	20%	24%	19%	26%	26%	15%	40%	100%	9%	32%	31%	22%	20%	38%	18%	22%	21%	23%	27%	29%
10	**GILD**	31%	21%	12%	27%	26%	16%	14%	15%	7%	9%	100%	20%	18%	21%	31%	25%	22%	26%	31%	14%	27%	34%
11	**CL**	49%	33%	33%	28%	36%	34%	46%	29%	18%	32%	20%	100%	59%	53%	27%	57%	30%	36%	32%	28%	28%	47%
12	**PG**	45%	35%	28%	33%	37%	37%	47%	25%	22%	31%	18%	59%	100%	51%	23%	51%	27%	33%	36%	29%	23%	35%

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
		SPX	**CMCSA**	**DIS**	**CSCO**	**T**	**VZ**	**MCD**	**SBUX**	**TGT**	**WMT**	**GILD**	**CL**	**PG**	**MDLZ**	**CVS**	**JNJ**	**LLY**	**MRK**	**PFE**	**MSFT**	**ORCL**	**BLK**
13	**MDLZ**	52%	42%	41%	36%	31%	28%	49%	31%	24%	22%	21%	53%	51%	100%	31%	56%	23%	30%	37%	26%	32%	37%
14	**CVS**	48%	33%	37%	37%	28%	30%	22%	27%	36%	20%	31%	27%	23%	31%	100%	33%	16%	23%	32%	25%	24%	40%
15	**JNJ**	58%	41%	37%	34%	32%	37%	49%	30%	25%	38%	25%	57%	51%	56%	33%	100%	44%	42%	50%	33%	37%	43%
16	**LLY**	34%	12%	15%	21%	16%	19%	16%	14%	11%	18%	22%	30%	27%	23%	16%	44%	100%	44%	34%	23%	21%	24%
17	**MRK**	38%	17%	23%	22%	32%	35%	32%	22%	19%	22%	26%	36%	33%	30%	23%	42%	44%	100%	36%	25%	28%	23%
18	**PFE**	49%	32%	23%	33%	29%	31%	33%	24%	21%	21%	31%	32%	36%	37%	32%	50%	34%	36%	100%	23%	27%	43%
19	**MSFT**	65%	31%	40%	45%	26%	35%	38%	34%	29%	23%	14%	28%	29%	26%	25%	33%	23%	25%	23%	100%	51%	53%
20	**ORCL**	66%	43%	44%	53%	33%	28%	34%	36%	33%	27%	27%	28%	23%	32%	24%	37%	21%	28%	27%	51%	100%	56%
21	**BLK**	75%	46%	50%	50%	37%	35%	40%	44%	38%	29%	34%	47%	35%	37%	40%	43%	24%	23%	43%	53%	56%	100%

In Table 2, there are 22 risky assets and the full correlation matrix has 22^2 entries. Since the matrix is symmetric (σij = σji) and the diagonal terms (σii) are the standard deviations (100% per set, and 22 in total), the number of correlation estimates for Markowitz model to use is 22(22-1)/2 = 231. Also, the total estimates, which is 22 + 22 + 231 = 275, is equal to the sum of number of expected returns, variances and correlations.

Table 3. Inputs table for Single-index model

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	SPX	**CMCSA**	**DIS**	**CSCO**	**T**	**VZ**	**MCD**	**SBUX**	**TGT**	**WMT**	**GILD**	**CL**	**PG**	**MDLZ**	**CVS**	**JNJ**	**LLY**	**MRK**	**PFE**	**MSFT**	**ORCL**	**BLK**
Annualized Average Return	9.54%	10.13%	9.59%	8.49%	6.12%	5.86%	14.32%	13.90%	10.41%	9.40%	14.00%	8.54%	7.96%	8.72%	7.97%	7.40%	17.10%	9.49%	4.49%	17.04%	15.70%	17.41%
Annualized StDev	14.95%	24.07%	25.44%	25.30%	18.72%	17.71%	16.09%	27.10%	27.40%	17.09%	24.56%	15.42%	15.44%	17.80%	23.10%	14.71%	22.67%	22.22%	21.36%	22.50%	24.24%	28.53%
Beta	100.00%	101.72%	120.24%	108.26%	57.99%	49.27%	57.55%	100.55%	97.10%	40.30%	50.56%	50.25%	46.49%	61.56%	73.61%	57.00%	52.17%	56.07%	69.88%	97.99%	107.37%	142.80%
Annualized Alpha	0.00%	0.43%	-1.87%	-1.83%	0.59%	1.17%	8.83%	4.31%	1.15%	5.55%	9.18%	3.75%	3.53%	2.85%	0.95%	1.97%	12.13%	4.15%	-2.18%	7.70%	5.46%	3.79%
Residual StDev	0.00%	18.65%	18.00%	19.44%	16.59%	16.10%	13.60%	22.55%	23.23%	15.99%	23.37%	13.46%	13.79%	15.24%	20.30%	11.99%	21.28%	20.57%	18.63%	17.08%	18.16%	18.92%

