A Comparative Study of the Markowitz Model and the Single Index Model under Bull and Bear Market Scenarios

Shiyun Chen *

Guangdong Country Garden School, Guangzhou, China

* Corresponding Author Email: shiyunchen087@gmail.com

Abstract. The quest for the optimal balance between risk and return remains a pivotal area of research within modern portfolio theory. As two traditional approaches, the Markowitz model and the single-index model embody distinct philosophies: precise calculation versus simplified approximation. The Markowitz model delves deeply into the covariance relationships between assets, laying the foundation for modern portfolio theory; the single-index model enhances practical applicability by streamlining the estimation process of the covariance matrix. This study constructs the efficient frontier, global minimum variance portfolio, and optimal risky portfolio under both the Markowitz model and single index model using data from 21 selected sample stocks. Empirical testing and comparisons are conducted across various scenarios using metrics such as the Sharpe ratio and Sortino ratio. Empirical results indicate that the single-index model demonstrates superior risk-adjusted returns and robustness across most market conditions. Whilst the Markowitz model theoretically achieves greater diversification, cumulative estimation errors may lead to comparatively weaker performance in empirical outcomes. These findings not only assist investors in model selection across dynamic market environments but also inform future research on expanding constraints and incorporating factor models.

Keywords: Markowitz Model, Single-Index Model, Portfolio Optimization, Bull and Bear Markets, Performance Evaluation.

1. Introduction

With the intensifying volatility and rapid expansion of global capital markets, portfolio optimization has emerged as a pivotal subject in contemporary financial theory and practice. Since Markowitz introduced the mean-variance model, investors have been able to effectively balance risk and return through rigorous mathematical reasoning, an approach widely regarded as the cornerstone of modern portfolio theory [1]. However, this model faces practical challenges in high-dimensional covariance matrix computation and estimation bias, revealing limitations in complex market conditions. To simplify covariance structures, Sharpe's single-index model introduced market elements, substantially reducing computational complexity and enhancing the model's operational feasibility [2].

Subsequent empirical studies have conducted multifaceted comparisons between the two models. Early research indicates that both models often yield similar decision outcomes when constructing investment portfolios. Although the mean-variance model provides a more comprehensive characterization of asset correlations through the covariance matrix, rendering it theoretically more precise, portfolios constructed using the single-index model demonstrate greater robustness when dealing with limited sample sizes or high-dimensional assets. Compared to the Markowitz model, it also exhibits less sensitivity to parameter estimation errors, thereby demonstrating greater practical utility [3].

However, existing literature predominantly focuses on comparisons within static market environments, with scant attention paid to how the two models perform under differing dynamic market conditions—such as bull and bear markets. Given the significant impact of market volatility on portfolio construction, this study not only compares the differences between the Markowitz model and the single-index model in portfolio construction at an aggregate level, but also extends the analysis to examine their performance specifically within bull and bear market scenarios.

Through comparative analysis, this study not only fills a gap in existing literature concerning market conditions but also provides empirical support for investors to rationally select portfolio construction methods under varying market circumstances, thereby forging a closer link between theory and practice.

The empirical section utilized Microsoft Excel's Solver and Solver table tools to construct portfolios under both the Markowitz model and the single-index model based on historical data. This process generated metrics including the efficient frontier and optimal risky portfolios. The objective was to demonstrate the differing risk-return trade-offs exhibited by these models. Furthermore, the sample was divided into bull and bear market periods for comparative analysis, thereby illustrating the applicability and robustness of both models under varying market conditions.

2. Literature Review

In 1952, Markowitz first proposed the mean-variance theory, laying the foundation for modern portfolio theory. He applied mathematical statistics to portfolio selection research, constructing asset combinations to maximise expected returns for a given level of risk (measured by variance), thereby establishing the theoretical framework of the efficient frontier [1]. However, this model exhibits significant limitations in practical application. The most substantial shortcoming lies in the substantial covariance matrix estimation requirements, which lead to computational complexity and cumulative error estimation. This often restricts the model's practicality within real investment environments. To mitigate the complexity of covariance estimation, Sharpe introduced the single-index model in 1963. This model assumes a stable linear relationship between security returns and market returns, thereby simplifying the extensive covariance estimation required into a small number of parameters related to market factors [2]. Through this approximation, the single-index model not only substantially reduces computational difficulty but also yields portfolio outcomes closely resembling those of the mean-variance model in most scenarios. Subsequently, Sharpe introduced the Capital Asset Pricing Model (CAPM), demonstrating that under market equilibrium, an asset's expected return depends solely on its systematic risk (β), thereby providing a theoretical foundation for the single-index model [4].

