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Geometric Analysis of Ballet Path Based on Least Squares Method

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Abstract: Mathematics and ballet are unrelated seemingly, but a closer look reveals profound similarities between them. Both are based on precision and strict control, and ballet movement gestures and mathematical equations demonstrate rigorous geometric design and systematic mechanisms together. This study aims to explore how geometric methods can be used to optimise the movement and step paths in ballet, thereby enhancing the symmetry and aesthetics of the dance. By applying combinatorial mathematics and geometry, this study analysed the movement arrangements and paths of the Bluebird choreography, using least squares to fit geometric shapes (e.g. squares, circles, ellipses and triangles) to optimise the dance movements. In the experiment, the dance movements were recorded and analysed using advanced mathematical modelling and motion capture techniques to measure the error between the geometric shapes and the actual dance movements. The results of the study showed that mathematical optimisation significantly enhanced the beauty and expressiveness of the dance choreography. This study revealed the mathematical aesthetics in the art of dance and provided a new perspective for future choreography

Keywords: Ballet, Dance Path Analysis, Geometry, Least Squares

1. Introduction

The relationship between choreography and geometry is very subtle and profound [1-2]. Choreography is not only the design and arrangement of movement, but also the artistic use of space. With the geometric shapes, choreographers can add new dimensions to dance, making it more expressive and structural [3].

First, the movements and formations in dance can often be described by geometric shapes. For example, a choreographer might arrange dancers in a circle, square or triangle; these geometric shapes help to create visual balance and harmony. Dancers moving through these shapes can enhance the rhythmic and visual impact of the dance.

Second, the geometric shapes also affect the fluidity and dynamics of the dance. Dance paths and formation changes often follow the geometric principles such as straight lines, curves or rotation. These shapes not only guide the movement paths of the dancers, but also determine the overall style of the dance. For example, spiral movements can convey a sense of rotation and constant change, while straight lines can demonstrate strength and stability. Finally, the use of geometric shapes can also help the choreographers create dance images with depth and layers in space. The carefully designed spatial layout, allows the choreographers to direct the audience's attention and make the dance more visually compelling. Through these geometric forms, dance is not just about the performance of movement, but also becomes a display of spatial art.

Ballet as a category of dance, and although mathematics and even geometry are seemingly unrelated to ballet, a closer look reveals fascinating similarities between the two. Both focus on precision and tight control, and this precision lays the foundation for creativity and free expression in the art [3]. Every gesture in ballet and every equation in mathematics is organised and guided by a system of symbols that represent rigorous assignments, precise geometric designs and systematic physical mechanisms. This relationship, revealed by observing the complex structure of nature, has not only inspired the mathematicians to develop theories to unravel the mysteries of the world, but has also allowed ballet to reflect the profound connection between humanity and the digital world via the meticulous reproduction of natural patterns.

In the art of ballet, the mathematical element is reflected in the order and details of human movement. Dancers must not only understand the segment and geometric patterns, but also have a keen sense of spatial awareness to ensure the harmonious beauty of the performance and to avoid unnecessary body collisions. When training complex techniques, dancers must follow the strict technical standards to ensure symmetry in posture, such as the octagonal method. Dance schools such as Cecchetti and Vaganova emphasise the symmetry and beauty, which further demonstrated the critical position of mathematics in ballet. While mathematics is commonly used to understand physics, biology and economics, it is rarely used in aesthetics. The current study aimed to fill this gap by exploring how mathematics can be used to improve choreography and develop new laws of choreography through the study of ballet. Using methods from combinatorial mathematics and graph theory, this study is dedicated to optimising the movement and formation choreography to improve the symmetry and beauty of dance. Taking the dance song "Bluebird" as a case study, the principles of this pleasant and beautiful choreography are summarised by analysing its movement arrangement and path.

In further exploration, the study will combine the dance and mathematics for a deeper understanding of the structure and inherent aesthetic principles of dance movements through geometric and mathematical analysis. With the help of advanced mathematical models, algorithms, and motion capture technology, we can accurately record and analyse dance movements with the aim of enhancing their aesthetics and performance [7]. This analysis not only supports the teaching and training of dance, but also provides a new perspective on creativity. The beauty of the science of dance art will ultimately be better revealed by geometric analysis.

