Mathematical Modeling and Algorithm Application
ISSN: 3006-0842 | Vol. 4, No. 1, 2025

Optimal Celebrity Endorsement Decision and Pricing Strategy with Strategic Consumers

Peng Du

Business School, Nankai University, Tianjin 300071, P.R. China.

Abstract: Given the importance of determining the traffic level in celebrity endorsement management and prominent impacts from strategic consumers, this paper utilizes mathematical modelling to study the optimal traffic level and pricing strategy as well as relevant influencing factors in the context of celebrity endorsement. The results show that optimal traffic level is highest under uniform high pricing strategy and lowest under uniform low pricing strategy, with that of dynamic pricing strategy lying in between. Aside from the proportion of strategic consumers, optimal pricing strategy is also determined by the joint effects of celebrity endorsement characteristics, such as the premium effect of celebrity endorsement, the market demand expansion effect, and the traffic using cost. It depends on the tradeoff between the positive effects (premium effect and market demand expansion effect) and the negative effects (endorsement fee of unit traffic) of the celebrity endorsement.

Keywords: Pricing strategy; Celebrity endorsement; Traffic level; Strategic consumers

1. Introduction

Celebrity endorsement is a widely used marketing strategy to promote brand and product sales. With the celebrity endorsement, the brands can attract the fans of the celebrity that are originally not their customers to purchase the products, thus expand the market demand. Further, with the loyalty of the fans to the celebrity, the brands can even set a high price of their products to earn a premium. Apparently, the higher the traffic level of the endorsing celebrity, the more profit the brands can harvest from the prominent market demand expansion effect and premium effect. However, employing celebrities with high traffic level to endorse the brand will incur a large sum of endorsement fee, which can be a heavy burden for the brands. Therefore, determining the traffic level of endorsing celebrity is a critical issue for the brands.

Meanwhile, with the increased value of the product for the fans of the celebrity, the brands have strong incentives to adopt dynamic pricing strategy to harvest the premium from the celebrity's fans and simultaneously secure a large market demand. However, consumers are trained and learned more from these "tricks played by the brands". They tend to form an expectation of the fluctuation of the price in the future, and make purchase decisions according to intertemporal utility comparison. This type of consumers are called strategic consumers. In the presence of strategic consumers, the brands' profit with dynamic pricing can drop abruptly, which may even urge the brands to switch back to uniform pricing strategy. Under uniform pricing, the brands also have to trade off the value of the endorsing premium and the sales volume. Besides, the incentives to adopt higher traffic level vary largely under different pricing strategies. Therefore, strategic consumers imposes prominent impacts on brand's decision of celebrity endorsement and pricing strategy.

In view of this, this paper is motivated to explore the optimal traffic level and pricing strategy and relevant influencing factors in the context of celebrity endorsement. The main findings are listed as follows: Uniform high pricing strategy is equipped with the highest optimal traffic level due to the brand's strongest incentive to adopt high traffic level caused by largest premium-paying group of consumers. Optimal traffic level is lowest under uniform low pricing strategy, with uniform high pricing strategy lying in between. For this reason, uniform low pricing strategy is always dominated by dynamic pricing strategy. As for the comparison between uniform high pricing strategy and dynamic pricing strategy, conventional wisdom suggests that uniform pricing strategy should be adopted when consumers are highly strategic, or the proportion of strategic consumers is large. However, this paper finds that the premium effect of celebrity endorsement, the market demand expansion effect, and the traffic using cost also count in the context of celebrity endorsement. The rationale behind lies in the tradeoff between those positive effects brought by celebrity endorsement and the traffic using cost it incurs.

The remainder of the paper is constructed as follows: Section 2 reviews relevant literature. Section 3 describes the problem. Sections 4 and 5 analyze the cases of dynamic pricing strategy and uniform high and low pricing strategies respectively. Section 6 make comparative analyses among the three pricing strategies. Section 7 concludes the paper.

2. Literature review

There are two streams of literature that are most related to this research, which are celebrity endorsement management, and operations management with strategic consumers.