However, the theoretical premises of these classical models have also faced persistent scrutiny and challenges: Roll argued that real market portfolios are unobservable, rendering the CAPM and single-index model untestable and casting doubt on their theoretical validity [5]; Fama & French discovered that factors such as firm size and book-to-market ratios explain stock returns more effectively than beta, challenging the explanatory power of the single-index model and driving the development of multi-factor models [6]; Michaud, Richard O further revealed another major flaw in the Markowitz model: the model exhibits extreme sensitivity to input parameters, with ‘optimized results’ potentially proving unstable or even inferior to simple strategies [7].

In response to these theoretical and practical challenges, scholars have continually refined models to enhance their real-world applicability: within the Chinese market context, Zhang Hua developed a dynamic single-index model building upon existing frameworks, permitting β coefficients to vary over time. This approach enabled more accurate estimation of the covariance matrix and improved portfolio performance in the Chinese stock market [8]; Wang, Lei, and Jing Zhao enhanced the Markowitz model by incorporating practical factors such as transaction costs and constraints, validating its effectiveness using Chinese data [9]; In recent years, with advances in artificial intelligence, Chen, Xiaohong et al. have attempted to apply deep learning methods (such as LSTM) to predict asset returns and covariances, subsequently integrating these into the Markowitz framework to significantly improve portfolio performance [10].

However, existing literature has primarily focused on comparing the theoretical structures and general performance characteristics of the two models, with relatively little attention paid to their differing performance under varying market conditions, such as bull and bear markets. This research gap has resulted in an insufficient understanding within the academic community regarding the applicability of these two models within dynamic market environments.

Therefore, further investigation into the validity of mean-variance models and single-index models across different market phases not only enriches existing portfolio theory but also holds practical application value. It further provides insights for subsequent research into expanding constraints and introducing factor models.

3. Research Method

3.1. Selection of Data

This study selects 21 constituent stocks from the S&P 100 Index as the research subjects. The sample selection is based on market capitalization and industry distribution (see Table 1 for details). It covers multiple core industries, including technology, financial, consumer cyclical, consumer non-cyclical and energy, to ensure diversity and representativeness. In order to measure the overall performance of the market, this study selects the S&P 500 Index as a proxy variable for the market portfolio. This index is widely used to measure the overall performance of the US stock market and provides a reasonable reference for risk and return. In addition, the risk-free interest rate is based on the US federal funds rate (FED01).

Table 1. the selected 21 constituent stocks and their sector distribution

stock	AAPL	MSFT	NVDA	INTC	JPM	BAC	BPK/B
industry	Technology	Technology	Technology	Technology	Financial	Financial	Financial
stock	JNJ	PFE	MRK	UNH	PG	COST	WMT
industry	Consumer,non-cyclical	Consumer,non-cyclical	Consumer,non-cyclical	Consumer,non-cyclical	Consumer,non-cyclical	Consumer,cyclical	Consumer,cyclical
stock	XOM	MCD	TXN	CVX	DIS	CRM	KO
industry	Energy	Consumer,cyclical	Technology	Energy	Communications	Technology	Consumer,non-cyclical

The data used in this study includes daily closing prices from 2004 to 2024, which were used to calculate stock returns and related statistical indicators. All data required for the study was provided by the supervisor and was organized and checked for consistency prior to use.

3.2. The Delineation of Bull and Bear Market Periods

This study classifies market conditions based on the cumulative percentage change in the S&P 500 Index. Following industry convention, a period is defined as a bull market when the index rebounds by over 20% from a recent low, and as a bear market when it declines by over 20% from a recent high. Drawing upon historical market data provided by Yardeni Research, this study identifies the period from 9 October 2007 to 9 March 2009 as a bear market phase (approximately a 56.8% decline) and the period from 9 March 2009 to 19 February 2020 as a bull market phase (approximately a 676.5% increase).

