Mathematical Modeling and Algorithm Application
ISSN: 3006-0842 | Vol. 3, No. 2, 2024

The Investigation of the Importance of Terrain and Tactical Strategy Based on the Lanchester Law Used in the Battle of Iwo Jima

Hongyuan Huang

The High School affiliated Renmin University of China International Curriculum Center, Beijing 100080, China

Abstract: This study addresses the significant yet understudied quantitative modeling of casualties in the Battle of Iwo Jima by integrating historical data with predictive frameworks like the Lanchester Model. By analyzing terrain, tactical strategy, and force interactions, we created a precise mathematical model to represent personnel losses. This approach yields valuable insights into the battle's dynamics, enhancing our understanding of historical and strategic military factors. The research advances the study of military history and the application of mathematical models in conflict analysis by refining battle unit usage and identifying critical warfare determinants.

Keywords: Lanchester Model; Terrain; Casualties.

1. Introduction

War is a large-scale organized conflict, usually between political entities (e.g., states, non-state actors, or coalitions),[1] involving sustained armed forces to achieve political, territorial, or ideological objectives. It is inherently political, continuing policy by other means, often resulting in widespread violence, destruction, and societal upheaval (Clausewitz, 1976[3]; Kaldor, 1999[2]; Howard, 1976). According to the statistics, research has found that there have been 2309 violent conflicts in Europe over the past 1100 years (Gat 2008) [4]. It is clear that while wars may aim to achieve strategic objectives, they often lead to significant casualties, destruction, and social or political instability, prompting efforts to avoid them. However, viewing dialectically, some positive consequences could be brought by wars. For example, war can drive technological innovation and progress (Chin, 2019)[5]. War often stimulates the progress of military technology, such as weapons and projectiles (Black, 2013) [6]. Additionally, it can attack the colonial rulers and weaken their control over the ruled nation, thus achieving national liberation (Morrock, 1973) [7].

Through the analysis and quantification of historical war data, we can have a clearer understanding of the root causes, progression, and influencing factors of a specific war. In other words, the modeling of data can make us better understand the nature and law of war to better prevent and avoid the occurrence of war. Based on the Lanchester model, this paper builds a mathematical model of the Battle of Iwo Jima in the Pacific War of World War II from a quantitative point of view, analyzes the relevant parameters, and plots the collected data to make it more intuitive. Then, I built accurate models of the casualties. This research has a very good constructive opinion on the deployment of troops and the understanding of influencing factors under the conditions of modern war.

2. Related work

Scholars have used a variety of mathematical models to analyze wars, such as the Markov Model, the Hidden Markov Model, or even the System Dynamic model.

Shallcross (2016)[8] combines logistic regression and the Markov Chain Model to predict violent conflicts and political transitions in nation-states. The integration of these two methods allows researchers to identify key factors that promote war, and it offers a more dynamic perspective on conflict transitions, including peace, tension, and conflicts. However, there are limitations to this modeling. The logistic regression assumes that relationships between variables are linear, which might oversimplify complex conflict dynamics since Conflicts are often influenced by non-linear and multi-dimensional factors.

Kiefer (1988)[9] applies a Markov model to study the dynamics of interstate wars in Third World countries. The Markov model provides a structured framework to study how third-world countries move between three states: tension and conflict. It also simplifies the complex nature of interstate wars into manageable probabilistic transitions, making it easier to quantitatively analyze war probability and durations. However, the model's assumption fails to capture the complex causes of wars in the Third World, such as foreign intervention, colonial legacies, or ethnic tensions. Additionally, the simplification may lead to the exclusion of socio-economic, political, or geopolitical conditions unique to the Third World, which play a significant role in the initiation and resolution of wars.

Shearer (2007)[10] investigates the application of Hidden Markov Models to predict changes in the Israeli-Palestinian conflict. The author focuses on how Hidden Markov Models track both observable events (such as ceasefires, peace talks, and conflicts) and hidden, underlying states (political pressures or secret negotiations). HMMs can easily capture explicit events (such as the bombing outbreak of conflict) and implicit events (such as tension within the Israeli government), which makes it highly appropriate for the Israeli-Palestinian conflict). However, the model's assumptions oversimplify some complex historical and cultural causes that led to this conflict. In this case, external influences like the US or Arab states' interventions are not considered. Besides, this model relies heavily on the accuracy of collected data. In the Israeli-Palestinian conflict, however, gathering accurate data can be challenging due to political sensitivities, biased reporting, or the fragmented nature of the region. Thus, the reliability of

this model may be reduced.