Management of celebrity endorsement has always been attracting attention from academia and practitioners. For comprehensive literature review, readers are referred to Schimmelpfennig & Hunt (2020)[1], Rocha et al. (2020)[2], and Bergkvist & Zhou (2016)[3]. Extant literature mainly focus on celebrity capital life cycle (Carrillat & Ilicic, 2019)[4], the impact of celebrity endorsement on consumers' purchase intention (Macheka et al., 2024; Osei-Frimpong et al., 2019)[5]-[6] and brand perception (Schartel & Nisbett, 2023)[7], the effect of celebrity endorsement on destination image shaping in tourism management (Lee & Jeong, 2023)[8], the impact of celebrity endorsement on product sales (Elberse & Verleun, 2012)[9]. However, how to determine the traffic level of celebrity endorsement and pricing strategy, as well as the impact from consumer

behavior are left untapped. Moreover, existing studies mainly use empirical research method to explore the issues of celebrity endorsement. Differently, this paper constructs and analyzes mathematical models to study the optimal endorsement traffic level and pricing strategy facing strategic consumers.

Nowadays, more and more consumers are trained by the enterprises' adjustment of price, and become more rational, which imposes non-negligible impacts on enterprises' decisions and profit. Some researchers focus on impacts of strategic consumers on supply chain co-opetition strategy (Yang et al., 2023)[10], group-buying mechanism design (Ke et al. 2017)[11], channel strategy (He et al. 2023)[12], product variety management (Parlaktürk, 2012)[13], supply chain coordination (Su & Zhang, 2008)[14], and capacity decision (Liu & Van Ryzin, 2008)[15], etc. More studies concentrate on effects of strategic consumers on product pricing, such as pricing of seasonal products (Aviv & Pazgal, 2008)[16], pricing with multi-unit demand (Elmaghraby et al., 2008)[17], pricing in a competitive environment (Levin et al., 2009; Liu & Zhang, 2013)[18]- [19], and pricing with speculators (Su, 2010)[20], etc. Different from the above literature, this paper explores optimal endorsement traffic level and pricing strategy with strategic consumers in the context of celebrity endorsement.

3. Problem description

Consider a brand introducing celebrity endorsement as a marketing strategy to promote the sales of her product in the market. The unit production cost of the product is c, and $c \in (0,1)$. When there is no celebrity endorsement, the market volume is normalized to 1, and the consumers' valuation w.r.t. the product is assumed as v and homogeneous among the consumers, where $v \in (c,1)$.

The brand has to decide the traffic of the celebrity providing the endorsement, denoted as α. The traffic α can capture the volume of the celebrity's fans and is described as the proportion of the fans among the market volume, hence $\alpha \in (0,1)$. The celebrity charges an endorsement fee $k\alpha^2$ from the brand, which represents the accelerating growing endorsement fee when the traffic level increases. To capture the reality of expensive endorsement, this paper assumes the coefficient of endorsement fee satisfies $k > \frac{(1+N)(v_H-c)}{2}$. The celebrity's fans hold higher valuation w.r.t. the brand's product due to the celebrity's endorsement, and is denoted as v_H, which satisfies $v_H \in (v,1)$. This can be regarded as the premium effect brought by the celebrity endorsement. The non-fans of the celebrity still hold valuation v w.r.t. the product. Moreover, there is also usually a market expansion effect raised by the endorsement. This paper characterize the effect by introducing a parameter N on the increase of the market demand, and $N \in (0,1)$. As the increased consumers are also the fans attracted by the celebrity endorsement, this paper also assume the increase of N is based on α and their valuation w.r.t. the product is v_H.

As there is valuation heterogeneity among consumers, the brand is motivated to adopt dynamic pricing strategy, i.e., the brand first sets the price as $p = v_H$, and then adjusts the price as $p = v$ after the consumers holding valuation v_H complete purchase. To respond this pricing strategy, some rational fans would rather strategically wait for the price reduction for the sake of higher consumer surplus, which are deemed as strategic consumers. This paper assumes β as the proportion of the strategic consumers among the total fans, indicating that $\beta \in (0,1)$. The strategical waiting behavior would undermine the brand's profit margin, perhaps even the total profit. In view of this, the brand has an incentive to adopt a uniform high pricing strategy to induce the strategic consumers to purchase at $p = v_H$ with the aim of securing the profit margin. Meanwhile, the brand can also adopt a uniform low pricing strategy, i.e. $p = v$, to seek for larger market demand instead of higher profit margin, which is even likely to have a chance to compete with the dynamic pricing strategy when empowered by a larger traffic level decision of celebrity endorsement. The notations used in this paper is summarized in Table 1.