3.3. Data Computation

3.3.1. Calculate the monthly return for each stock and for the market portfolio, proxied by the S&P 500 Index.

$$R_{x,t} = \frac{P_{x,t} - P_{x,t-1}}{P_{x,t-1}} \tag{1}$$

R_t: the return of asset x in month t

$P_{x,t}$: the closing price of asset x at the end of month t

$P_{x,t-1}$: the closing price of asset x at the end of month $t-1$

When performing mean-variance optimization and other models, assumptions often rely on approximate normality. Daily and weekly returns often deviate from normal distribution (with more severe fat tails, skewness, etc.). In order to make return distributions closer to normal, while reducing noise and asynchronous trading issues, this study uses monthly returns as one of the statistical indicators.

3.3.2. Calculate the monthly excess return for each stock and for the market portfolio, proxied by the S&P 500 Index.

$$R_{i,t}^{excess} = R_{i,t} - R_{f,t} \tag{2}$$

$R_{i,t}^{excess}$: the excess return of asset i in month t
$R_{i,t}$: the total return of asset i in month t
$R_{f,t}$: the risk-free return rate in month t

3.3.3. Calculate the residual return for each stock and for the market portfolio, proxied by the S&P 500 Index.

$$\varepsilon_i = (R_i - R_f) - \left(\alpha_i + \beta_i(R_m - R_f)\right) \tag{3}$$

ε_i: the residual of stock i
$(R_i - R_f)$: the excess return of stock i
α : the intercept of regression
β: the sensitivity of stock i to market movements
$(R\_m - R\_f))$: the excess return of market portfolio, proxied by the S&P 500 Index.

3.3.4. Calculate other input variables.

the annualized average return, annualized standard deviation, coefficient of regression (β), intercept of regression (α), residual standard deviation and correlation for each stock and for the market portfolio, proxied by the S&P 500 Index were computed using built-in formulas and Data Analysis Toolpak in Microsoft Excel.

3.4. Formula and Input Variables for the Markowitz Model

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i) \tag{4}$$

$E(R_p)$: expected return of the portfolio
$E(R_i)$: expected return of asset i
w_i: weight of asset i in the portfolio

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{5}$$

σ_p^2: variance of the portfolio return (measure of risk)
σ_{ij}: covariance between the returns of asset i and asset j
w_i: weight of asset i in the portfolio
w_j: weight of asset j in the portfolio
Input Variable: annualized average return; annualized standard deviation

3.5. Formula and Input Variables for the Single-Index Model

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i \tag{6}$$

R_i: return of asset i
α_i: asset-specific return (intercept, not explained by the market)
β_i: sensitivity of asset i to the market
R_m: return of the market portfolio (systematic factor)
ε_i: idiosyncratic error term of asset i\

$$\sigma_p^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon i}^2 \tag{7}$$

σ_m^2: variance of the market return

$\sigma_{\varepsilon i}^2$: variance of the idiosyncratic error term

σ_p^2: variance of the portfolio return (measure of risk)

β_i: sensitivity of asset i to the market

Input Variable: residual standard deviation; return of the market portfolio (R_m); intercept of regression (α); sensitivity of asset i to the market (β)

3.6. Introduction of Constraints

Furthermore, to examine the impact of different constraints on the results of portfolio optimization, this study sets up three scenarios under both models: Prob.1: Unconstrained scenario, where asset weights can theoretically take any value, allowing readers to see the model's potential optimal performance as a benchmark result; Prob.2: Non-negative constraint, where asset weights cannot be negative, simulating real-world investor operations (most investors cannot short sell); Prob.3: Weight upper bound constraint (single asset weight≤20%), to avoid excessive concentration of the portfolio in a few assets and enhance diversification.