2. Related works.

2.1. Analysis of geometric elements in Zapin Pat Lipat dance

This study focused on the floor patterns in the Zapin Pat Lipat dance [4], aiming to identify and analyse the geometric elements in them, and exploring how these geometric shapes contribute to the harmony, rhythm and visual appeal of the dance. The research methodology included constructing floor patterns from observations of the dance, identifying the geometric elements in the dance, and analysing the dance with a focus on polygons and straight lines using types of geometry such as site geometry, architectural geometry, differential geometry, descriptive geometry, and analytic geometry. The results of the study show that the Zapin Pat Lipat dance has unique geometric element that highlight the unique connection between mathematics and dance. Floor patterns are critical to the orderliness and coordination of the dancers' movements, and the floor patterns of the dance contain a variety of geometric shapes, especially 2D polygons and straight lines, which play a key role in the formation and execution of the dance.

2.2. Choreography inspired by mathematical concepts

This study examined how to create a dance performance based on mathematical concepts and their historical development, and explored how mathematical ideas can be expressed by dance [5]. The study includes the process of developing the "point without a part" of a dance piece based on mathematical concepts. By analysing the development of mathematical concepts and their impact on the modern world, the study included the performance in the Serbian Mathematics Month, involving public participation and science communication to broaden the attention and understanding of mathematics. The results of the study show that the dance performance "Dot No Part" effectively combines mathematical concepts with artistic expression and demonstrates the artistic manifestation of mathematical ideas. The performance demonstrated the development of important mathematical concepts and their impact on today's technology-driven world, and emphasised the value of interdisciplinary collaboration, successfully achieved by bringing together mathematicians, choreographers, dancers, science communicators and designers. In addition, presentations at Maths Month events will highlight the effectiveness of innovative approaches in increasing public interest in and understanding of mathematics.

2.3. Dance as a medium for expressing abstract mathematical concepts

This study explored how dance can be used as a medium to effectively represent abstract mathematical concepts, and in particular how Sarah Chase presents mathematical ideas such as number theory by dance teaching [6]. The research methodology included analysing Sarah Chase's approach to teaching dance, observing and describing her cross-modal movement and verbal narrative approach, and presenting her teaching through short films that provide hands-on demonstrations and visual presentations. The results of the study show that dance effectively interacts with abstract mathematical concepts on a sensory and physical level, providing an alternative to traditional abstract teaching methods. Chase's work demonstrates how dance can be integrated with mathematical concepts to enhance understanding and retention of abstract concepts with multi-sensory experiences.

The three studies described above demonstrated the rich interaction between mathematics and dance. The first study revealed the geometric elements in dance floor patterns and their impact on dance performance; the second study examined how mathematical concepts could inspire dance creations and disseminate mathematical knowledge among the public; and the third study demonstrated how dance can be used as a medium to effectively express abstract mathematical concepts. Although these studies provided insights, precise computational tools have not yet been employed to analyse the relationship between dance movements and pathways. Therefore, this work aims to perform a more precise analysis using computer software such as MATLAB to further explore the mathematical structure and representation of dance movements.

3. Research Methods

3.1. Different types of geometric shapes

Ballet choreography often involves complex dance formations and sequences that can be described and analysed using geometric shapes. Circular formations: The dancers can form circular or spiral formations, and these geometric shapes can create visual balance and harmony in the dance. Squares and rectangles: In some dances, dancers form square or rectangular arrangements, and this geometric arrangement helps to show the symmetry and stability of the dance. Straight lines and diagonals: Arranging dancers along straight lines or diagonals can enhance the sense of dynamism and space in the dance.

In ballet solos, we walk across the stage, taking specific paths to express emotions and situations. However, the paths we have taken are often irregular. At this point, we need to split the paths and use different regular geometries to fit the trajectories of the dance paths in order to analyse the deviation between the actual paths of the dance and the regular geometries. In this section we choose four types of regular geometries to analyse.

3.1.1. Square fitting:

A square is a special type of quadrilateral. A square has four equal sides. The length of the sides determines the size of the square; the longer the sides, the larger the area and perimeter of the square. In terms of angles, all four corners of a square are right angles, that is, 90 degrees. This makes a square stable and regular.

Assume the side length of a square is a, with the central point is located at (x_c, y_c). If the sides of a square are parallel to the coordinate axis, the equation of the square is as below:

$$x_c - \frac{a}{2} \le x \le x_c + \frac{a}{2} \quad (1)$$

$$y_c - \frac{a}{2} \le y \le y_c + \frac{a}{2} \quad (2)$$

3.1.2. Rectangular fitting:

A rectangle is a special type of quadrilateral of which all four angles are right angles (90 degrees). Rectangles have important properties and applications in geometry. The diagonals of a rectangle are equal in length and bisect each other. The lengths of the diagonals can be calculated from the lengths of the sides of the rectangle.

