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Study on the Influence of Gaussian Window Shaping Parameters on Amplitude Accuracy

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Abstract: To address the issue of accuracy in extracting the amplitude of electronic pulses outputted by photon-counting imaging detectors based on microchannel plates, this paper combines deconvolution technology with Gaussian window filtering to achieve Gaussian shaping of the pulses. Simulations were conducted using MATLAB to model the exponentially decaying signals outputted by a charge-sensitive amplifier. By adjusting the pulse width (L) and Gaussian shape parameter (α), the amplitude-frequency response of the shaping system was observed. The amplitude-frequency response patterns of the Gaussian window were obtained.

Keywords: Gaussian Window, Deconvolution, Photon Counting, Pulse Shaping.

1. Introduction

Photon-counting imaging detection plays a crucial role in detecting low-light environments, such as those in space and the ocean. When a low-light target is incident on a photoelectron multiplier device (such as a microchannel plate) in front of the detector, single photons undergo photoelectric multiplication, and the emitted electron cloud strikes a position-sensitive detector. The charge is then amplified by a preamplifier (CSA), generating a sequence of exponentially decaying pulses. By processing these signals, low-light targets can be reconstructed. The entire signal processing involves noise filtering, pileup identification, and amplitude extraction. Digitizing and shaping the signal pulses can achieve superior processing results for these tasks.

Currently, commonly used shapers include the CR-RCn shaper and the Sallen-Key shaper. Both of these shapers can produce asymmetric, Gaussian-like pulses with smoother tops compared to the original input signal, enabling more accurate pulse amplitude extraction. Additionally, synthetic shaping methods, as designed in references, are also common. By adjusting geometrically meaningful shaping parameters, various shapes such as trapezoidal, triangular, and Gaussian can be designed.

In different photon counting rate environments, an appropriate shaping pulse width can reduce both pulse pileup and the impact of noise on pulse amplitude accuracy. Under fixed pulse width conditions, selecting the optimal pulse shape can achieve the highest pulse amplitude accuracy. Therefore, this paper analyzes the influence of different shaping parameters on Gaussian shaping from a frequency domain perspective and examines the relationship between Gaussian shape parameters and amplitude accuracy when the Gaussian window length is fixed.

2. Gaussian Shaping Algorithm

The pulse shaping method employed in this paper combines deconvolution technology with Gaussian window filtering. The first step involves deconvolving the double-exponential signal outputted by the detector to obtain an impulse signal. The second step involves convolving the impulse signal with a Gaussian window to obtain a Gaussian pulse shape, as shown in the figure 1.

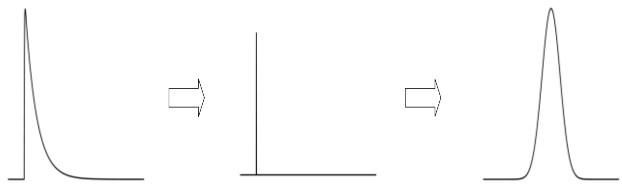

Figure 1. Pulse Shaping Process Flowchart

2.1. Double Exponential Decay Signal

It is known that the exponentially decaying signal outputted by the CSA can be approximated using a double exponential signal model, which features a rapid rise and a slower decay, as shown in Equation (1).

$$v(t) = A\left(e^{-\frac{t-t_0}{\tau_1}} - e^{-\frac{t-t_0}{\tau_2}}\right)u(t-t_0) \tag{1}$$

In this equation, τ1 and τ2 represent the decay time constants of the detector signal, with the general requirement that τ1 ≫ τ2. A denotes the signal amplitude, t is the signal duration, u(t-t0) is the step signal, and t0 is the signal arrival time. Subsequently, the signal is discretized, as represented by Equation (2), with Ts denoting the sampling period.

$$v(t) = A\left(e^{-\frac{t-t_0}{\tau_1}} - e^{-\frac{t-t_0}{\tau_2}}\right)u(t-t_0) \tag{2}$$

2.2. Deconvolution Processing

Through deconvolution operations, the double exponential signal can be converted into an impulse signal. The position of this impulse signal corresponds to the starting point of the double exponential signal, enabling precise localization of the pulse. The transfer function of this system is represented by

Equation 3.

$$H_1(z) = \frac{(1 - M_1 z^{-1})(1 - M_2 z^{-1})}{(M_1 - M_2) z^{-1}} \quad (3)$$