3.7. The Evaluation Indicators Employed in this Study

This study selected the efficient frontier, global minimum variance portfolio, optimal risky portfolio, Sharpe ratio, minimum variance frontier, Sortino ratio, and downside variance as evaluation metrics. The efficient frontier represents the set of portfolios that can achieve the highest expected return for a given level of risk, or the lowest risk for a given return target. It serves as the benchmark for comparing the performance of portfolios constructed using different models. The global minimum variance portfolio represents the combination with the lowest risk (variance or standard deviation) among all possible portfolios. It frequently serves as the starting point for portfolio optimisation. The optimal risky portfolio is the combination located on the efficient frontier that delivers maximum utility after accounting for investor risk preferences. It represents the theoretically optimal choice for investors. The minimum variance frontier is the curve formed by all risk-minimizing portfolios on the risk-return plane, aiding investors in identifying portfolios with the lowest risk for a given return or the highest return for a given risk. The Sortino ratio measures excess return per unit of downside risk. It improves upon the Sharpe ratio by avoiding penalizing returns due to upward volatility; the downside standard deviation calculates only the volatility of portfolio returns below the target return. This forms the core of the Sortino ratio's denominator. (Minimal Risk or Variance Frontier, range for portfolio returns from -30% to 70% with step of 0.5%. Maximal Return or Efficient Frontier, range for portfolio standard deviation: from 11% to 50% with step of 0.5%.)