In Table 3, for each asset among the 22 risky assets, there is one αi (the intercept or non-market component), one βi (slope and sensitivity to market) and σ^2ϵi (the variance of residual risk or the square of the standard deviation of residual risk). Additionally, for market index, there are two more inputs: the expected market return E(Rm) and variance of market return σ^2m. Thus, for single-index model, there are 3*22 + 2 = 68 estimated in total.

By comparison, for 22 risky assets, Markowitz model needs 275 estimates and Single-index model needs 68 estimates. As the number of assets increases, the calculation speed of single-index models will be faster and more accurate, while the computational pressure of Markowitz models will greatly increase. Due to the reliance of the Markowitz model on the correlation model of all assets, it is more sensitive to the input data. The expected return and correlation matrix of the Markowitz model input are predicted from past data, and any small erroneous guess or data change will amplify the errors in the model.

3.1.2 The Accuracy of Capturing the Trade-off between Risk and Return

The precision in reflecting the balance between risk and return is another essential criterion for Markowitz and single-index model comparison in this study. In the case of the identical sample, the impact of estimation error on the two models will be displayed by comparing the composition and effective frontier of the investment portfolio. Firstly, it is the comparison of the return and risk estimates generated for the selected sample of 22 risky assets, which include 21 OEX constituent stocks and one S&P 500 index.

Table 4. The return estimate and risk estimate of 22 risky stocks under Markowitz model

	minvar	**maxsharpe**
SPX	33.260%	-11.672%
CMCSA	-4.821%	4.101%
DIS	-3.538%	-3.909%
CSCO	-2.734%	-6.617%
T	4.530%	-0.508%
VZ	10.906%	-1.876%
MCD	11.119%	46.732%
SBUX	2.584%	7.444%
TGT	-0.222%	-0.016%
WMT	18.181%	18.641%
GILD	10.077%	17.534%
CL	15.530%	0.022%
PG	8.636%	0.142%
MDLZ	1.948%	0.100%
CVS	4.103%	1.313%
JNJ	4.352%	-7.014%
LLY	5.283%	28.947%
MRK	-5.339%	-3.728%
PFE	1.719%	-14.444%
MSFT	2.972%	18.537%
ORCL	1.130%	6.326%
BLK	-19.677%	-0.055%
Return	8.639%	17.852%
StDev	10.125%	13.409%
Sharpe	85.3%	133.131%

Table 5. The return estimate and risk estimate of 22 risky stocks under Index model

	minvar	maxsharpe
SPX	-2.438%	-37.653%
CMCSA	-2.928%	-0.085%
DIS	-7.726%	-5.507%
CSCO	-2.919%	-3.968%
T	8.934%	0.168%
VZ	11.846%	2.010%
MCD	13.349%	32.056%
SBUX	-1.160%	3.192%
TGT	-0.264%	-0.036%
WMT	14.504%	15.313%
GILD	5.458%	11.589%
CL	16.512%	13.136%
PG	17.208%	12.100%
MDLZ	9.528%	6.379%
CVS	3.342%	0.107%
JNJ	17.563%	6.513%
LLY	6.329%	18.655%
MRK	2.725%	6.040%
PFE	4.637%	-2.650%
MSFT	-1.339%	14.146%
ORCL	-3.199%	7.299%
BLK	-9.961%	1.198%
Return	7.833%	14.295%
StDev	8.105%	10.210%
Sharpe	96.644%	140.012%

Based on Table 4 and 5, the investment portfolio generated using the Markowitz model framework has slightly higher expected returns and risks compared to single-index models under the constraint: $\sum|wi| <= 2$. The expected return and risk of minimum-variance portfolio of two models are 0.086 vs. 0.078 and 0.101 and 0.081 respectively. However, the single-index model provides higher sharpe ratios, which mean that the investment portfolio receives more excess returns per unit of risk, with 0.966 and 1.14 respectively.