In the equation, $M_1 = \exp(-T_s/\tau_1)$ and $M_2 = \exp(-T_s/\tau_2)$. By performing the inverse Z-transform on Equation (3), we obtain the recursive equation for impulse shaping, as represented by Equation (4).

$$\delta[n] = \frac{v[n+1] - (M_1 + M_2)v[n] + M_1 M_2 v[n-1]}{M_1 - M_2} \quad (4)$$

2.3. Gaussian Window Shaping

The standard Gaussian window function is given by:

$$w\left(n + \frac{(L-1)}{2}\right) = \exp\left(-\frac{1}{2}\left(\alpha \frac{n}{\frac{(L-1)}{2}}\right)^2\right) = exp\left(-\frac{n^2}{2\sigma^2}\right) \quad (5)$$

Where the window length is L-1, and there are L sampling points.

$$-\frac{(L-1)}{2} \leq n \leq \frac{(L-1)}{2} \quad (6)$$

α is the shape parameter of the Gaussian window, which determines the overall shape of the Gaussian window. σ is the standard deviation of the Gaussian curve.

$$\sigma = \frac{(L-1)}{2\alpha} \quad (7)$$

Gaussian windows with different shaping parameters are illustrated in the figure.

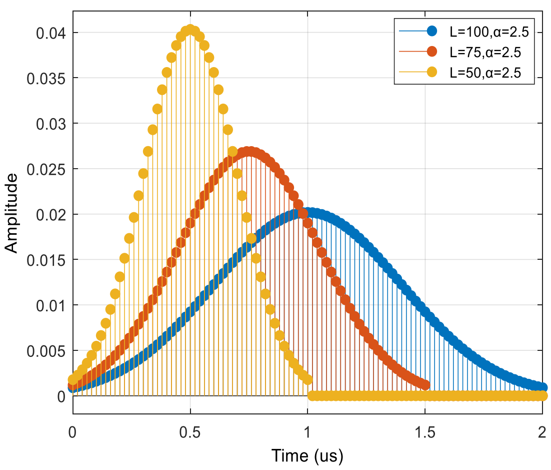

Figure 2: Gaussian Window Impulse Response with Different Shaping Widths L

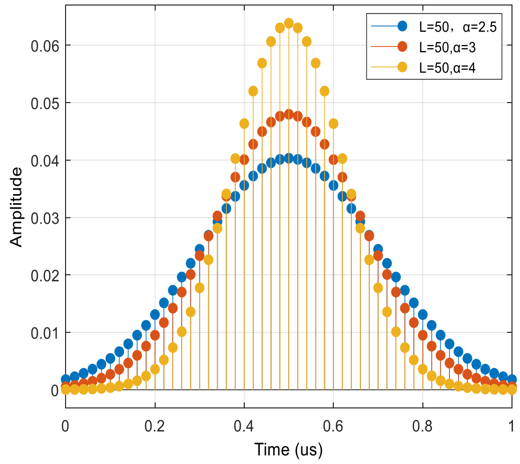

Figure 3: Gaussian Window Impulse Response with Different Shaping Parameters α

3. Simulation Experiment

In the simulation, different pulse amplitudes A were set, following a Gaussian distribution with a mean of 10, a standard deviation of 2, and a minimum amplitude of 1. A total of 50,000 pulse samples were generated. White noise was added with a standard deviation of 0.1 and a mean of 0. The sampling frequency was set to 5x10^7 Hz, resulting in a sampling interval of T = 20 ns. The input pulse count rate was 250 kcps, and the time constants were set to 83T and 1.8T, respectively. The pulse intervals followed a Poisson distribution, with a minimum pulse interval limited to 2 times the sampling interval. Pulses with intervals less than 1 times the sampling interval were considered as the same pulse, while those with intervals greater than 1 but less than 2 times the sampling interval were deemed as indistinguishable.

The simulation results are shown in Figure 4. The Gaussian-shaped pulses exhibited smoother and more symmetrical characteristics compared to the noisy double-exponential signals. The slower decay edge of the double-exponential pulses was compressed, and pulse shaping achieved good pulse separation.