Schrodt (2006)[11] uses hidden Markov models (HMM) to forecast conflict in the former Yugoslavia for the period January 1991 through January 1999. The model is capable of forecasting potential future conflict transitions based on historical patterns, offering early warning signals that can inform diplomatic or military interventions. However, the inherent simplifications of the model—such as its assumption of discrete hidden states—and the challenges of data quality in the Balkans present significant obstacles. In the Balkans- a region with such a complex web of historical, ethnic, and geopolitical influences- the utility of HMMs depends on the ability to collect accurate data, and since it is difficult in this situation, the effectiveness is limited.

Spirling (2007)[14] examines the use of Bayesian change-point detection methods, specifically reversible-jump Markov Chain Monte Carlo, to identify structural breaks in a time series of civilian casualties during the Iraq conflict. These turning points correspond to shifts in the level of violence. The study highlights four such turning points, all associated with increasing violence. However, his study requires significant computational resources, and the accuracy of change-point detection depends heavily on the quality of civilian casualty data, which would be difficult to obtain or be reported with biases in conflict zones.

Lindquist (2017)[16] discussed how to use Markov-Chain models to analyze the dynamic and unpredictable nature of combat environments, focusing on troop movements and enemy engagements. The paper demonstrates how this model can help forecast battlefield outcomes, evaluate the effectiveness of different strategies, and optimize resource allocation. It can effectively model random events and decision-making processes in combat, which is crucial in military operations. However, the Markov chain framework typically assumes that the future state depends only on the current state, potentially **ignoring deeper, long-term dependencies** or complex, non-linear interactions. The simplification inherent in this model limited the applicability of it to all combat situations.

Duncan and Siverson (1975)[12] explore the application of Markov chain models to analyze conflict dynamics between China and India. It can model the escalation and de-escalation patterns in Sino-Indian relations by tracking the probability of transitions between conflict states (such as peace, tension, and war), which may be helpful for policymakers to understand future scenarios for possible conflict escalation or resolution. However, the model assumes that future states depend only on the current state and does not account for deeper historical or geopolitical factors that influence decision-making, which oversimplifies the complexity of long-standing historical tensions between China and India. Besides, it requires reliable historical data, which is difficult to obtain. Thus, the reliability of the model in forecasting real-world conflicts is reduced.

Schrodt (1997)[13] used Hidden Markov Models to predict and analyze potential conflicts in the Southern Lebanon region. It is advantageous in capturing hidden or unobservable states that influence observable events, such as political or military actions. It helps anticipate flare-ups in Southern Lebanon by identifying underlying patterns in conflict behavior that may not be directly visible. However, Markov Models rely heavily on assumptions regarding hidden states and their transitions[14], which may oversimplify the actual complexity of conflict drivers in Southern Lebanon. The actual factors have a wider range (social, political, and regional) than the model expected. While this model is intended to improve the early warning system that uses the traditional Markov model, the model's predictive power and effectiveness are reduced since it heavily relies on reliable input data, which is hard to obtain in volatile regions like Southern Lebanon.

In conclusion, the Markov model and its ramification model analyze wars from a macro perspective. Markov models assume "memorylessness," meaning the next state depends only on the current state and not on past states. What's more, Markov models often simplify reality by assuming probabilistic state transitions without accounting for more complex variables or nonlinear dynamics. Besides, it may overlook rare but impactful events, such as sudden tactical changes or external interventions. Additionally, the accuracy of the Markov model heavily relies on the preciseness of input data, which could be hard to obtain. Thus, there could be limitations in complex scenarios, like warfare, where historical effects and strategic adjustments are significant.

However, the micro-perspective modeling of the Battle of Iwo Jima is still a blank, and this is the reason why this research is trying to comprehensively analyze it.

3. Data Collection and Preprocessing

3.1. The Battle of Iwo Jima

From February 19 to March 6, 1945, the U.S. Marine Corps and the U.S. Navy engaged the Imperial Japanese Army in the Battle of Iwo Jima in the Pacific. [23]After the United States recovered the Marshall Islands and bombed the Caroline Islands, Japan attempted to establish a defense perimeter extending from the Caroline Islands to the Marianas and Iwo Jima and westward to Palau and the Philippines.[24][25] After the Marianas incident, U.S. bombers targeted the Japanese mainland, prompting the Japanese to view Iwo Jima as an important early warning station and air defense base. After the Battle of Leyte, Iwo Jima's strategic value as a Japanese airfield and supply point increased, and the U.S. goal was to occupy it to eliminate the threat to the Marianas, support continental bombing raids, and cut off Japan's supply lines in preparation for an invasion of Japan.

3.2. Gathering Primitive Data

Initial Force Strengths: The US Army had an initial force of 110000 people.The Japanese Army had an initial force of 23000 people.