In this case, the investment portfolio generated by the Markowitz model has higher expected returns, but higher returns come with higher risks. Due to the faster growth of risk than return, the sharpe ratio has decreased. For the single index model, it needs to bear less risk. Even if the total return is low, the higher sharpe ratio of the model indicates that the single index model can provide better risk adjustment performance by achieving a higher unit risk return rate in this case.

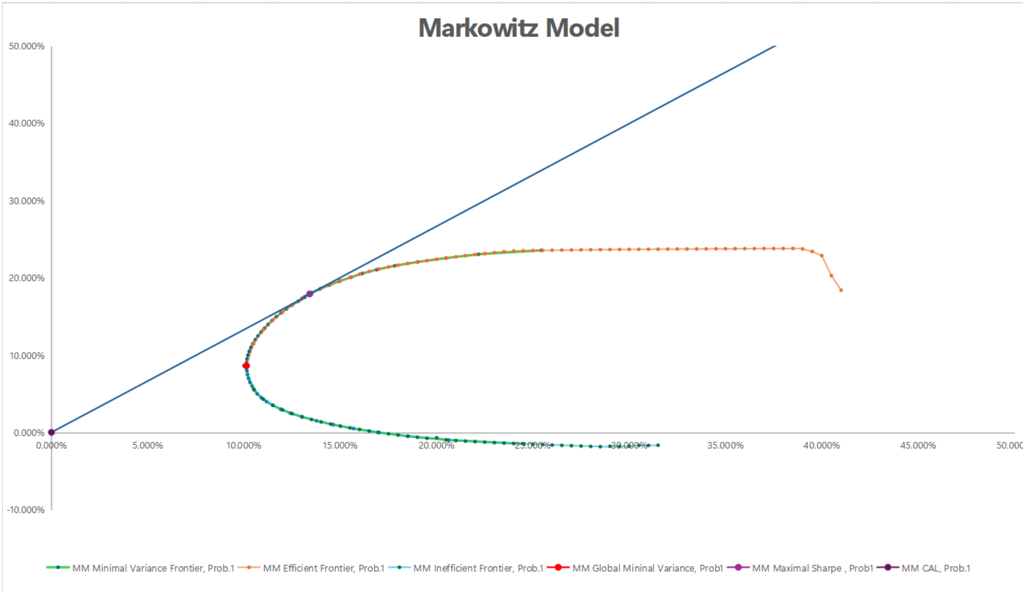

Fig. 2 Permissible portfolios of Markowitz model under constraint: ∑|wi| <= 2

Fig. 3 Permissible portfolios of Index model under constraint: ∑|wi| <= 2

Furthermore, the accuracy of capturing the trade-off between risk and return can be illustrated in the visualizations of permissible portfolios. Based on figure 2 and 3, the efficient frontier obtained from the single index model is smoother, while the efficient frontier of the Markowitz model has a clear turn at approximately 40% standard deviation. This indicates that the single index model is less sensitive to covariance estimation errors. In the visualizations above, comparing the tangency portfolio and the minimal-variance portfolio, it can be observed that the Markowitz efficient frontier is located above the single-index efficient frontier. This indicates that under the same level of risk, the Markowitz model is more efficient in achieving higher expected returns.

The figure 2 and 3 show that the single index model generates a simplified efficient frontier based on system and company specific risks: its effecient frontier is smaller than that of the Markowitz model. The effecient frontier of the Markowitz model extends up to approximately 42% of the standard deviation, while the effecient frontier of the single index model is approximately 40%. This indicates that the diversification benefits are actually approximate, rather than being fully captured.

In this case, when examining the tangent investment portfolios under both models, Markowitz generates higher expected returns but also greater variance. In contrast, the single index model tangent investment portfolio provides a milder balance of risk and return, with lower volatility than Markowitz model, but at the cost of slightly lower expected returns.