For a rectangle with centre in coordinates(x_c, y_c), width ofw, height of h, with the side parallel to the coordinate axis, the standard equation is:

$$|x - x_c| \leq \frac{w}{2} \quad (3)$$

$$|y - y_c| \leq \frac{h}{2} \quad (4)$$

This means that the horizontal boundary of the rectangle is between $x_c - \frac{w}{2}$ and $x_c + \frac{w}{2}$, and the vertical boundary is between $y_c - \frac{h}{2}$ and$y_c + \frac{h}{2}$.

3.1.3. Circular fitting:

A circle is a flat figure. Morphologically, a circle is an enclosed figure surrounded by a curve. This curve is called the circumference of the circle, and any point on the circumference is equidistant from the centre of the circle; this distance is called the radius. The radius determines the size of the circle; the larger the radius, the larger the circle. This shape of a circle makes it uniquely advantageous in many situations, such as the ability to remain smooth and fluid when rolling.

The standard equation is applicable when the centre and radius are known. The standard equation for a circle is:

$$(x - x_c)^2 + (y - y_c)^2 = r^2 \quad (5)$$

In which, (x_c, y_c) is the coordinate of the centre of the circle,r is the radius of a circle. This equation represents all points on the plane (x_c, y_c) at a radius of r from the centre of the circle.

3.1.4. Ellipse fitting:

An ellipse is a flat figure. Shape-wise, an ellipse is enclosed by a closed curve. It has two foci, and the sum of the distances from any point on the ellipse to the two foci is a constant value. The long axis is the longest diameter in the ellipse and the short axis is the diameter perpendicular to the long axis. The lengths of the long and short semi-axes determine the shape and size of the ellipse. The unique shape of an ellipse makes it valuable for special applications in certain fields.

When the long and short axes of an ellipse are parallel to the coordinate axes, the standard equation of the ellipse is:

$$\frac{(x - x_c)^2}{a^2} + \frac{(y - y_c)^2}{b^2} = 1 \quad (6)$$

In which, (x_c, y_c) is the centre of the ellipse, a is half the length of the long axis of the ellipse (i.e. long semi-axis), b is half the length of the short axis of the ellipse (i.e. short semi-axis), $a \geq b$, if $a = b$, the ellipse degenerates into a circle.

3.2. Least Squares Optimisation

The Least Squares Method (LSM) [7] is a mathematical optimisation technique for mathematical fitting and parameter estimation widely used in statistics, engineering and economics. Its basic purpose is to find the best parameter estimates by minimising the error between the predicted values and the actual observed values.

3.2.1. Basic principle

The core idea of least squares is to minimise the sum of the squares of the errors between the model predicted values and the actual observed data by adjusting the model parameters. The error usually refers to the difference between the predicted and actual values for each data point.

It finds the best functional fit to the data by minimising the sum of the squares of the errors. Given a set of data points, the actual observed values and the values predicted by the model, for a linear model of general form (a simple linear model is used here as an example), then the error. The objective of the least squares method is to find appropriate sum of parameters that minimises the sum of squares of the errors.

Curve fitting: When a functional relationship needs to be determined from a set of data points, the least squares method can be used for fitting. For example, in experimental data processing, trends and patterns in the data can be better understood by fitting a curve [8].

3.2.2. "Curve fitting performs two tasks:

First, when the functional relationship between a physical quantity and a physical quantity has been determined, but only the constants (and their specific forms) have not been determined, the optimal values of the constants are fitted based on the data points.

Second, if the functional relationship between the physical quantity and is unknown, an empirical formula for the relationship is fitted from the data points and the optimal values of the constants are found.

Given that the values of the known function $f(x)$ at several data points are $x_i\ (i = 1,2,\cdots,n)$ y_i , an approximation of the interpolation polynomial can be established $f(x)$ based on the interpolation principle. However, in scientific experiments and production practices, it is common that the function values on nodes are not very accurate, and these function values are obtained through experiments or observations, inevitably with measurement errors. If the approximate function curve obtained is required to pass through all points (x_i, y_i)accurately and without error, it will preserve all test errors in the curve. Therefore, we expect to construct an approximate function based on the given data, (x_i, y_i) without requiring the function to completely pass through all data points, but only requiring the obtained approximate curve to reflect the basic trend of the data.