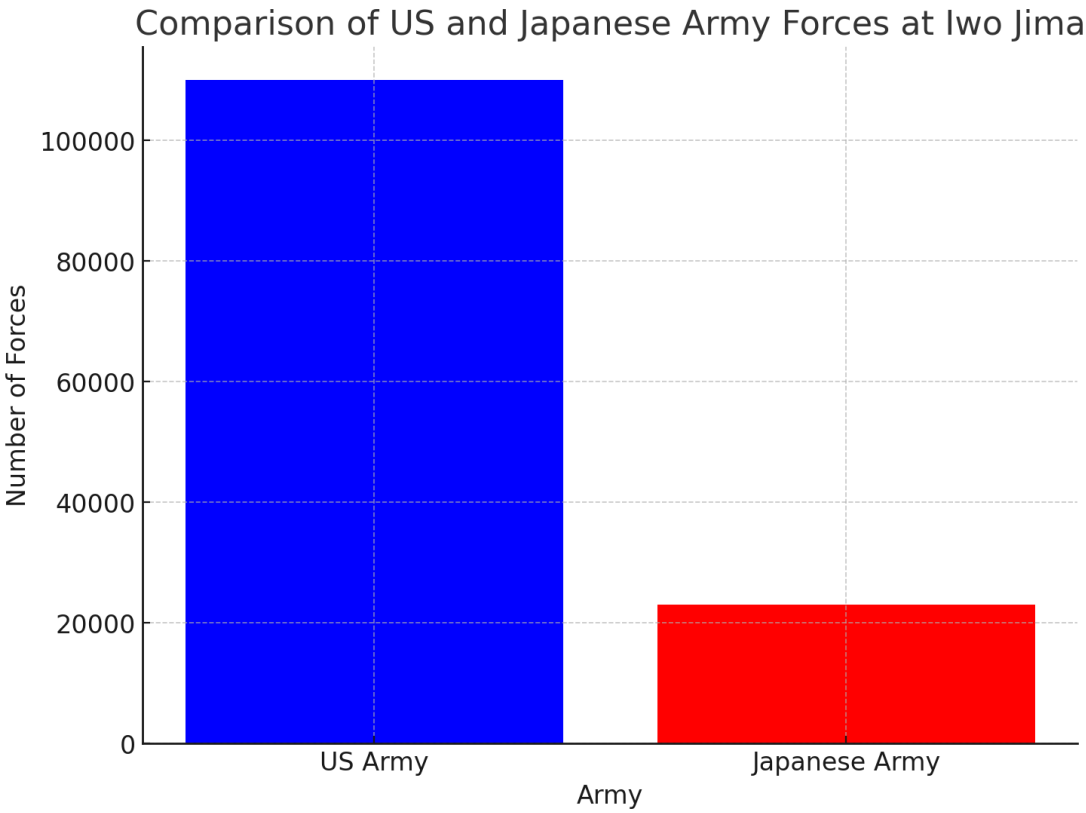


Figure 1 Initial Force Strengths

The number of warcraft: The number of the US Army warcraft was 30 warcraft was 2000.The number of the Japanese Army