3.8. Verification of Constraints and Final Selection

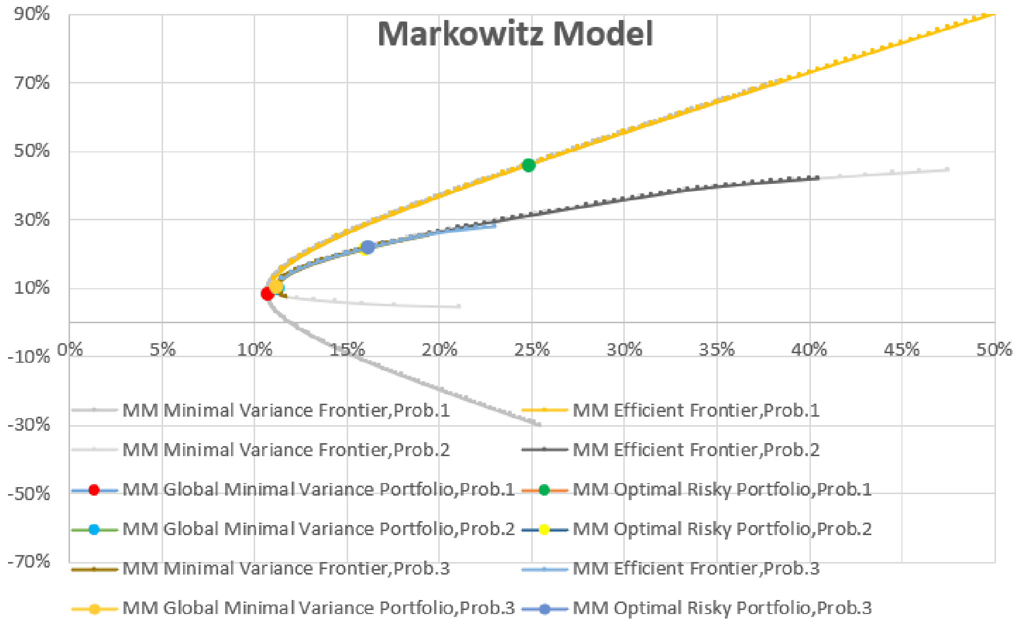

Figure 1. indicators of Markowitz Model

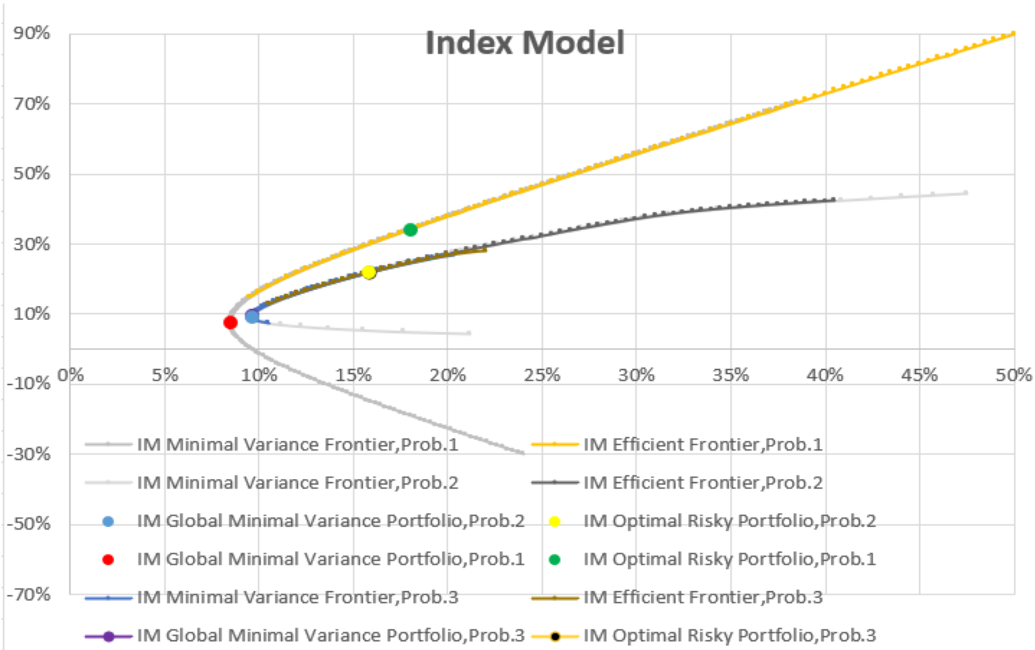

Figure 2. indicators of Index Model

As shown in figure 1 and figure 2, under unconstrained conditions, the efficient frontier reflects the theoretically optimal state (i.e., under the same return or risk level, unconstrained models often achieve lower risk or higher returns). However, it includes negative weights, i.e., it allows short selling, which is often difficult to achieve in real markets; Under non-negative constraints, the efficient frontier shrinks outward as a whole, limiting the risk-return space but enhancing operability; Under the weight limit constraint (single asset weight≤20%), the efficient frontier is further restricted. Overall, constraints have a significant impact on the shape of the efficient frontier: As constraints increase, the efficient frontier shows a trend of becoming tighter. Although the introduction of constraints results in 'efficiency losses,' it brings 'practical feasibility' in return. The differences between the three indicate that portfolio optimization research should not only pursue mathematical optimality, but also take into account practical feasibility and robustness.

To ensure the practical applicability of the research findings, this study employs only portfolio scenarios under specified constraints in model comparisons and subsequent market scenario analyses. The efficient frontier underweight cap constraints nearly coincide with that under non-negative constraints, indicating that weight caps impose no additional constraints on the optimal solution. Consequently, this study adopts non-negative constraints as the primary discussion pathway to maintain the analytical framework's conciseness and readability (Prob.2). Concurrently, the lower portion of the minimum variance frontier comprises the inefficient frontier. Compared to the efficient frontier, this segment yields lower returns for the same level of risk. Rational investors should select portfolios along the efficient frontier; consequently, the inefficient frontier is not discussed further herein.

