The Impact of Liquidity Constraints on Risk-Return Characteristics of Investment Portfolios

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Abstract. In the financial market, the investment portfolio is constructed in various ways, with different components, and under some constraint factors. The Markowitz model, invented by Harry Markowitz, as well as the modern portfolio theory, contributes to the effective construction of portfolios. While the model points out the most efficient portfolios, which are greatly balanced between volatility and return, the change in constraints, which strongly affects the results of the solver, can make some of those potential portfolios impossible, or unfeasible. This study focuses on the setting of constraints on each component's weight in the whole portfolio, as well as the evaluation of the differences of risk and return rate between the two portfolios under different constraints. The Markowitz model is used to calculate the most efficient portfolios and the efficient frontier under each setting of the constraints. The results of the study show that additional constraints on weights, which are based on the total value of the portfolio and average daily trading value of the stock, make the range of potential volatility rate more limited by controlling the weights of some stocks in the portfolio.

Keywords: Liquidity constraints, average trading value, Markowitz model, portfolio selection, efficient frontier.

1. Introduction

Liquidity control is an essential issue for portfolio management, and it increases its relevance in stocks which present lower trading volumes. In major global trading markets such as those in the United States or the United Kingdom, most stocks have high liquidity, and thus their average daily trading volume is quite large. However, even within S&P500, there are many stocks with relatively low average daily trading volume and liquidity, which stands in contrast to some top-tier stocks.

This study primarily selects four stocks with relatively low liquidity from the S&P500 as the core research objects. It explores their overall impact on the return and volatility of the entire portfolio--consisting of seven stocks in total-- under different constraints settings.

In market trading, models like the Markowitz model or index model can generally inform investors about optimal weight allocation to construct a reasonable portfolio. However, in practical operations, stocks are constantly being bought or sold. For instance, the "7% rule" suggests that when a stock drops to a certain value, it must be sold immediately [1]. Therefore, a more comprehensive portfolio management process needs to account for changes in stock holdings. As such, portfolio liquidity should be incorporated into the model, as it determines whether investors can buy or sell stocks at a given time--ensuring favorable returns and avoiding excessive losses on unstable stocks.

In many other liquidity-related studies, various methods for setting constraints are included, including the one adopted in this study, as well as a range of other types of factors [2]. In contrast, this study focuses solely on a single constraint-setting method and intuitively presents the outcomes of its impact.

This study selects three stocks from the S&P500 with relatively high liquidity and large daily trading volume as the basic components of the entire portfolio, while the other four stocks are chosen from those with relatively low liquidity. Specifically, these four stocks are selected from the group with lower trading value to serve as the variables subject to constraints.

In the pre-filtering process, stocks with different trading values are distinguished [3]. Then, the first scenario is constructed, where the weight of each of the seven stocks is set to be greater than or equal to zero (short-selling is prohibited) and less than or equal to 40% (excessively high weight is prohibited), serving as the basic weight constraints [4]. Under such circumstances, the weight allocation for min variance and max Sharpe ratio was calculated using the Markowitz model. Additionally, the max return was computed across the entire range of possible standard deviations, so as to construct the efficient frontier [5].

In the second scenario, the weight constraints for the latter four stocks were adjusted. Due to their low liquidity, further and stricter restrictions were imposed on their weight allocation based on the total value of the portfolio, the average daily trading volume, and the daily turnover of each individual stock [6]. Specifically, drawing on the common practice in liquidity-constrained portfolio optimization, this study adopts a 10% liquidity multiplier: the market value of each stock in the portfolio (i.e., total portfolio value multiplied by the stock's weight) is constrained to not exceed 10% of the stock's average daily turnover [7]. This restriction ensures that the target positions are feasible to execute within a reasonable time frame without causing significant price impact, thereby aligning the theoretical portfolio construction with practical trading conditions. Then, the same model was applied to perform the same calculations for the stocks under this scenario, so as to derive the efficient frontier and the weight allocations under specific conditions.

Finally, the calculation results of the two scenarios are compared, including the ranges of standard deviation and the corresponding return intervals under the two scenarios. The study demonstrates that liquidity-based weight constraints hold certain significance in portfolio selection.

2. Literature Review

2.1. Markowitz Model

The Markowitz model is to make a portfolio of stocks, which gives optimal returns with reasonable risk to investors. Markowitz's stock diversification model works on the returns of portfolio assets by combining the returns that have less than positive correlation and the returns to decrease portfolio variance without reducing the return.

This study utilizes the Markowitz Mean-Variance Model to optimize portfolio allocation among 7 selected stocks, leveraging an existing Excel template of the model.