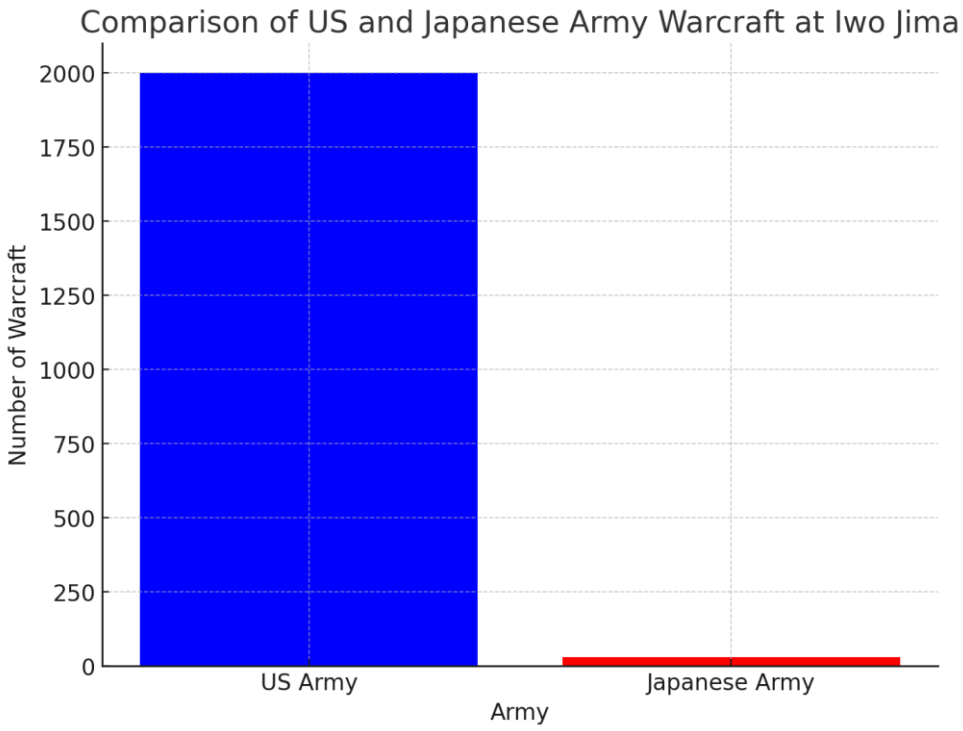


Figure 2. The number of warcraft

The number of tanks: The number of the US Army's tanks was 200.The number of the Japanese Army's tanks was 23

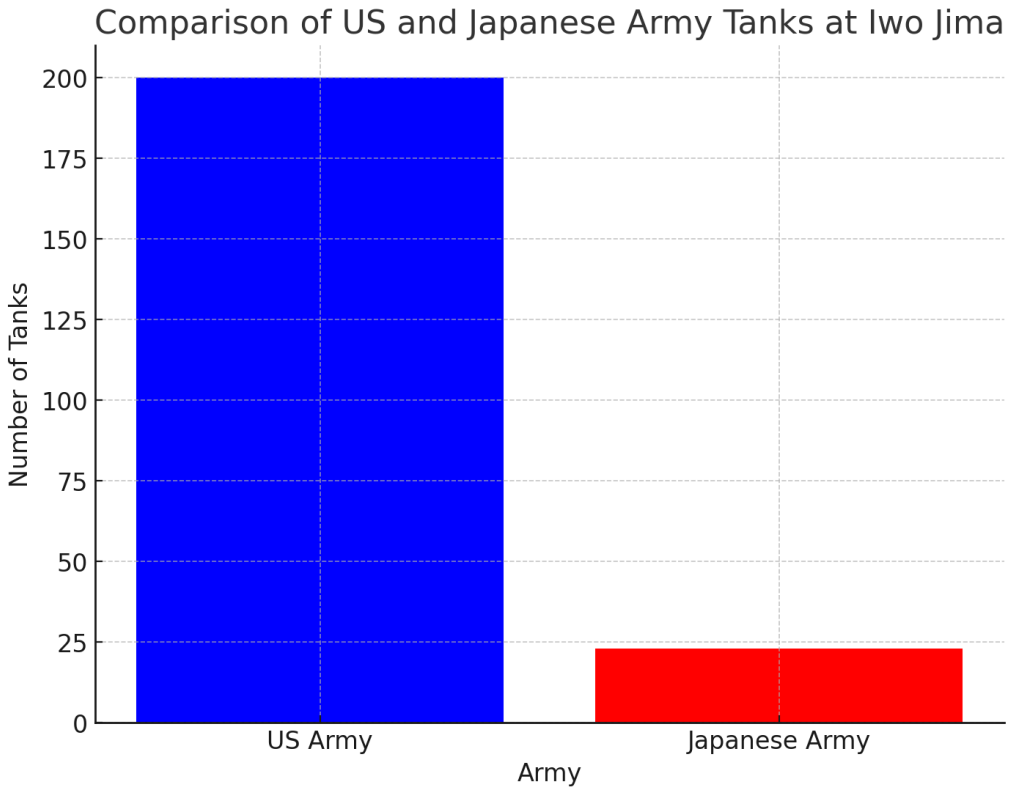


Figure 3. The number of tanks

3.3. Possible Factors

Most of the US infantry were equipped with M1 Garand, which is a semi-automatic rifle that has eight bullet magazines. On the other side, the Japanese infantry was equipped with a Type 38 rifle, which is bolt bolt-actioned. Thus, the firepower provided by the US standard arm is much more powerful than the Japanese one. Besides, the US Army equipped a large number of flamethrowers, which was effective in combating strongly fortified and entrenched Japanese positions.

The U.S. Army was equipped with the M4 Sherman tank, which had an armor thickness of up to 63 millimeters and was armed with a 75mm main gun. In contrast, the Japanese Army used the Type 97 Chi-Ha tank, whose armor thickness ranged from only 8 to 25 millimeters and was equipped with a 57mm main gun. The Japanese tanks were noticeably inferior in armor protection and firepower than the American ones. This disparity gave the M4 Sherman a significant advantage on the battlefield against the Type 97 Chi-Ha, allowing it to withstand more enemy fire and effectively destroy targets at greater distances.