4. Empirical Findings and Analysis

4.1. Comparison of the Two Models over the Entire Sample Period

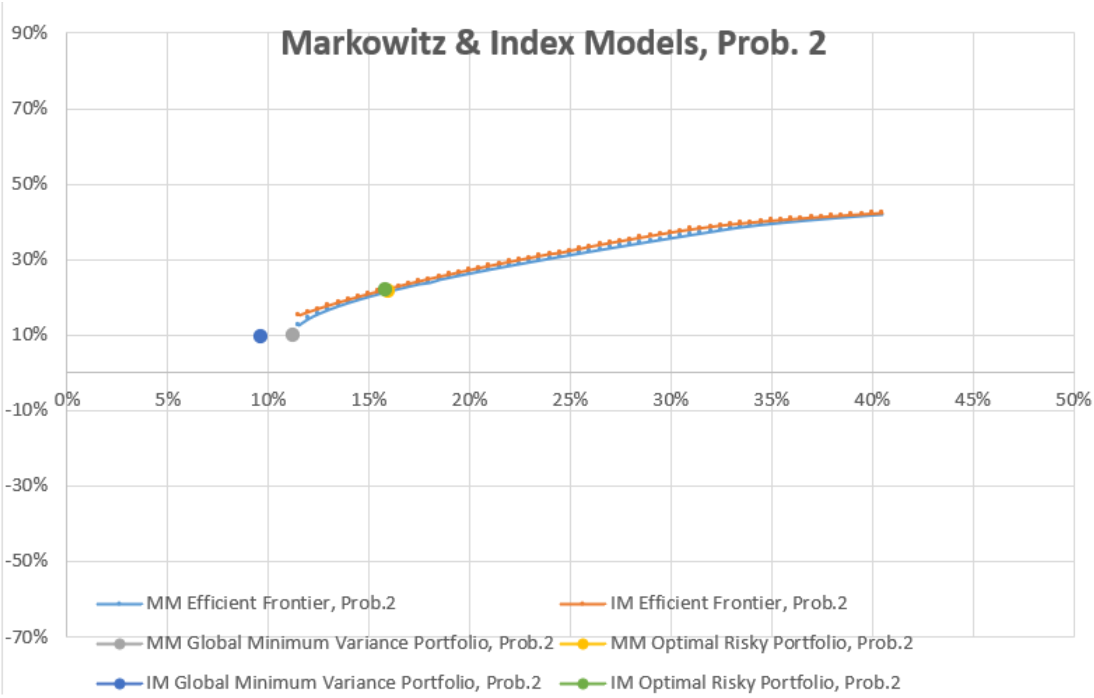

Figure 3. indicators of Markowitz Model and Index Model

Table 2. The Sharpe ratio within the optimal risky portfolios under two models

	Sharpe ratio
Markowitz Model	1.352
Index Model	1.388

According to figure 3 and table 2, this study conducted a three-dimensional comparison of the output results from the Markowitz mean-variance model (MM) and the single-index model (IM) Empirical findings reveal a high degree of consistency between the two approaches: across all performance indicators, IM consistently demonstrated a marginal advantage over the MM.

Specifically, firstly, in comparing the efficient frontiers, the efficient frontier constructed by the IM lies entirely above that of the MM (particularly in the low-risk region). This implies that at the same risk level, it offers a higher expected return. This indicates that IM provides a superior portfolio of investment opportunities across the entire risk-return plane.

Secondly, in comparing optimal risky portfolios, the Sharpe ratio of IM portfolio is marginally higher than that of MM. This indicates that lM delivers slightly superior risk-adjusted returns.

Finally, at the risk control level (global minimum variance portfolio), the standard deviation of IM portfolio is lower than that of MM. This indicates that for extremely risk-averse investors, IM can also construct portfolios with lower risk.

MM requires the complete estimation of all pairwise correlations between assets. For N assets, this necessitates estimating N(N-1)/2 covariance terms. As the number of assets increases, estimation errors progressively accumulate and amplify within the portfolio's risk-return performance [5]. This estimation uncertainty directly undermines the model's precision in characterizing portfolio risk and return, thereby adversely affecting investment performance.

IM substantially simplifies the estimation process of the covariance matrix by incorporating market indices as common factors. This simplification not only reduces the dimensionality of parameters but also mitigates the cumulative effect of estimation errors to a considerable extent. Despite its more streamlined structural assumptions compared to the MM, it enhances the model's robustness and operational feasibility, thereby yielding superior performance.

Empirical results indicate that IM outperforms MM in terms of the portfolio efficient frontier, optimal risky portfolio, and minimum variance portfolio. This phenomenon suggests that model complexity is not the sole determinant of portfolio performance; moderate simplification enhances both applicability and robustness. Consequently, IM not only offers a more interpretable theoretical approach but also provides investors with a more efficient and robust practical tool.

4.2. Comparison of the Two Models under Bull And Bear Market Scenarios

Building upon a comparative analysis of the overall sample period, this study further examines the applicability and robustness of the two models under divergent market conditions, such as bull and bear markets.