Through the model, two key portfolio allocations are derived: the maximum Sharpe ratio portfolio and the minimum variance portfolio. Additionally, by taking portfolio standard deviation as the X-axis, the model calculates the corresponding maximum return rate, as the Y-axis, for portfolios with different risk levels, reflecting their risk-return trade-off.

2.2. Constraints

2.2.1. Portfolio Weight Summation Constraint

The sum of weights of all stocks in the portfolio must be equal to 1.

$$\sum_{i=1}^{7} w_i = 1 \tag{1}$$

Where w_i represents the weight of the i-th stock in the portfolio.

2.2.2. Non-negativity Constraint

The weight of each individual stock is required to be non-negative, which prohibits short-selling. It is formulated as:

$$w_i \geq 0 \tag{2}$$

2.2.3. Upper Limit Constraint on Individual Stock Weight

To avoid excessive risk concentration in a single stock, the weight of each individual stock is restricted to not exceed 40%. This constraint is expressed as:

$$w_i \leq 0.4 \tag{3}$$

Where w_i has the same definition as above.

2.2.4. Special Constraints on the latter Four Stocks

There are various types of constraints on the weight of individual stocks in a portfolio based on trading volume. This study adopts a relatively straightforward formula as follows:

$$x_i \cdot TPV \leq l_i \tag{4}$$

Where x_i is the weight allocated in each asset i, TPV is the total portfolio value and l_i the daily turnover of each asset, which is calculated as:

$$l_i = ADTV_i \cdot AP_i \tag{5}$$

Where $ADTV_i$ is the average daily trading volume of stock i, and AP_i is the average price of stock i.

However, this study still chooses to multiply the right-hand side of this inequality by 10%, so the inequality becomes:

$$x_i \cdot TPV \leq l_i \cdot 0.1 \tag{6}$$

Therefore,

$$x_i \leq \frac{l_i \cdot 0.1}{TPV} \tag{7}$$

And when x_i equals the value on the right, it is called the weight boundary, which is the maximum weight allocated for the certain stock.

3. Data Collection

This study sets up two scenarios, A and B, for comparison. The portfolios in both scenarios consist of the same 7 stocks. This study used Yahoo's database to obtain historical daily return data for the 7 stocks that are all included in S&P500. Among them, the first one of SPX index, and then Intel Corporation, Apple Inc., International Business Machines Corporation, Morgan Stanley, Oracle Corporation, Citigroup Inc. The data includes monthly average return rates, monthly standard deviations, and the correlation matrix of the seven stocks, with data spanning from 2004 to 2024. According to Table 1and Table 2, all daily data were aggregated to the monthly observation to reduce non-Gaussian effects. Based on that monthly observation, it's calculated all the optimization input for the full Markowitz Model. The basic data are shown as follows:

Table 1. Return and Volatility of Stocks

	SPX	INTC	AAPL	IBM	MS	ORCL	C
Monthly Average Return	9.535%	5.392%	33.9%	8.448%	10.458%	15.699%	0.204%
Standard deviation	14.952%	27.976%	32.458%	20.983%	33.102%	24.238%	42.741%

Table 2. Correlation Matrix

	SPX	INTC	AAPL	IBM	MS	ORCL	C
SPX	100%	55%	57%	58%	67%	66%	68%
INTC	55%	100%	42%	41%	34%	43%	35%
AAPL	57%	42%	100%	33%	38%	42%	27%
IBM	58%	41%	33%	100%	42%	48%	36%
MS	67%	34%	38%	42%	100%	44%	57%
ORCL	66%	43%	42%	48%	44%	100%	40%
C	68%	35%	27%	36%	57%	40%	100%

The only variable between scenario A and B is that certain restrictions have been added to the weight setting of the last four companies: IBM, MS, ORCL, and C. These specific weight restrictions, according to Table 3, are derived from the formula mentioned above. Average daily trading volume, average daily price, and the total value of the portfolio, which is set to one billion, are used in this calculation.