Additionally, the significantly greater number (200: 23) of U.S. tanks provided several important advantages in battle. The abundance of M4 Sherman tanks ensured superior firepower, allowing for overwhelming force against enemy positions. This numerical advantage enhanced strategic and tactical flexibility, enabling U.S. forces to execute simultaneous offensives on multiple fronts and concentrate armor for decisive breakthroughs. The presence of numerous tanks boosted troop morale and confidence while exerting psychological pressure on the enemy, potentially lowering their combat effectiveness. Additionally, the tanks offered protection and support to infantry units, reducing casualties by shielding them from enemy fire and helping to destroy Japanese defensive structures effectively. All in all, the advantage of tanks that the US Army had ultimately allowed the US to seize and maintain the initiative on the battlefield.

3.4. Preliminary analysis

In the battle of Iwo Jima, assuming the result of the fight depends on the forces put into the war and the combat coefficient, and the casualties are only driven by fighting. At the same time, according to Lanchester Laws, combat abilities participating force x combat effectiveness.

Simplification of the Battle of Iwo Jima: first, soldiers from both sides

Forces are exposed to each other. Second, this is a fight between man and man or weapons and weapons. Third, the average loss rate of the two sides of the war is constant. Fourth, for both sides, non-combat attrition is negligible.

3.5. Lanchester Law:

$$\begin{cases} \dfrac{dx(t)}{dt} = -a \\ \dfrac{dy(t)}{dt} = -b \end{cases} \tag{1}$$

In function 1, x and y are the remaining forces at time t, and a and b are the combat coefficients.

According to the situation, factors that can influence the battle are listed below:

Table 1 Related Factors

forces	Quality of weapons	Armor forces	Quality of armor forces	Supply guarantee
m	l	t	o	n

Based on these factors and the Lanchester Law, we can establish the following equations:

$$\begin{cases} \dfrac{dm(t)}{dt} = -k_m n - m_n(t) - l_n(t) - t_n(t) - o_n(t) \\ \quad -u_n(t) - s_n(t) - h_n(t) - z_n(t) - v_n(t) \\ \dfrac{dn(t)}{dt} = -k_n m - m_m(t) - l_m(t) - t_m(t) \\ -o_m(t) - u_m(t) - s_m(t) - h_m(t) - z_m(t) - v_m(t) \end{cases} \quad (2)$$

In function 2, m(t) and n(t) represent the instantaneous forces. kmn 、knm each represents the combat coefficient of the US Army and the Japanese Army.

According to the above statements, the US Army was superior to the Japanese Army in all dimensions. Thus, in the battle of Iwo Jima, the US Army was an absolute advantage.

Based on the above analysis, we can make some assumptions.

Assumption 1: the advantages of the US Army have been evenly matched by the Japanese Army.

In other words, the coefficient of m,l,t,o,n is 1.

Function 2 can be simplified as:

$$\begin{cases} \dfrac{dm(t)}{dt} = -k_m n \\ \dfrac{dn(t)}{dt} = -k_n m \end{cases} \quad (3)$$

Assumption 2: the combat coefficients are equal. kmn = knm.

Solve for function 2:

$$\begin{cases} m = m_0 - k_m n \times t \\ n = n_0 - k_n m \times t \end{cases} \quad (4)$$

After substituting the number of forces of the Japanese and the US Army, which m0=23000, n0 =11000, we can calculate that the casualty of this battle deviated from the historical data. The calculated casualty ratio of the US VS Japan is 1:5.5, while the virtual casualty ratio is 1.5:1.

Some essential factors are disregarded.

Introduce factor R, which combines the influence of terrain and tactical strategy.

While the assumption 1 and 2 remain the same, replace function (4) with the following one:

$$\begin{cases} m = m_0 - k_m n \times t \\ n = n_0 - (k_n m \times t) R_M \end{cases} \quad (5)$$

Table 2 Effect of R

Japanese Casualty (ascertained)	Theoretical Casualty ratio	Theoretical casualty of US	Virtual casualty of US	Effect of R
22703	1:5.5	4127	28686	6.95

According to the above assumptions, while not considering the effect of R, the casualty ratio between the US and Japan should be 1: 5.5, which is not accurate.

R, the effect of terrain and tactical strategy, should have raised the Japanese Army combat efficiency by 6.95 times. (28686÷4127=6.95)

3.6. Combat model based on Lanchester Square Law

Compared with the Lanchester linear Law, the Lanchester Square law could align with the modern battle environment better. The condition of its uses is limited. First, both sides need to have weapons that have similar firepower. Second, both sides are exposed to each other. Third, each side can concentrate its power on a small force of the other side.

Lanchester Square Law:

$$\begin{cases} \dfrac{dx(t)}{dt} = -ay(t) \\ \dfrac{dy(t)}{dt} = -bx(t) \end{cases} \quad (6)$$

In function (6), x and y represent the remaining forces at time t. a and b are the number of casualties that 1 unit of force could cause per unit time.