Table 1 Notations

Notations	Definition
c	The unit production cost of the product, $c \in (0,1)$
v	The valuation w.r.t. the product held by the non-fans of the celebrity, $v \in (c,1)$
v_H	The valuation w.r.t. the product held by the fans of the celebrity, $v_H \in (v,1)$
β	The proportion of the strategic consumers among the total fans, $\beta \in (0,1)$
N	The increase of the market demand based on traffic, $N \in (0,1)$
α	The celebrity's traffic level, defined as the fans' proportion among the original market volume, $\alpha \in (0,1)$
p	The retail price of the product
k	The coefficient of endorsement fee charged by the celebrity, $k > \frac{(1+N)(v_H-c)}{2}$
Π	The brand's profit

4. The model with dynamic pricing strategy

When the brand adopts the dynamic pricing, the retail price is set as $p = v_H$, and then $p = v$ after the strategic fans have completed the purchase. Therefore, the brand decides the traffic level α of the endorsing celebrity with the problem

$$\text{Max}_{\alpha}\Pi^{D} = (v_H - c)(1-\beta)(1+N)\alpha + (v-c)(\beta(1+N)\alpha + 1 - \alpha) - k\alpha^2$$
$$\text{s.t. } 0 < \alpha < 1$$

Proposition 1 is utilized to characterize the optimal traffic level of celebrity endorsement and the brand's corresponding

profit under dynamic pricing strategy.

Proposition 1 Under dynamic pricing strategy, the brand's optimal traffic level of celebrity endorsement is $\alpha^{D*} = \frac{(1+N)v_H-(v+Nc)-(1+N)(v_H-v)\beta}{2k}$, and the brand's optimal profit is $\Pi^{D*} = \frac{((1+N)v_H-(v+Nc)-(1+N)(v_H-v)\beta)^2+4(v-c)k}{4k}$.

Proof: All the proof in this paper is delegated to the Appendix.

It can be easily inferred from Proposition 1 that the optimal traffic level α^{D*} and the optimal profit Π^{D*} both decrease in the unit production cost c, the proportion of strategic consumers β, and the coefficient of endorsement fee k. Meanwhile, the optimal traffic level α^{D*} and the optimal profit Π^{D*} both increase in the fans' valuation v_H, and the market expansion rate N. These results are all quite intuitive: When the level of factors rises such as the unit production cost, the proportion of strategic consumers, and the coefficient of endorsement fee, the value of premium effect and market demand expansion effect is diminished caused by the endorsement of celebrity traffic. Therefore, the incentive to employ higher traffic is curbed. With the erosion of the profit margin of each unit product sold and the shrinkage of market demand expansion, the brand's profit is also undermined. On the contrary, the increase of the fans' valuation and coefficient of market demand expansion directly push up the premium effect and the market demand expansion effect, hence the optimal traffic level of celebrity endorsement and corresponding profit increases. However, the impact of the non-fans' valuation is not that simple, which is characterized by Corollary 1.

Corollary 1 Under dynamic pricing strategy, the brand's optimal traffic level of celebrity endorsement increases in the non-fans' valuation (i.e. $\frac{\partial \alpha^{D*}}{\partial v} > 0$) when the proportion of strategic consumers is large (i.e. $\beta > \frac{1}{1+N}$), and decreases otherwise. The brand's optimal profit always increases in the non-fans' valuation (i.e. $\frac{\partial \Pi^{D*}}{\partial v} > 0$).

Corollary 1 shows that the increase of non-fans' valuation w.r.t. the product enhances the motivation to adopt higher endorsement traffic level when there are more strategic consumers, and undermines the motivation otherwise. However, the brand always benefits from a larger non-fans' valuation regardless of the proportion of strategic consumers. The reason behind can be interpreted as follows: When the proportion of strategic consumers is sufficiently large, the increase of non-fans' valuation means more generated premium can be saved, hence the incentive to adopt higher endorsement traffic is enhanced. Otherwise, the saved premium is outweighed by the larger incurred endorsement fee, and the willingness to adopt higher traffic is undermined. Anyway, no matter what the proportion of strategic consumers is, the increase of non-fans' valuation can save the valuation loss caused by consumers' strategic behavior, and improve the profit margin of sales to the non-fans. Therefore, the brand can always be the beneficiary.