4. Discussion

4.1. Discussion of Estimation Errors and Simplification Benefits

According to the results, although the Markowitz model framework theoretically captures covariance and correlations between assets more accurately, the number of estimates required also increases significantly as the asset range expands. This heavy reliance on the correlation matrix introduces a significant amount of estimation error, as even small inaccuracies in the correlation can lead to significant distortion of the effective frontier. In contrast, the single index model reduces estimation errors by replacing pairwise correlations with market beta coefficients, although at the cost of ignoring non market correlations between assets.

4.2. Limitations of the Research

This research method has limitations. Firstly, all the research in this article was conducted under the constraint: $\sum|wi| <= 2$. However, the estimated values and visualizations generated by the two models will be different under different constraints (e.g., the weight of the first asset, usually the market index must be 0 if investment in the market index is not allowed). Similarly, when ESG (environmental, social, and governance) constraints are applied, the feasible investment scope will narrow, thereby changing the effective frontier, leading to different estimates and visualizations of the Markowitz model and the single index model. Therefore, different constraints can affect the comparability of the model and limit the generalizability of research results to unconstrained optimization problems.

Moreover, this study relies on a fixed sample size of 22 risk assets (21 OEX component stocks and one stock index). Although this choice ensures the feasibility of calculations and clear comparisons between models, it cannot capture the broader dynamics of a more diversified asset portfolio. In practice, portfolio managers typically consider hundreds or thousands of securities, among which estimation errors, computational difficulties, and different constraints may interact in different ways. Expanding the sample or using alternative asset sets for robustness testing can provide stronger validation for conclusions.

4.3. Discussion and Suggestion for Portfolio Managers, Researcher and Investors

For professional portfolio managers, practical considerations such as data availability, time constraints, and calculations are crucial. The single index model provides a practical solution that enables managers to generate reasonable approximations of effective boundaries with less data demand and faster computation speed. However, if based on a theoretical benchmark that understands the precise properties of diversity, covariance structure, and risk reward trade-off, the Markowitz model has advantages. For investors weighing between two models, Markowitz aims to maximize absolute returns, while a single index model can achieve effective risk adjusted returns. From an academic perspective, the Markowitz model has a richer covariance structure, which makes it highly valuable in exploring portfolio theory, developing new optimization techniques and new model theories.

For institutional investors who achieve optimal performance in theory, have sufficient funds, strong risk tolerance, and possess powerful computing capabilities, such as large fund companies or asset management institutions, they are suitable for using the Markowitz model because they can better absorb the costs of computation and estimation, and they also require a large amount of high-quality data as support. In contrast, individual investors lack the resources to calculate and maintain the complete covariance structure of their investment portfolio. Therefore, this research suggests that a single index model is more practical for individual investors, providing a simple diversification method and considering company specific risks.

5. Conclusion

This study compared the Markowitz model and the single index model from the perspectives of computational complexity, accuracy of estimated numerical pairs, and portfolio visualization. The research results indicate that there is a clear trade-off between mathematical efficiency and practical simplicity of these two models.

First and foremost, in terms of data and computational requirements, research has shown that the Markowitz model requires a complete correlation or covariance matrix, and as the asset range expands, the Markowitz model brings higher computational costs and becomes increasingly sensitive to estimation errors. In contrast, the single index model achieves significant simplification, improves robustness, and reduces estimation requirements by replacing pairwise correlations with market beta values.

In addition, regarding the trade-off between risk and return, research has shown that the Markowitz model generates investment portfolios with higher expected returns but also higher variances (risk). Although the single index model generated slightly lower returns, it achieved a higher Sharpe ratio in the sample, indicating better performance after risk adjustment. The visualization of effective boundaries further indicates that the Markowitz model captures more diverse opportunities, but is also more sensitive to estimation errors, while the single exponential model provides smoother and more stable effective boundaries.

Therefore, the Markowitz model performs well in theoretical accuracy, capturing the complete covariance structure and maximizing potential returns for large-scale, resource-rich institutional investors. However, its dependence on a large number of inputs makes it computationally intensive and prone to error amplification. In contrast, the single index model provides practical simplicity, faster computation, and more robust approximations, making it suitable for individual investors or situations with limited data and computing power.

Future research may expand the sample size beyond 22 risky assets to test whether these findings are applicable to more diversified investment portfolios. Incorporating additional constraints, such as ESG factors or short selling restrictions, may further clarify the relative advantages of each model in the real investment environment. Moreover, combining Markowitz's covariance accuracy with the robustness and simplicity of index-based methods is also beneficial for future in-depth research on this topic.

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