The remaining force at time t:

$$\begin{cases} x(t) = x_0 ch(\sqrt{abt}) - (\sqrt{\dfrac{a}{by_0}} sh\sqrt{abt}) \\ y(t) = y_0 ch(\sqrt{abt}) - (\sqrt{\dfrac{a}{ba_0}} sh\sqrt{abt}) \end{cases} \quad (7)$$

Specifically:

$$sh(x) = (e^x - e^{-x})/2 \quad (8)$$

$$ch(x) = (e^x + e^{-x})/2 \quad (9)$$

We can establish the following function:

$$\begin{cases} \dfrac{dm(t)}{dt} = -k_1 \cdot n(t) \cdot \sum n \\ \dfrac{dn(t)}{dt} = -k_2 \cdot m(t) \cdot \sum m \end{cases} \quad (10)$$

Σm and Σn is the sum of factors that can influence the result.

$$\sum \mathrm{m} = m_m(t) + l_m(t) + t_m(t) + o_m(t) + u_m(t) + s_m(t) + h_m(t) + z_m(t) + v_m(t) \quad (11)$$

$$\sum n = m_n(t) + l_n(t) + t_n(t) + o_n(t) + u_n(t) + s_n(t) + h_n(t) + z_n(t) + v_n(t) \quad (12)$$

According to the first prerequisite, we still can assume that the ratio of the US and the Japanese's coefficient of weapons

is 1:1. Intended to control other factors, we weakened the advantage that the US Army has, giving those the coefficient of 1.

Turn function (8) into:

$$\begin{cases} \dfrac{dm(t)}{dt} = -k_1 \cdot n(t) \\ \dfrac{dn(t)}{dt} = -k_2 \cdot m(t) \end{cases} \tag{13}$$

T is the battle time, and n and m each represent the Japanese army and the US Army's remaining force and time t. K1 and K2 are the average combat coefficient of each side's soldier.

Integral function (11):

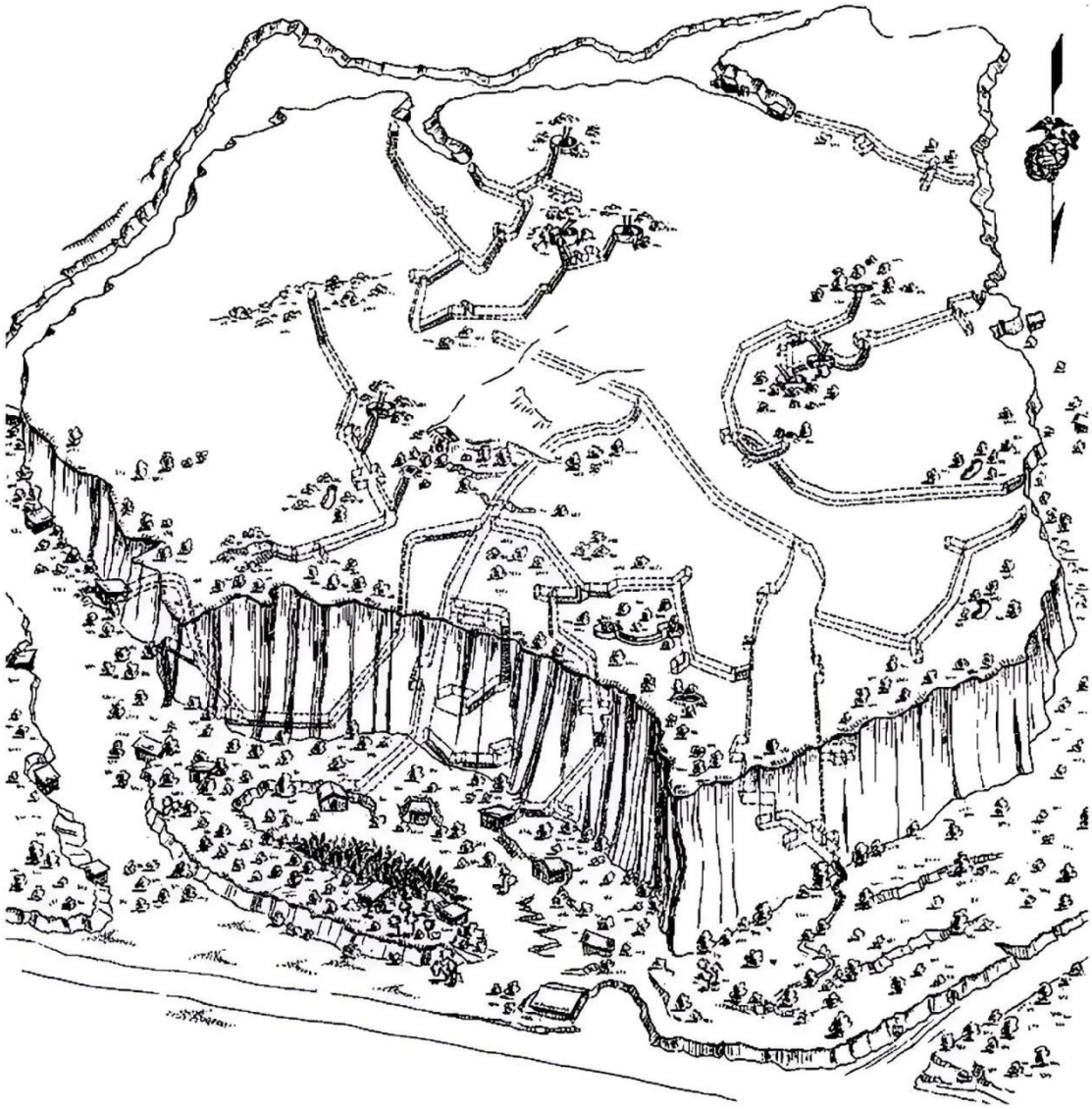
$$k_1 \cdot \left(n_0^2 - n^2\right) = k_2 \cdot \left(m_0^2 - m^2\right) \tag{14}$$