5. The model with uniform pricing strategy

Due to the consumers' strategic purchasing behavior, the brand has incentives to adopt uniform pricing strategy. The brand can either seek for high profit margin with the price $p = v_H$ (uniform high pricing strategy), or seek for high market demand with the price $p = v$ (uniform low pricing strategy). This section will analyze these two strategies sequentially.

When the brand adopts uniform high pricing strategy, she faces the problem

$$\text{Max}_\alpha \Pi^{UH} = (v_H - c)(1+N)\alpha - k\alpha^2$$
$$\text{s.t. } 0 < \alpha < 1$$

Proposition 2 is utilized to characterize the optimal traffic level of celebrity endorsement and the brand's corresponding profit under uniform high pricing strategy.

Proposition 2 Under uniform high pricing strategy, the brand's optimal traffic level of celebrity endorsement is $\alpha^{UH*} = \frac{(1+N)(v_H-c)}{2k}$, and the brand's optimal profit is $\Pi^{UH*} = \frac{(1+N)^2(v_H-c)^2}{4k}$.

Proposition 2 shows that the optimal traffic level α^{UH*} and the optimal profit Π^{UH*} both decrease in the unit production cost c and the coefficient of endorsement fee k, and increase in the fans' valuation v_H, and the market expansion rate N. The rationale behind is quite similar to that of the situation with dynamic pricing strategy. Note that the non-fans of the celebrity are not served under uniform high pricing strategy, and all strategic consumers are forced to purchase the product at high price $p = v_H$. Therefore, neither the non-fans valuation v nor the proportion of strategic consumers β imposes impact on the brand's decision and performance.

When the brand adopts uniform low pricing strategy, she faces the problem

$$\text{Max}_\alpha \Pi^{UL} = (v - c)(1+N\alpha) - k\alpha^2$$
$$\text{s.t. } 0 < \alpha < 1$$

Proposition 3 is utilized to characterize the optimal traffic level of celebrity endorsement and the brand's corresponding profit under uniform low pricing strategy.

Proposition 3 Under uniform low pricing strategy, the brand's optimal traffic level of celebrity endorsement is $\alpha^{UL*} = \frac{N(v-c)}{2k}$, and the brand's optimal profit is $\Pi^{UL*} = \frac{(N^2(v-c)+4k))(v-c)}{4k}$.

Proposition 3 shows that the optimal traffic level α^{UL*} and the optimal profit Π^{UL*} both decrease in the unit production cost c and the coefficient of endorsement fee k, and increase in the non-fans' valuation v, and the market expansion rate N. The rationale behind is quite simple and intuitive. Different from the situation of uniform high pricing strategy, al the consumers are served at the same low price $p = v$. Therefore, neither the fans' valuation v_H nor the proportion of strategic consumers β imposes impact on the brand's traffic level decision and profit.

6. Comparative analyses

This section compares the brand's optimal decision of endorsement traffic and profit among the three pricing strategies, to provide insights and guidance for selection of pricing strategy and endorsement traffic level. In what follows, Proposition 4 is utilized to compare optimal endorsement traffic level among the three pricing strategies. Propositions 5-7 are utilized to characterize the conditions to adopt each pricing strategies.

Proposition 4 The optimal endorsement traffic level of the three pricing strategies satisfy $\alpha^{UH*} > \alpha^{D*} > \alpha^{UL*}$.

Proposition 4 indicates that the uniform high pricing strategy has the highest endorsement traffic level among the three pricing strategies, and uniform low pricing strategy is

equipped with the lowest. The optimal endorsement traffic level under dynamic pricing strategy stays in between. In general, the optimal endorsement traffic level is determined by the premium and expanded market demand generated by the celebrity endorsement. When the uniform high pricing strategy is adopted, all the fans of the celebrity, including the fans from the expanded market, complete their purchase at the high price, which drives the brand to employ the highest endorsement traffic level among the three pricing strategies. Although the uniform low pricing strategy achieves to induce all the consumers to make the purchase, yet no consumers pay the premium. This then leads to the lowest optimal endorsement traffic level. Under dynamic pricing strategy, the irrational fans pay the premium when purchasing the product, which means the sales generating the premium fall in between the two uniform pricing strategies. The strategic fans and non-fans make purchase at low price, hence the total market demand is more than that under the other two pricing strategies. As a result, the optimal endorsement traffic level under dynamic pricing strategy stays in between the other two.