Curve fitting function $\varphi(x)$ is not strictly required to pass through all data points, i.e. deviation (also known as residual) of fitting function $\varphi(x)$ at x_i:

$$\varepsilon_i = \varphi(x_i) - f(x_i)\ \ (i = 0,1,\cdots,n) \quad (7)$$

It is not strictly equal to zero, i.e. a system of contradictory equations. However, in order to make the approximate curve reflect the changing trend of the given data points as possible, it is necessary $|\varepsilon_i|$ to minimize it according to some measurement standard. If we denote a vector, $e = [\varepsilon_0, \varepsilon_1, \cdots, \varepsilon_n]^T$ we require that e its certain norm be minimized, $\|e\|$ such as the e1-norm $\|e\|_1$ or ∞- norm.

For the calculation, analysis, and application, $e$the 2-norm is usually solved.

$$\|e\|_2 = \left(\sum_{i=0}^{n} \varepsilon_i^2\right)^{\frac{1}{2}} = \left\{\sum_{i=0}^{n} [\varphi(x_i) - f(x_i)]^2\right\}^{\frac{1}{2}} \quad (8)$$

i.e.

$$\|e\|_2^2 = \sum_{i=0}^{n} \varepsilon_i^2 = \sum_{i=0}^{n} [\varphi(x_i) - f(x_i)]^2 \quad (9)$$

minimum. This type of fitting that requires the minimum sum of squared errors (deviations) is called the least squares method of curve fitting.

The essence is still to solve the least squares solution of the system of contradictory equations.

3.2.3. Taking a circle as an example

Using the least squares method to optimise the parameters of the circular equation. The standard equation for a circle is:

$$(x-a)^2+(y-b)^2=r^2 \tag{10}$$

In which, (a,b) is the coordinate of the centre of the circle,r is the radius of a circle. We need to find the most suitable a,b and r, to minimize the distance error between the given data point (x_i,y_i) and the fitted circle. The equation of a circle is a nonlinear second-order polynomial. For solving, we can rearrange the equation:

$$x_i^2+y_i^2-2ax_i-2by_i+a^2+b^2-r^2=0 \tag{11}$$

The objective function is to minimize the sum of squared residuals:

$$L(a,b,r)=\sum_{i=1}^{n}(x_i^2+y_i^2-2ax_i-2by_i-D)^2 \tag{12}$$

In which$D=a^2+b^2-r^2$,

Derive the three parameters separatelya,b,r, the linear equation system obtained is as follows:

$$\frac{\partial L(a,b,r)}{\partial a}=0 \tag{13}$$

$$\frac{\partial L(a,b,r)}{\partial b}=0 \tag{14}$$

$$\frac{\partial L(a,b,r)}{\partial r}=0 \tag{15}$$

Solve the linear equation system above, obtain the optimal parameters, $a,b,r$and the function expression of the circle.

3.3. Capturing Dance Movement

In this paper, the authors captured the dance movement trajectories by recording a dance movement themselves. The author's dance piece was recorded by himself using the front camera of his mobile phone and shows the classical ballet variation - The Bluebird. This dance is taken from the ballet Sleeping Beauty, which is also one of the common test pieces in the Grade 8 Northern Dance Ballet Exam. It presents the image of the lively and lovely Bluebird with a cheerful rhythm and beautiful melody.

At the beginning of the performance, the author entered in a leisurely and relaxed manner, and gradually as the music progresses, the author's emotions became more and more elevated. The dance contains many complex movements, such as fast turns, jumps and toe tapping techniques, requiring excellent body control and virtuoso skills to accurately portray the style and emotion of this dance. To perform this variation to perfection requires not only a deep understanding of the music and meaning of the dance, but also a focus on the expression of emotion. Solid training in basic skills, improvement in body flexibility and strength, and a precise understanding of the rhythm of the music are all key to a successful performance. Figure 1 below shows some screenshots of the dance movements.

Figure 1. Dance Movement Show Chart

4. Experiment

In this section, we provided the process of extracting dance movement data and local movements slicing, as well as the experimental design process and experimental data analysis results.

4.1. Extraction and slicing of dance movement data

In this experiment, with MediaPipe, the author extracted the dance movement data for analysis and optimisation of dance movements. MediaPipe provides excellent posture estimation and motion capture capabilities, especially human posture tracking, which can identify key points of the body (such as joints), which can be used for analysis and optimisation of dance movements. Here are the steps for extracting dance movement data using MediaPipe:

First, install MediaPipe and OpenCV libraries for image processing;

Import necessary libraries such as MediaPipe and OpenCV into the code;

Load dance video files with OpenCV and process the images frame by frame;

MediaPipe provided 33 key points of human posture, such as shoulders, elbows, knees, etc. You can obtain the coordinates(x, y, z) and visibility of each key point. In this experiment, we extracted the trajectory of the left foot landing point of the dance movement, and the corresponding motion trajectory diagram is shown in Figure 2.

