Game Analysis and Bidding Strategy Optimization in First-Price Sealed-Bid Auctions: A Bayesian Nash Equilibrium Perspective

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Abstract. The first-price sealed-bid auction (FPSB) is a prevalent mechanism for selling high-value assets. However, its susceptibility to inefficiencies like the winner's curse and strategic underbidding can deter market participation and lead to revenue fluctuations, as observed in recent global auction market trends. This paper aims to derive optimal bidding strategies that approximate the Bayesian Nash Equilibrium (BNE), thereby mitigating these inefficiencies. Through a case study of a 1961 Alaska oil and gas lease auction, this study identifies the format's advantage in deterring collusion alongside its problems of pricing inaccuracy and resource misallocation. To solve these problems, two solutions are provided in this paper. Provided the cumulative distribution expected value of bidders follows the cubic function, a function that can achieve the Bayesian Nash Equilibrium is provided. This paper also provides different strategies based on the genre of the auction item. Bidders can shade more when bidding for artworks and antiques, and shade less when bidding for real estate and gas fields.

Keywords: First-Price Sealed-Bid Auction, Bayesian Nash Equilibrium, Optimal Bidding Strategy, Bid Shading, Winner's Curse, Allocative Efficiency.

1. Introduction

1.1. Research Background

The first-price sealed-bid auction is a type of auction mechanism. Customers submit their bids at the same time, and bidders who bid the highest can pay their bid and get the object. Meanwhile, others pay nothing, and they receive nothing, which means that their payoff is zero. This is usually used to sell high-value goods such as art, antiques, and real estate. Recently, the art and antiques auction market worldwide has experienced fluctuations. In the last five years (2019-2024), the aggregate sales value fluctuated between 21.2 and 31.3 billion US dollars. It peaked in 2021 and dramatically decreased, and ended up at only 23.4 billion in 2025 [1]. This phenomenon underscores the prevalence of the 'winner's curse' - a situation where the winning bidder overpays due to incomplete information and aggressive bidding. Due to the mechanism of the first sealed price auction, the market downturn might be partially attributed to the exorbitant premiums that bidders are required to pay in such auctions, which could deter market participation. As a result, people may prefer fixed price sales rather than auctions. In order to solve this problem of paying exorbitantly, this paper will identify several positive influences and problems of the first-price sealed-bid auction and provide several bidding strategies, aiming to mitigate the winner's curse and alleviate the financial burden on bidders by proposing more efficient bidding strategies. This may help bidders compete with a smart strategy, reducing the waste of money. Thus, an analysis of the first sealed price bidding is pivotal and necessary. However, real-world bidding behaviors often deviate from the theoretical Bayesian Nash Equilibrium (BNE), necessitating a more nuanced analysis.

1.2. Literature Review

The existing literature on first-price sealed-bid auctions can be broadly categorised into several sectors. In terms of the best bidding strategy, William Vickery pointed out that the best bidding strategy under a linear utility function is $b_i = \frac{N-1}{N} v_i$, where N, b_i, and v_i represent the number of bidders, bidding price, and expected value of the item, respectively. He also noticed that a second-price sealed-bid auction, known as the Vickery auction, can be more effective than the first-price sealed-bid auction [2]. Then, Chao Wang and Peijun Guo indicated that buyers are not always rational; they use the one-shot decision theory to explain the irrational behaviours in the first-price sealed-bid auction, such as overbidding or throwing away. Their model is scenario-based, which means it does not rely on the Nash Equilibrium [3]. Doron Klunover proposed that in a two-player first-price sealed-bid auction, if one of them has a "head start", they are able to reach a unique Bayesian Nash equilibrium. Under this equilibrium, the player who has the head start always wins, and they bid less than the expected value [4]. Other scholars focused on the comparison of the payoffs of different auction formats in several industries, which shows the current situation of the first sealed price auction. Stephen Buschbom, Carolyn Dehring, Neil Dunse, and Henry Munneke reveal the outcome of the fixed price sale and the first-price sealed-bid auction in the real estate market. They analysed the data of the Scottish housing market and found that there is no significant price difference between these approaches, even though there are auction discounts and auction premiums [5]. In terms of the rare flower market, Yongming Wang and Chen Xiang pointed out that the first-price sealed-bid price auction is not the best way to sell. They built a model with 1 supplier and 2 risk-neutral resellers, and proposed that the total revenue can be higher by using the second price sealed auction compared with the first sealed price auction [6]. The aforementioned studies provide a robust foundation for understanding bidding behaviors, primarily within the framework of linear utility assumptions. To be specific, it is assumed that a linear utility function ranging from 0 to 1 can be used to describe the distribution of expected value. In contrast, few papers noticed that the distribution of expected value can be different from the linear utility function due to the evaluation of bidders. For example, in some cases, the expected value of the item by different bidders can be similar, which means that the density of bids in a certain price range can be larger.