Proposition 5 Uniform high pricing strategy dominates uniform low pricing strategy in the following conditions:

(1) $\frac{(\sqrt{2}-1)((3+2\sqrt{2})v-c)}{2} < v_H < 2v-c,\ N > \frac{(v-c)\sqrt{2(v_H-c)(v_H-v)}-(v_H-c)(v_H-v)}{2}$ and $k < \frac{((v_H+v-2c)N+v_H-c)((v_H-v)N+v_H-c)}{4(v-c)}$;

(2) $v_H \geq 2v-c$ and $k < \frac{((v_H+v-2c)N+v_H-c)((v_H-v)N+v_H-c)}{4(v-c)}$.

Otherwise, uniform high pricing strategy is dominated by uniform low pricing strategy.

Proposition 5 shows that when dynamic pricing strategy is not available, pricing high with a premium requires low traffic using cost and strong premium effect. If the premium effect is not sufficiently prominent, the market demand expansion effect should also be remarkable. Otherwise, higher profit margin is less attractive than higher market demand for the brand. Moreover, the coefficient of endorsement traffic also plays an important role in determining the price of high/low. This is because the brand has more incentives to employ an endorsing celebrity with higher traffic, due to the utilization of premium effect and higher market demand expansion effect. Thus, a low using cost of celebrity traffic is more in favor of dynamic pricing strategy.

Proposition 6 Uniform low pricing strategy is always dominated by dynamic pricing strategy.

As Proposition 4 shows, the optimal endorsement traffic level under dynamic pricing strategy is higher than that under uniform low pricing strategy. Thus the market demand expansion effect is more prominent, resulting in a larger market demand. Meanwhile, under dynamic pricing strategy, irrational fans of the celebrity pay premium when purchasing the product, only strategic consumers and non-fans of the endorsing celebrity make the purchase at low price. Under uniform low pricing strategy, all consumers buy the product at the low price. Therefore, the profit margin under dynamic pricing strategy is also higher than that under uniform low pricing strategy. As a result, dynamic pricing strategy always dominates uniform low pricing strategy.

Proposition 7 Uniform high pricing strategy dominates dynamic pricing strategy in the following conditions:

(1) $v_H \geq 2v-c,\ k < \frac{(1+N)^2(v_H-c)^2}{4(v-c)}$, and $\beta > \frac{(1+N)v_H-(v+Nc)-\sqrt{(1+N)^2(v_H-c)^2-4(v-c)k}}{(1+N)(v_H-v)}$;

(2) $v < v_H < 2v-c,\ N > \frac{2v-c-v_H}{v_H-c},\ k < \frac{(1+N)^2(v_H-c)^2}{4(v-c)}$ and $\beta > \frac{(1+N)v_H-(v+Nc)-\sqrt{(1+N)^2(v_H-c)^2-4(v-c)k}}{(1+N)(v_H-v)}$.

Otherwise, uniform high pricing strategy is dominated by dynamic pricing strategy.

Conventional wisdom on operations management with strategic consumers suggests that uniform pricing strategy should be adopted when consumers are highly strategic, or the proportion of strategic consumers is large. Different from that, Proposition 7 shows that, aside from the proportion of strategic consumers, whether to adopt uniform high pricing strategy or dynamic pricing strategy in the context of celebrity endorsement is also impacted by the premium effect of celebrity endorsement, the market demand expansion effect, and the traffic using cost. When the traffic using cost is low and the proportion of strategic consumers is large, uniform high pricing strategy should be adopted when the premium effect is sufficiently prominent. However, if the premium effect is not sufficiently large, considerable market demand expansion effect is also required. This is attributed to the tradeoff between the positive effects brought by celebrity endorsement and the traffic using cost it incurs.

7. Conclusion

This paper uses the method of mathematical modelling to explore the optimal traffic level and pricing strategy in the context of celebrity endorsement. Dynamic pricing strategy, uniform high pricing strategy and uniform low pricing strategy are analyzed and compared. The results show that the optimal traffic level is highest under uniform high pricing strategy and lowest under uniform low pricing strategy. Dynamic pricing strategy is equipped with the optimal traffic level lying in between. This is determined by the incentive to adopt high traffic level caused by premium effect and market demand expansion effect. For this reason, dynamic pricing strategy always dominates uniform low pricing strategy. When it comes to the comparison between uniform high pricing strategy and dynamic pricing strategy, conventional wisdom suggests that uniform pricing strategy should be adopted when consumers are highly strategic, or the proportion of strategic consumers is large. This paper finds that, aside from the characteristics of strategic consumers, the premium effect of celebrity endorsement, the market demand expansion effect, and the traffic using cost also play important roles in the context of celebrity endorsement. This can be explained by the tradeoff between positive effects of celebrity endorsement and the traffic using cost it incurs.