n0 and m0 each represent the initial force of the US and the Japanese.

3.7. Reality Consideration

3.7.1. Terrain

During the Battle of Iwo Jima, the Japanese forces constructed an extensive underground tunnel network that played a pivotal role in their defense against the U.S. invasion. This tunnel system spanned over 18 kilometers (approximately 11 miles), connecting more than 1,500 underground rooms, bunkers, and pillboxes across the island (Ross, 1985[21]). Some tunnels reached depths of up to 25 meters below the surface, allowing Japanese troops to withstand intense naval bombardments and aerial attacks (Garand & Strobridge, 1971[20]). Approximately 21,000 Japanese soldiers utilized these subterranean passages to move troops and supplies covertly, launch surprise attacks, and coordinate defensive strategies (Ross, 1985). The tunnels provided protection and significantly prolonged the battle by enabling the Japanese to continue fighting even after surface positions were compromised, making it exceedingly challenging for U.S. forces to secure the island (U.S. Marine Corps Historical Division, 1954[22]).



Tunnels dug by the Japanese Army (Sketch of Hill 362A, made by the 31st U.S. Naval Construction Battalion. Dotted lines show the Japanese tunnel system.)

3.7.2. Tactic

One consequential strategy was the decision not to engage U.S. forces during the initial amphibious landing. Under the command of General Kuribayashi, the Japanese troops waited for the Americans to move further inland before launching their attacks. This tactic, often referred to as "defense-in-depth," allowed the Japanese to conserve their firepower and maximize their defensive capabilities from well-fortified positions. The Japanese defenders, hidden in caves, bunkers, and an intricate 18-kilometer tunnel network, launched their counterattacks only after U.S. forces had advanced into more vulnerable terrain (Britannica, 2023; Warfare History Network, 2023). This approach contrasted sharply with previous Pacific battles, where defenders typically tried to repel forces directly at the beaches. By holding their fire, the Japanese inflicted heavier casualties on the Americans, as they were able to concentrate their fire on exposed troops already bogged down by difficult terrain like volcanic ash (Warfare History Network, 2023). The strategy was effective in slowing the U.S. advance and prolonging the battle.

Additionally, following the landings, the Marines anticipated the customary Japanese banzai attack (suicidal attack) at night. This had been the typical Japanese last defense tactic in past conflicts with enemy troops in the Pacific, like during the Battle of Saipan. During those attacks that the Marines were ready for, most of the Japanese attackers were killed, and the Japanese force was significantly weakened. Nevertheless, General Kuribayashi had completely banned the "human wave" assaults by Japanese infantrymen due to his belief that they were pointless (Allen, [25]).

Based on the quantitative analysis that is based on Lanchester's Law, I found out that the effect of terrain and tactical strategy could raise combat effectiveness by 6.95 times.

3.8. Limitations and Expectations:

To make calculations easier and idealize the model, I made some assumptions that might be impractical in reality. For instance, the US Army was almost in advantageous in any aspect. Although I assigned the same combat coefficient for the US Army and the Japanese Army, terrain and tactical strategy still had a positive effect 6.95 times. The virtual effect of terrain and tactical strategy may be higher than 6.95. Also, the model can be improved by incorporating additional factors, such as climate conditions and political context, which also play critical roles in warfare outcomes.