1.3. Research framework

This paper will be organised as follows. To begin with, a case will be introduced where the distribution of bids does not follow the linear equation. This case will serve as an empirical foundation to demonstrate the limitations of linear models and the deviation from BNE, thereby justifying our subsequent analysis. Then, positive influences and problems will be presented to illustrate the pros and cons of the first sealed price auction. After that, suggestions will be given under the assumption that the bidders' bids follow the cubic power function.

2. Case study

In this section, a real auction example will be introduced and analyzed. This first-price sealed-bid auction happened in Alaska, Dec 10th, 1959. Bidders bid on oil and gas fields, which covered 88055 acres. Initially, bidders were asked to pay a bid deposit to guarantee they would pay their bid. Then, they offered their bid for a certain amount of the field rather than the whole field. As a result, this auction was slightly different from the orthodox form of a first-price sealed-bid auction. Bidders could bid on different sections of the field so that they could submit multiple bids. To ensure comparability across heterogeneous bids, a standardized measure of bidding behavior is necessary. This paper will assume that the quality of the oil field was homogeneous, oil and gas were distributed evenly. Therefore, the bid per acre (total bonus divided by acreage) is adopted as the standardized metric for comparing bidding strategies across all participants. For instance, Bidder No.590 bid 564 acres with a total bonus of 126534 dollars, indicating its final bid was 224.173 dollars per acre.

In this auction, there were 37 bids made by 30 firms. It is worth noticing that bids made by the same firm can be different due to their bidding strategy or the diverse expected value of the field. For example, a firm invested jointly by The Ohio Oil Company and Union Oil Company of California bid 2587.2, 1137.236, and 2353.756 for different oil fields, respectively. There are two significant traits of the data. To begin with, there is a big gap between the upper and lower limits. The highest bid is 2353.756 dollars per acre, while the lowest is merely 1.1 dollars per acre, which is only about 0.05% of the highest one. Moreover, the mode of the data is 25; there are 10 bidders who bid 25 dollars per acre from different firms.

To clarify the bids, this paper provides Figure 1 as follows. The horizontal axis represents the bid, and the vertical axis represents the percentage of bids equal to or less than this bid. For instance, the lowest bid is 1.1, then the value of the horizontal axis is 1.1, and the vertical value is 0.027 (1/37).