Table 3. Special Weight Constraints

	Daily Trading Volume (million)	Average Daily Price (dollar per share)	Daily Turnover (billion)	Weight Boundary (percentage)
IBM	5.39	243	1.3097	13.09
MS	4.45	148	0.6586	6.58
ORCL	11.81	235	2.7753	27.75
C	11.03	95.3	1.0511	10.51

And finally, all the constraints for Scenario A and B are listed in Table 4:

Table 4. Constraints of Scenario A and B

Component	A	B
SPX	$0 \leq w_1 \leq 0.4$	$0 \leq w_1 \leq 0.4$
INTC	$0 \leq w_1 \leq 0.4$	$0 \leq w_1 \leq 0.4$
AAPL	$0 \leq w_1 \leq 0.4$	$0 \leq w_1 \leq 0.4$
IBM	$0 \leq w_1 \leq 0.4$	$0 \leq w_1 \leq 0.130977$
MS	$0 \leq w_1 \leq 0.4$	$0 \leq w_1 \leq 0.06586$
ORCL	$0 \leq w_1 \leq 0.4$	$0 \leq w_1 \leq 0.277535$
C	$0 \leq w_1 \leq 0.4$	$0 \leq w_1 \leq 0.105116$
Total Weights	1	1

4. Analysis of Comparative Portfolios

This study obtained, through calculations, the efficient frontier graphs under two scenarios, according to Figure 1, as well as the specific data for each coordinate point on the graphs, including the return rate and the corresponding standard deviation. Additionally, the weight allocation corresponding to each data point was derived.

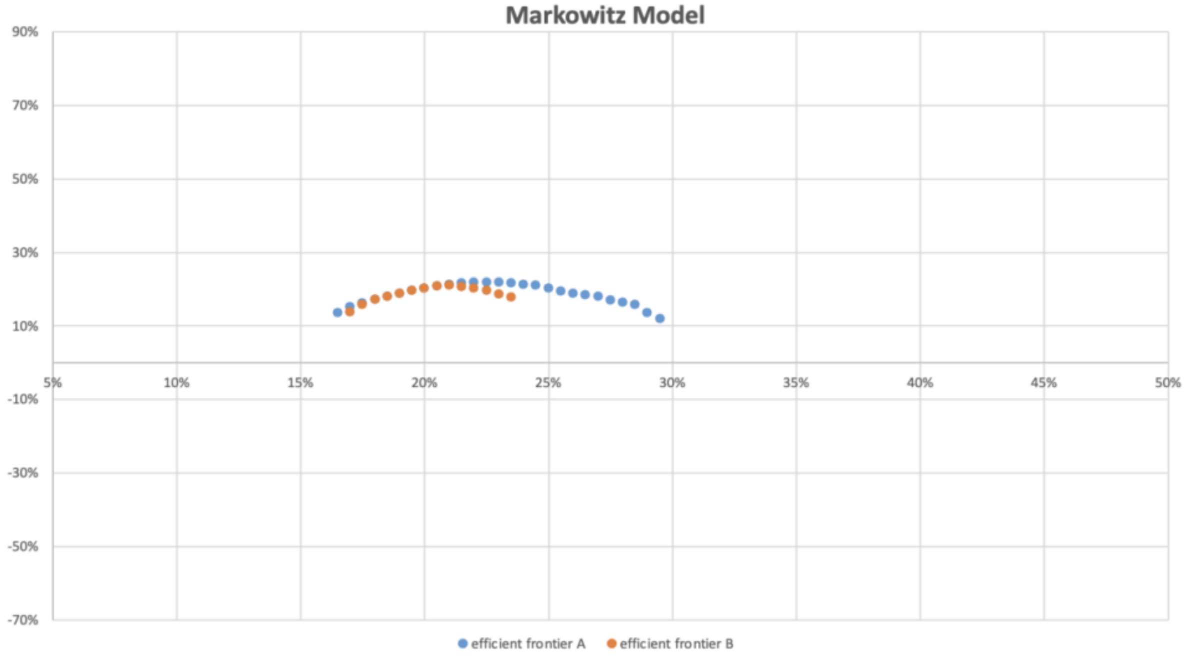

Figure 1. Efficient Frontier under Markowitz Model for each scenario

The weight allocations when the Sharpe Ratio reaches the maximum under the two scenarios are exactly the same, according to Figure 2, which is also shown below:

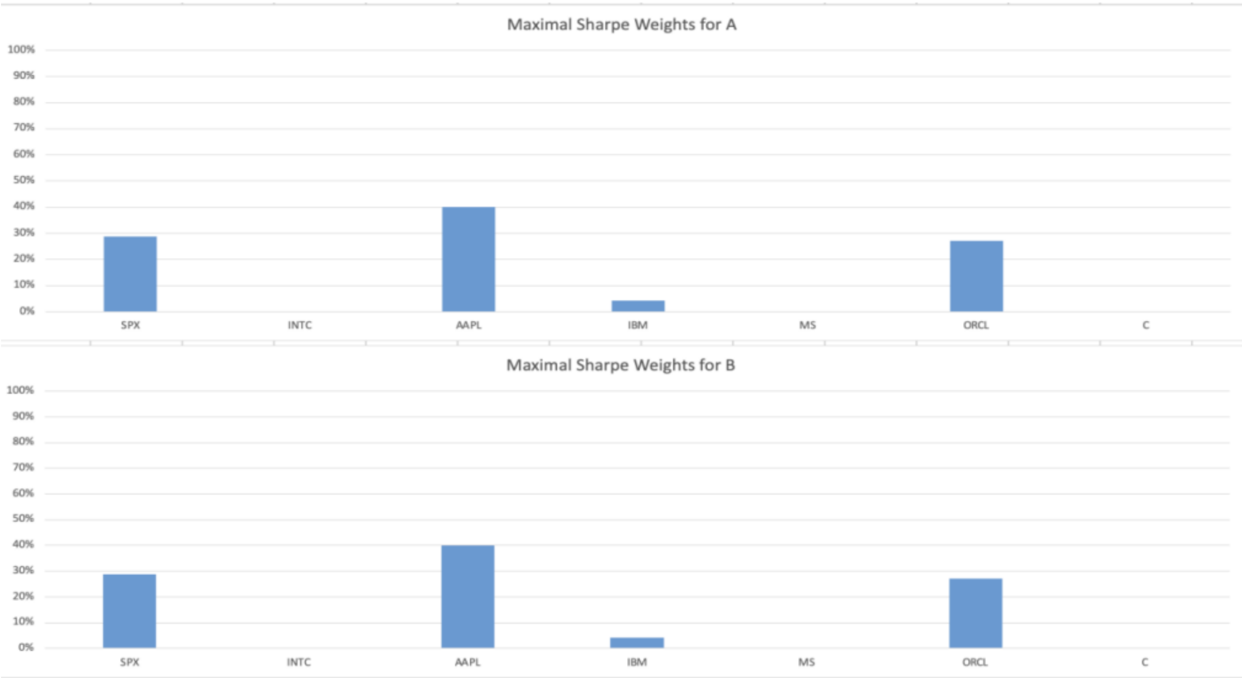

Figure 2. Max Sharpe Weights for A and B

Thus, in this case, if an investor selects these stocks and does not necessarily seek the optimal combination of return and standard deviation, not setting additional constraints will not have any impact on the actual results. However, the role of constraints lies in limiting the possible range of standard deviation, which is shown in Table 5.

Table 5. The Range of Std for Scenario A and B

A	16.5%	....	29.5%
B	17%	....	23.5%

Although there is a certain difference in the initial points of standard deviation (the minimum standard deviation) between the two scenarios, the maximum standard deviation in Scenario A is higher than that in Scenario B, with a difference of nearly 6 percentage points. This indicates that without special constraints, the volatility of the entire portfolio could reach a higher level, but the additional constraints have restricted such a situation from occurring.

Even ordinary investors tend not to consider points with relatively high standard deviations, as their corresponding return rates have already deviated from the highest levels. However, if the composition of stocks changes, investors may potentially opt for options with higher risk coefficients and slightly higher return rates, in an attempt to find their desired optimal balance.

Yet, actual volatility is not limited to standard deviation alone; there are also unaccounted-for liquidity constraints that arise when buying or selling stocks. Consequently, the real standard deviation may be even higher, exceeding the limits of the data provided by the model itself.

5. Conclusion

This study highlights the substantial impact that liquidity constraints have on portfolio construction and its associated risk-return dynamics. By comparing two scenarios—one with standard weight limitations and another with liquidity-based constraints—it becomes evident that imposing stricter limits on low-liquidity assets narrows the feasible range of portfolio volatility. While the maximum Sharpe ratio remains unaffected, the constrained scenario notably reduces the upper bound of standard deviation, thereby limiting exposure to high-risk configurations.

Such findings highlight the practical importance of integrating liquidity considerations into portfolio optimization models. Investors who overlook these constraints may inadvertently expose themselves to elevated risks that are not captured by traditional mean-variance frameworks. Moreover, the study illustrates that liquidity constraints can serve as a safeguard, especially in volatile markets where the ability to execute trades efficiently becomes paramount.

Looking forward, future research could expand on this foundation by exploring dynamic constraint models that adjust in real-time based on market conditions or investor behavior. Additionally, incorporating transaction costs, bid-ask spreads, and market impact metrics could offer a more holistic view of portfolio feasibility under liquidity stress. As algorithmic trading and high-frequency strategies continue to evolve, the relevance of liquidity-aware portfolio design will only grow, demanding more sophisticated tools and models to ensure robust investment decisions.

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