4. Conclusion and expectations:

By reviewing existing research that employs the Markov Model to analyze wars, I identified its limitations in capturing the full complexity of conflict dynamics. After collecting and processing relevant historical data, I visualized key aspects of the Battle of Iwo Jima, providing clearer insights into the war's dynamics. By applying the Lanchester Model, I quantitatively modeled the casualties and interactions between the US Army and the Japanese Army during the battle of Iwo Jima and emphasized the importance of terrain and tactical strategy. Moving forward, it is essential to develop a more comprehensive model that accounts for these harder-to-quantify variables, allowing for broader application across different historical and modern conflicts.

Reference

[1] Clausewitz, C. von. (1976). *On War* (M. Howard & P. Paret, Eds. and Trans.). Princeton. University Press.

[2] Kaldor, M. (1999). *New and Old Wars: Organized Violence in a Global Era*. Stanford University Press.

[3] Howard, M. (1976). *War in European History*. Oxford University Press.

[4] Gat, A. (2008). *War in human civilization*. Oxford University Press.

[5] Chin, W. (2019). Technology, war and the state: past, present and future. International Affairs, 95(4), 765–783. https://doi.org/10.1093/ia/iiz106

[6] Black, J. (2013). War and Technology. Indiana University Press.

[7] Morrock, R. (1973). Heritage of Strife: The Effects of Colonialist "Divide and Rule" Strategy upon the Colonized Peoples. Science & Society, 37(2). https://philpapers.org/rec/MORHOS

[8] Shallcross, N. (2016). A Logistic Regression and Markov Model for the Prediction of Nation-State Violent Conflicts and Transitions. ResearchGate, https://doi.org/10.13140/RG.2.1.1299.1762

[9] Kiefer, D. (1988). Interstate Wars in the Third World: A Markov Approach. Conflict Management and Peace Science, 10(1), 21–36. https://doi.org/10.1177/073889428801000102

[10] Shearer, R. (2007). Forecasting Israeli-Palestinian Conflict with Hidden Markov Models. Military Operations Research, 12(4), 5–15. https://doi.org/10.5711/morj.12.4.5

[11] Schrodt, P. A. (2006). Forecasting Conflict in the Balkans using Hidden Markov Models. In Springer eBooks (pp. 161–184). https://doi.org/10.1007/1-4020-4390-2_8

[12] Duncan, G. T., & Siverson, R. M. (1975). Markov Chain Models for Conflict Analysis: Results from Sino-Indian Relations, 1959-1964. International Studies Quarterly, 19(3), 344. https://doi.org/10.2307/2600315

[13] Schrodt, P. A. (1997, January 1). Early Warning of Conflict in Southern Lebanon using Hidden Markov Models. In American Political Science Association. http://eventdata.parusanalytics.com/papers.dir/schro97b.pdf

[14] Spirling, A. (2007). "Turning Points" in the Iraq Conflict: Reversible Jump Markov Chain Monte Carlo in Political Science. The American Statistician, 61(4), 315–320. https://doi.org/10.1198/000313007x247076

[15] Schrodt, P. A. (2006b). Forecasting Conflict in the Balkans using Hidden Markov Models. In Springer eBooks (pp. 161–184). https://doi.org/10.1007/1-4020-4390-2_8

[16] Lindquist, J. M., Teutsch, C., & Torres, E. (2017). A New Use for an Old Tool: Markov Chains and the Afghanistan Warfighter. Military Operations Research, 22(2), 5–18. http://www.jstor.org/stable/26296091

[17] Burrell, R. S. (2004). Breaking the Cycle of Iwo Jima Mythology: A Strategic Study of Operation Detachment. The Journal of Military History, 68(4), 1143–1186. https://doi.org/10.1353/jmh.2004.0175

[18] Leckie, R., & Burwell, T. (2004). The Battle for Iwo Jimahttp://ci.nii.ac.jp/ncid/BA74057303

[19] Kress, M. (2020). Lanchester Models for Irregular Warfare. Mathematics, 8(5), 737. https://doi.org/10.3390/math8050737

[20] Garand, G. W., & Strobridge, T. R. (1971). *History of U.S. Marine Corps Operations in World War II, Volume IV:*

Western Pacific Operations. Washington, DC: U.S. Marine Corps Historical Division.

[21] Ross, B. D. (1985). *Iwo Jima: Legacy of Valor*. New York, NY: Vintage Books.

[22] U.S. Marine Corps Historical Division. (1954). *Iwo Jima: Amphibious Epic*. Washington, DC: U.S. Government Printing Office.

[23] Battle of Iwo Jima | Facts, Significance, Photos, & Map. (2023). Britannica.

[24] The Battle of Iwo Jima: Red Sun, Black Sand. (2023). Warfare History Network.

[25] Allen, R. E. (2004). *The first battalion of the 28th Marines on Iwo Jima: A day-by-day history from personal accounts and official reports, with complete muster rolls*. McFarland & Company.