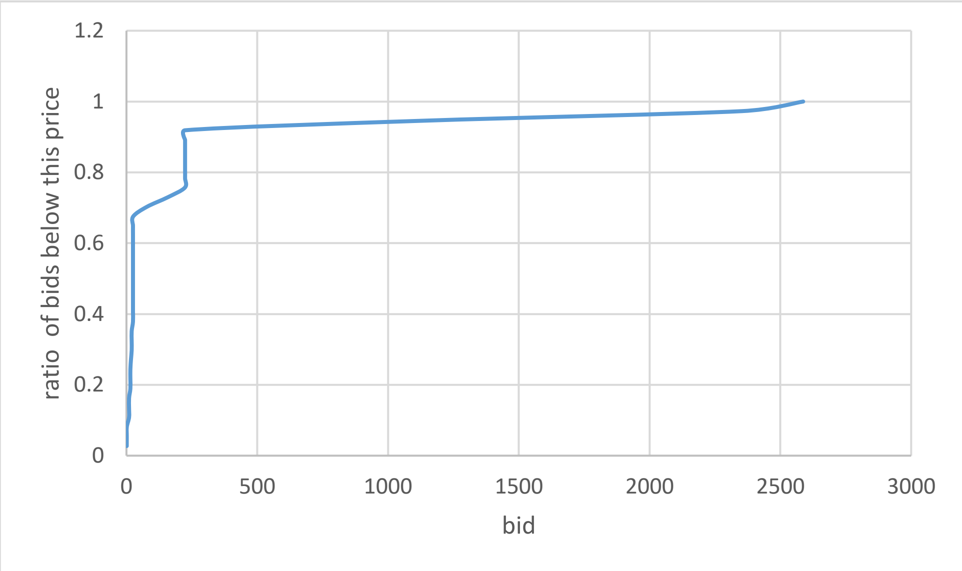

Fig 1. Cumulative Bidding Form [7]

This case exhibits two classic deviations from theoretical predictions: the winner's curse (overbidding by high-valuation bidders) and strategic underbidding or non-serious bidding by low-valuation participants. Also, the function of this auction is totally different from a linear function; it is hard to describe. The observed bidding patterns - extreme overbidding, widespread clustering at a low price point, and the absence of a smooth, monotonic strategy - are inconsistent with the predictions of the Bayesian Nash equilibrium model for symmetric bidders with independent private values. Overall, they do not achieve the Bayesian Nash equilibrium.

3. Analysis of the Problem

3.1. Positive Attributes: Prevention of Collusive Bidding

Many auctioneers choose a first-price sealed bid auction due to its effectiveness in preventing cheating and collusion. Cheating and collusion are seen in other formats of auction, such as the English auction. Several bidders can form a ring, sharing information and design bidding strategies to manipulate price or customise a winner. In a typical English auction, bidders bid openly and continuously, which allows ring members to manipulate the bidding process. By contrast, the first-price sealed-bid auction involves bidding simultaneously and secretly; bidders cannot observe others' behaviour, which makes it hard for rings to coordinate strategies [8]. Moreover, the ring in a first-price sealed-bid auction is not stable, since a Nash equilibrium that provides a positive revenue does not exist in the auction. This implies that members in a cartel have incentives to cheat, bidding higher than the designed winner in order to gain a positive revenue [9]. Thus, if a cartel desires to manipulate the price or winner, it is urged to design a more complex mechanism, such as PAKT, to ensure members follow the bidding rule. This process itself is costly and unstable [8]. So, as a result of its effective mechanism for preventing cheating and collusion, the government usually uses it as a form to sell state-owned assets such as real estate, gas and oil fields, and spectrum.

3.2. Problems

3.2.1 Winner's curse

In terms of bidders, the winner's curse is one of the most significant factors that impede them from maximising their profits. Winner's curse can be defined as overbidding. Since bidders bid based on their own evaluation, the winner is the one who overestimated the value of the products the most [10]. This can result in a loss for the winner of the auction. From the data, it is evident that compared with other bidders, Ohio Oil Company and The Oil Company of California bid over $2000 per acre. This highest bid was an order of magnitude greater than the next highest bid of approximately $220 per acre. This extreme overbidding is a textbook example of the winner's curse, which can be driven by several factors beyond simple valuation errors. Traditionally, scholars believe this effect is caused by the limitation of cognitive ability, as bidders cannot estimate the true value accurately. However, experiments demonstrate that social competence is the key factor that contributes to overbidding [11]. It is believed that victory has its own value; firms that win the auction can show their financial status or improve their reputation. Moreover, Risk aversion can contribute to overbidding. Research shows that bidders tend to bid higher than the risk-neutral Nash Equilibrium because they want a higher winning probability [12]. To be more specific, based on the Cobb-Douglas utility function, if bidders have a higher preference for winning rather than making a profit, they have a higher chance of overbidding, which can result in the revelation of the winner's curse [13]. Overall, there are several factors that can contribute to the winner's curse; they are the wrong estimation of value, social competence, and the preference for winning due to the aversion to risk. It leads to the excessive bonus of the auctioneer and a loss for the auction winner.