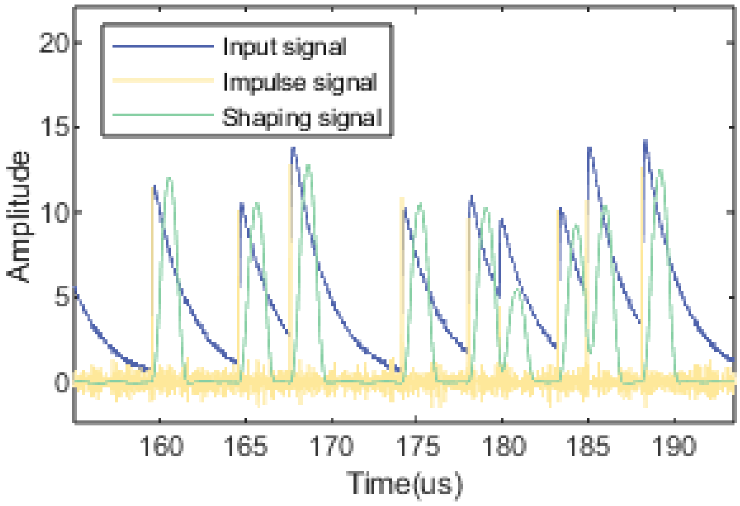

Figure 4: Simulated Exponential Decay Pulse Signal, Deconvolved Signal, and Shaped Pulse Signal

The simulation results indicate that deconvolution can achieve precise localization of pulses while also highlighting high-frequency noise signals. By setting a threshold, the location of pulse occurrence can be determined, enabling the central localization and amplitude extraction of the synthesized shaped pulses. After pulse shaping, the tailing of the exponential signal is significantly shortened, and the shaping width of each pulse is equal and satisfies the set Gaussian width.

Pulse localization can be achieved using the impulse signal from deconvolution, and the degree of pile-up can be inferred through pulse intervals to implement pile-up rejection. However, the deconvolution process has high-pass filtering properties, which amplify the noise in the impulse signal. Therefore, an appropriate pulse identification threshold A ts needs to be preset. Pulses with excessively small amplitudes, after deconvolution, may have their impulse signals submerged in noise, resulting in missed pulse identification. Conversely, setting the identification threshold too low may lead to false identification of pulse signals due to excessive noise. As shown in Figure 3, when the identification threshold is at position a, it may result in missed identifications, and when the threshold is at position b, it may lead to false identifications.

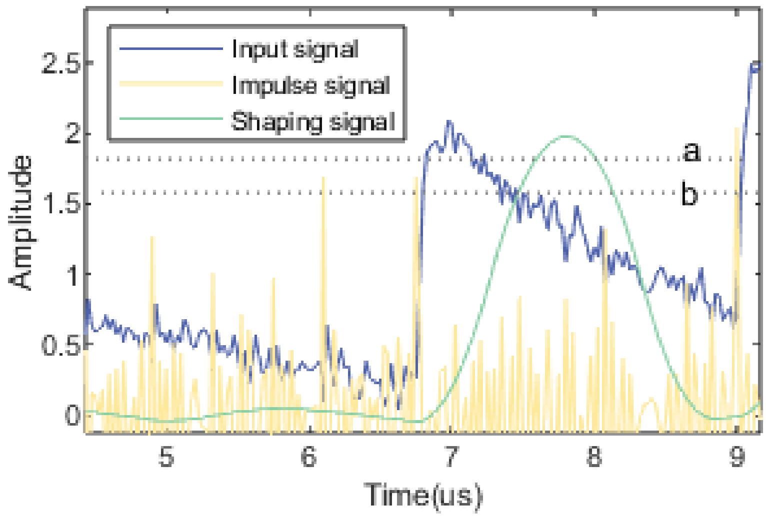

Figure 5: Comparison of Recognition Thresholds for Deconvolved Signals

Excessive missed identifications can lead to incorrect judgments of pile-up states, where pulses that should be considered as piled-up are mistakenly identified as not piled-up and subsequently included in the counting, which reduces the overall amplitude extraction accuracy. False identifications increase the count of invalid zero-amplitude pulses, and if they affect the pile-up judgment, it may result in valid pulses being mistakenly identified as piled-up and excluded from counting. Under the current simulation conditions, the number of missed identifications and false identifications under different threshold conditions was analyzed using prior true pulse information. See Table 1 for details.

Table 1: Relationship Between Recognition Thresholds, Leakage Misidentification Rate, and False Identification Rate

Recognition Threshold	Leakage Rate	False Rate
2	0.012%	0.23%
2.5	0.014%	0.044%
3	0.018%	0
3.5	0.050%	0
4	0.164%	0

4. Amplitude-Frequency Response of Gaussian Window Shaping

By adjusting the width L of the Gaussian window and the shaping parameter α, we can observe the amplitude-frequency response of the Gaussian window shaping system, as shown in Figure 6.