Key point data can be saved in CSV files or other formats for further analysis and processing.

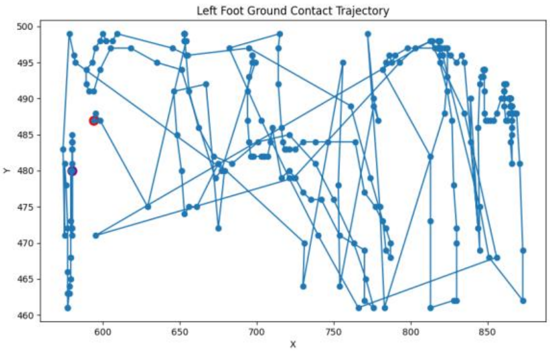
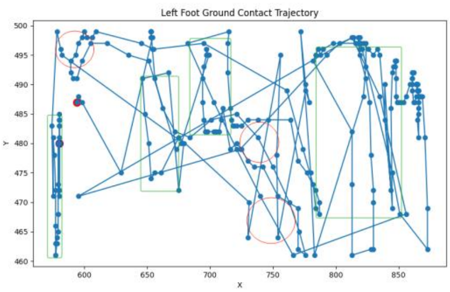


Figure 2. Ballet left foot path

4.2. Experimental procedure

From the trajectory diagram of the ballet dance movement obtained in Figure 2, the overall dance movement path is still a very complex curve. Therefore, it is very difficult to analyse the geometric law of the dance movement globally, so in this experiment, we extracted several local segments of the dance movement respectively, and used the regular geometric equations to fit the shapes of the movement segments to obtain the error data of each local movement segment compared with the regular geometric shapes, and the corresponding experimental results are shown in Figure 3 and Table 1.



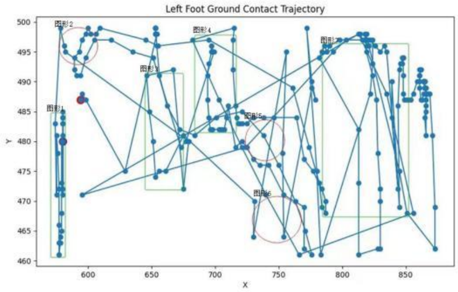


Figure 3. Geometric fitting of local movement segments

Table 1. Geometric fitting error of local movement segments

No.	Geometry	Movement segment frame rate	Fitting error
1	Rectangle	125-245	12.2%
2	Circle	270-370	7.8%
3	Rectangle	600-720	13.4%
4	Rectangle	1200-1500	14.1%
5	Circle	1620-1740	9.5%
6	Circle	1740-1960	12.5%
7	Rectangle	2200-2700	18.7%

From the experimental data above, we can see that we give a class of geometric quantification methods that can be used to calculate the error and the relationship between the actual dance trajectory and the regular geometry. In this experiment, we chose 7 groups of movement segments that are fitted with different geometries. Figure 3 shows a visualisation of the movement fitting, demonstrating the fitted shapes between the local movements and the regular geometries. The experimental data in Table 1 correspond to the fitting errors between different movement segments and regular geometries; the smaller the error, the closer the movement segments are to the geometries.

In addition, this method can fit any type of relationship or error between dance movement trajectories and standard geometric shapes. It can provide quantitative mathematical metrics for dance choreography, which can be used to provide mathematical guidance for dance choreography based on the assessed geometric errors, thus enhancing the mathematical beauty of dance movements.

5. Conclusion

This study used mathematical methods to explore the geometric aesthetics in the art of ballet, revealing a deep connection between mathematics and dance. The precision and specification of dance movement trajectories was improved by fitting geometric shapes (e.g. squares, circles, ovals and triangles) using the least squares method. The study combines the motion capture technology and mathematical modelling to provide a detailed analysis of the movement alignments and trajectories of the Bluebird dance piece, which not only accurately captures the dance movements, but also reveals the discrepancies between the geometric model and the actual dance segments. The results show that mathematical optimisation can significantly enhance the visual impact and expressiveness of choreography, providing a systematic optimisation solution for dance creation.

In the future, further research can be carried out to more ballet repertoires and dance styles to verify the applicability and effectiveness of mathematical optimisation methods in different dance forms. In addition, the combination of artificial intelligence and machine learning technologies for real-time dance movement analysis and optimisation can further enhance the quality of dance creation and performance. It is expected to reveal more mathematical laws in dance art with in-depth interdisciplinary research in the future, and promote the integration and development of dance art and science and technology.

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