3.2.2 Underbidding

Conversely, auctioneers may suffer from suboptimal revenue if bidders engage in severe bid shading, leading to underbidding. This can result in underbidding, which is defined as the bidder winning the auction at a relatively low price, less than the real value of the auction items. In the Alaska oil and gas auction, bidder No.608 won 2867 acres for 71675 dollars, only 25 dollars per acre. This is only 1% compared with the bid offered by Ohio Oil Company and The Oil Company of California. This phenomenon is also observed in other auctions. Timber auctions held between 1996 and 2003 in British Columbia, Canada, demonstrate that underbidding is not as common as overbidding, but it still exists. Based on the two-tier model, bidders are underbidding by 12% on average [14]. In New Mexico, underbidding is commonly seen in oil and gas tract auctions. Research uses net revenue to show the real value of tracts and finds that it is four times the bids. Martin proposed that this may be attributed to collusion and cheating because bids are not consistent with the Bayesian Nash Equilibrium [15]. Tan and Liu point out that, based on the Cobb-Douglas utility function, if bidders value the importance of profit more than the probability of winning, they have a higher chance of underbidding [13]. As firms tend to maximise their profits in raw material auctions, companies usually underbid in these auctions. This may lead to a loss for auctioneers.

3.2.3 Misallocation of resources

Bidders who participate in a first-price sealed-bid auction tend to shade their bid, which means their bid is lower than their evaluation. Different bidders shade their bid to different extents, which means bidders with low expected value may win the auction if they shade less. Bidders with low expected value are unlikely to maximise the value of auction items, such as an oil field, due to limitations of techniques or capital. As a result, this can lead to the misallocation of resources. In the Alaska oil and gas auction, Standard Oil Company of California and Richfield Oil Corporation won six oil and gas fields and ended up abandoning two fields, which can be seen as a waste of resources. Empirical data show that in sealed-bid auctions, the highest-valuation bidder wins with a probability of 88%, lower than the 95% in English auctions [16]. Also, this may result in deviating from Pareto efficiency. Experimental studies show that in two-bidder settings, if one bidder uses the equilibrium strategy (e.g., bidding 50% of valuation) while the other overbids (e.g., 80% of valuation), the lower-valuation bidder may win, reducing allocative efficiency to approximately 81.25%, compared to 100% in English auctions [16]. Therefore, the strategic shading of bids in first-price auctions not only affects individual outcomes but also has significant implications for overall market efficiency. Thus, bidding strategy can contribute to the effective allocation of resources, maximising the total profit for auctioneers, bidders, and society.

4. Strategic Bidding Recommendations: Towards Bayesian Nash Equilibrium

4.1. Optimizing Bid Shading Under Non-Linear Value Distributions

Bidders can avoid overbidding by shading their bid. However, the canonical model proposed by Vickrey et al. relies on the assumption of a uniform value distribution, which, as our case study illustrates, often fails to hold in practice. A more generalized approach is therefore necessary. If bidders can first know others' expected value of the auction items, they will be able to achieve the Nash equilibrium. Vickery points out that, suppose the expected value of other bidders is distributed evenly and consistent with the linear utility function, the best strategy is to bid $\frac{N-1}{N}v_i$. This is the bidding strategy that follows the Bayesian Nash Equilibrium [2]. Conversely, the distribution of others' expected value may not be distributed evenly; bidders are less likely to expect a very low price than a high price, as research reveals that the extent of overbid is higher than underbid, 15% against 12% [13]. It reveals that bidders tend to bid higher, which can be attributed to a higher degree of risk aversion than risk preference. In other words, the density of the expected value of bidders around the high price can be higher than around the low price. So, this study can assume that the cumulative distribution of bidders follows a cubic function. To simplify, this paper assumes that the cumulative distribution expected value of bidders follows the cubic function of $y = x^3$, its domain is 0 to 1. Suppose there are n bidders; the possibility for a random bidder to win is x^{4n-4}, where x is the bidder's bid. Its profit is v-b(x), where v is the true value of the auction item and b(x) is the bidder's bid. So, it's expected profit $[v - b(x)] * x^{4n-4}$. In this case, the best strategy for each player is to bid according to this function: $b(x) = \frac{4n-4}{4n-3}v$, where v represents the expected value of the auction item, and $b(x)$represents the price bidders should bid. By using this function, bidders can maximise the expected profit provided that the cumulative distribution of the expected value of bidders follows a cubic function.