During bear market phases, to assess portfolio resilience under adverse market conditions, this study also compared downside risk metrics such as downside standard deviation and the Sortino ratio. Conversely, in bull market phases, where overall market appreciation renders downside risk events rare, the focus primarily shifted to profitability and risk-adjusted metrics, with downside risk indicators excluded from comparative analysis.

Unlike the traditional Sharpe ratio, the Sortino ratio considers only downside risk (i.e., volatility where returns fall below a target yield), thereby more accurately reflecting a portfolio's loss control performance. As its core metric, downside standard deviation avoids treating upward volatility as risk in its measurement.

$$\sigma_d = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\min(R_t - T,\ 0))^2} \tag{8}$$

$R_{p,t}$: return of the portfolio at time t

R_{target}: target return or minimum acceptable return

σ_d: downside deviation

$$\text{Sortino Ratio} = \frac{E(R_p)-R_f}{\sigma_d} \tag{9}$$

$E(R_p)$: expected return of the portfolio

R_f: risk-free rate

σ_d: downside deviation

All the aforementioned yield and risk metrics are derived by applying the portfolio weights generated by each model to historical yield sequences, rather than representing the models' theoretical expectations. This approach primarily avoids over-reliance on unstable parameters, thereby reducing the interference of estimation errors on empirical results and enabling a more accurate assessment of portfolio performance under real market conditions.

Table 3. Indicators of two models in the optimal risky portfolio under a bull market

	Annualised average rate of return	Standard deviation	Sharpe ratio
Markowitz Model	22.838%	14.461%	1.58
Index Model	22.632%	13.017%	1.74

Table 4. Indicators of two models in the optimal risky portfolio under a bear market

	Annualised rate of return	Standard deviation	Sharpe ratio	Downward standard deviation	Sortino ratio
Markowitz Model	13.168%	19.806%	0.665	14.119%	0.933
Index Model	13.735%	20.524%	0.67	14.540%	0.945

According to table 3 and table 4, during bull market phases, IM portfolio exhibited an annualized average return marginally lower than MM, yet with markedly reduced volatility. This indicates superior risk-adjusted returns for IM during market uptrends. Conversely, in bear market phases, IM portfolio achieved a higher annualized average return than the MM. Although volatility and downside standard deviation were slightly elevated, risk-adjusted metrics (Sharpe ratio and Sortino ratio) remained superior to those of MM. In summary, IM portfolio outperforms MM in risk-adjusted returns across both bull and bear markets. This demonstrates its robustness and applicability across varying market conditions, fully validating the practical effectiveness of IM within the selected sample period. IM demonstrates superior risk-adjusted returns to MM in both bull and bear markets. The fundamental reason aligns with the preceding analysis: by approximating the covariance structure through market factors, IM model requires estimating only a small number of parameters (the beta of each asset, residual variance, and market variance). This significantly reduces the complexity of parameter estimation and the accumulation of errors, thereby enhancing the model's robustness [5].

5. Conclusion

This study systematically compares the Markowitz model with the single-index model across diverse market scenarios. Empirical findings indicate that the single-index model consistently outperforms in risk-adjusted returns, both at the aggregate level and within bull and bear market conditions. This observation suggests that while the Markowitz model theoretically captures asset correlations more precisely through its covariance matrix, its practical efficacy may be diminished by estimation errors and data instability. In contrast, the simplified structure proposed by the single-index model demonstrates greater resilience and applicability across varying market conditions. Consequently, investors should balance theoretical superiority with practical feasibility when selecting models, implementing appropriate dynamic adjustments based on prevailing market circumstances.

However, this study also has certain limitations. Firstly, the performance of each model may be influenced by sample size and the selection of time periods; secondly, this research disregards more complex market factors such as behavioral finance and liquidity constraints, focusing solely on traditional return and risk metrics. Future research could examine the robustness of different models within larger-scale and cross-market data contexts, while further exploring the application of multi-factor models and machine learning methods in portfolio optimisation. This would serve to broaden and deepen our understanding of portfolio optimisation theory and practice.

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