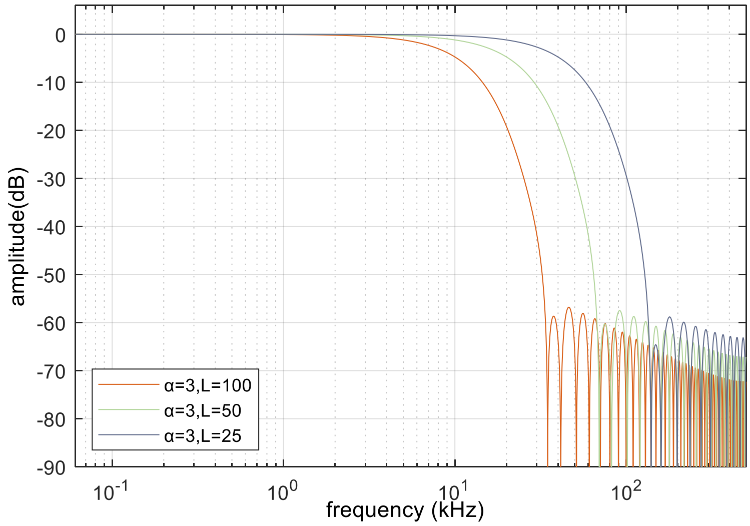

Figure 6: Amplitude-Frequency Response of Gaussian Window at 1 MSa/s with (α = 3) and (L = 100, 50, 25)

Under the same condition of α, the cutoff frequency (Fc) decreases as the pulse width (L) increases. While increasing the shaping width can enhance the suppression of high-frequency noise and improve the signal-to-noise ratio (SNR), it also leads to an increased degree of pulse pile-up.

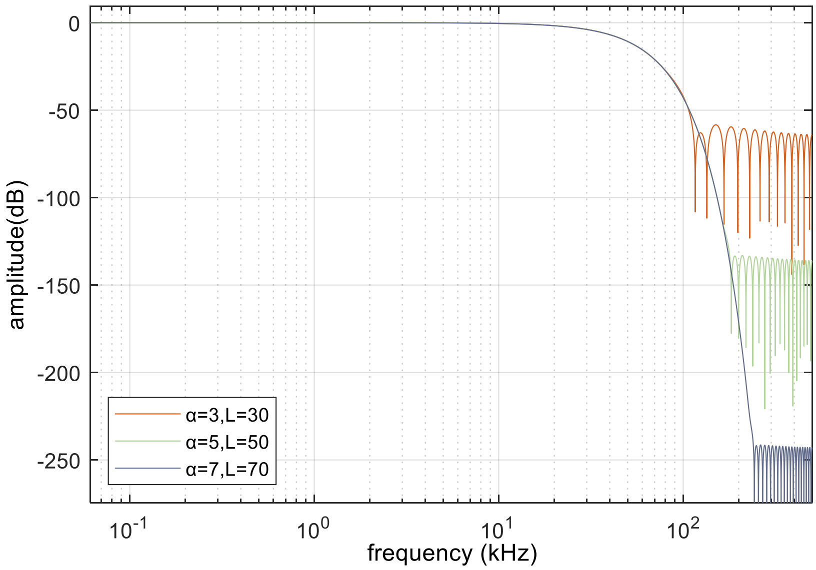

Figure 7: Amplitude-Frequency Response of Gaussian Window with Fixed Ratio of (α) to (L) at 1 MSa/s

Under the condition of a fixed ratio between (α) and (L), the cutoff frequency (Fc) shows minimal variation, while the transition band widens and the attenuation in the stopband is enhanced. This indicates that the shaping width (L) and the shaping parameter (α) can be determined based on the high-frequency component of the actual detection signal. In other words, the accuracy of pulse amplitude extraction is jointly influenced by both (α) and (L). Considering computational efficiency, it is advisable to fix (L) and adjust (α) to achieve optimal amplitude accuracy.

5. Conclusion

In this study, we implemented Gaussian shaping of simulated double exponential signals using deconvolution techniques and Gaussian window filtering. This approach effectively mitigated the effects of pulse pile-up and provided robust noise suppression. Additionally, by employing deconvolution for pulse localization and identification, we achieved an optimal recognition threshold that resulted in a leakage misidentification rate of only 0.0018% and a false identification rate of 0%. Through frequency domain analysis, we examined the amplitude-frequency response curve of the

Gaussian shaping system and found that increasing the Gaussian shaping width can lead to improved suppression of high-frequency noise.

Acknowledgment

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