4.2. Context-Dependent Bidding: Aligning Strategy with Asset Type and Bidder Motives

The theoretical model above provides a mathematically precise strategy under specific assumptions. In practice, however, bidders must incorporate contextual factors that theoretical models abstract away. A critical factor is the fundamental nature of the auction item, which dictates the primary motive for bidding—whether it is utility-derived (e.g., aesthetic enjoyment) or profit-driven. Bidders need to recognise the purpose of the auction, whether it is to cater to bidders' own aesthetic preferences or to make profits. Different bidding strategies can be utilised in different scenarios, depending on the auction items. The previous paper proposed that if bidders value the winning probability more than the profit of winning, which usually happens in artwork or antiques, bidders tend to overbid [13]. Under these circumstances, bidders can shade their bid to a larger extent. They can apply a shading factor (e.g., 0.9) to their pre-defined valuation to make sure that they are bidding rationally. Conversely, bidders tend to underbid when they want to make a profit from winning the auction. They give up the higher probability of winning the auction in order to gain more benefits. However, they will receive a negative benefit if they do not win the auction because their preparation will go down the drain. Thus, it is wise for them to bid higher than the original bid; they may apply a premium factor (e.g., 1.1) to their calculated Nash bid or intrinsic valuation to increase their winning probability. This may result in a bigger profit. Overall, bidders should bid based on the type of auction items. They may bid lower to ensure benefits when bidding for artworks and bid higher to increase winning probability when bidding for profitable items such as gas fields.

5. Conclusion

This study investigates bidding strategy optimization in first-price sealed-bid (FPSB) auctions from a Bayesian Nash Equilibrium (BNE) perspective, aiming to address prevalent market inefficiencies such as the winner's curse and suboptimal resource allocation. This paper mainly analyses the gas and oil fields auction in Alaska in 1961, identifying the benefits and several problems of the first-price sealed-bid auction. Compared with the English auction, the first-price sealed-bid auction exhibits superior efficacy in mitigating collusion and deterring fraudulent bidding practices, requiring a more complex strategy to collude with other bidders. However, the first-price sealed bid auction can result in overbidding or underbidding, causing profit loss to bidders or auctioneers. Moreover, it may lead to the misallocation of resources, referring to bidders who can make the most profit out of the auction item, usually the one who has the highest expected value, losing the auction due to a radical shading strategy.

In terms of the solution, this paper evaluates the best bidding strategy provided that the cumulative distribution of bidders follows a cubic function. Under this prerequisite, the best bidding strategy that follows the Bayesian Nash Equilibrium is to bid according to the function: $b(x) = \frac{4n-4}{4n-3}v$, where n and v refer to the number of bidders and expected value, respectively. Additionally, bidding strategy can be diverse depending on the type of auction item. Bidders can shade more when bidding for artwork and antiques, and bid more when bidding for raw materials that can make a profit, such as real estate or gas fields.

This paper hopes to figure out a better bidding strategy that can be utilised in the first-price sealed-bid auction. It enables bidders to mitigate the winner's curse, enhance their surplus, and improve allocative efficiency. In terms of the auctioneers, they may choose a more appropriate format of auction to increase their profits.

Several limitations should be acknowledged. First, the primary reliance on a single case study may affect the generalizability of the findings. Second, the assumption of a cubic value distribution, while more flexible than uniformity, may not hold universally, which presents a limitation to the generalizability of the proposed model. Future research may focus on the cumulative distribution by studying more auction samples and determining the real distribution model.

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