Acknowledgement

This research is supported by China Postdoctoral Science Foundation [grant number bs6820020].

Reference

[1] Schimmelpfennig C, Hunt J B. Fifty years of celebrity endorser research: Support for a comprehensive celebrity endorsement strategy framework[J]. Psychology & Marketing, 2020, 37(3): 488-505.

[2] Rocha P I, Caldeira de Oliveira J H, Giraldi J M E. Marketing communications via celebrity endorsement: an integrative review[J]. Benchmarking: An International Journal, 2020, 27(7): 2233-2259.

[3] Bergkvist L, Zhou K Q. Celebrity endorsements: A literature review and research agenda[J]. International journal of advertising, 2016, 35(4): 642-663.

[4] Carrillat F A, Ilicic J. The celebrity capital life cycle: A framework for future research directions on celebrity endorsement[J]. Journal of Advertising, 2019, 48(1): 61-71.

[5] Macheka T, Quaye E S, Ligaraba N. The effect of online customer reviews and celebrity endorsement on young female consumers' purchase intentions[J]. Young Consumers, 2024, 25(4): 462-482.

[6] Osei-Frimpong K, Donkor G, Owusu-Frimpong N. The impact of celebrity endorsement on consumer purchase intention: An emerging market perspective[J]. Journal of marketing theory and practice, 2019, 27(1): 103-121.

[7] Schartel Dunn S, Nisbett G S. When celebrity endorsements collide with social activism: exploring athlete celebrity endorsements, social issues and brand perception[J]. International Journal of Sports Marketing and Sponsorship, 2023, 24(3): 558-569.

[8] Lee S J, Jeong E H L. An integrative approach to examining the celebrity endorsement process in shaping affective destination image: A K-pop culture perspectives[J]. Tourism Management Perspectives, 2023, 48: 101150.

[9] Elberse A, Verleun J. The economic value of celebrity endorsements[J]. Journal of advertising Research, 2012, 52(2): 149-165.

[10] Yang W, Wu Y, Gou Q, et al. Co-opetition strategies in supply chains with strategic customers[J]. Production and Operations Management, 2023, 32(1): 319-334.

[11] Ke C, Yan B, Xu R. A group-buying mechanism for considering strategic consumer behavior[J]. Electronic Commerce Research, 2017, 17: 721-752.

[12] He P, He Y, Zhou L. Channel strategies for dual-channel firms to counter strategic consumers[J]. Journal of Retailing and Consumer Services, 2023, 70: 103180.

[13] Parlaktürk A K. The value of product variety when selling to strategic consumers[J]. Manufacturing & Service Operations Management, 2012, 14(3): 371-385.

[14] Su X, Zhang F. Strategic customer behavior, commitment, and supply chain performance[J]. Management Science, 2008, 54(10): 1759-1773.

[15] Liu Q, Van Ryzin G J. Strategic capacity rationing to induce early purchases[J]. Management Science, 2008, 54(6): 1115-1131.

[16] Aviv Y, Pazgal A. Optimal pricing of seasonal products in the presence of forward-looking consumers[J]. Manufacturing & service operations management, 2008, 10(3): 339-359.

[17] Elmaghraby W, Gülcü A, Keskinocak P. Designing optimal preannounced markdowns in the presence of rational customers with multiunit demands[J]. Manufacturing & Service Operations Management, 2008, 10(1): 126-148.

[18] Levin Y, McGill J, Nediak M. Dynamic pricing in the presence of strategic consumers and oligopolistic competition[J]. Management science, 2009, 55(1): 32-46.

[19] Liu Q, Zhang D. Dynamic pricing competition with strategic customers under vertical product differentiation[J]. Management Science, 2013, 59(1): 84-101.

[20] Su X. Optimal pricing with speculators and strategic consumers[J]. Management Science, 2010, 56(1): 25-40.

Appendix

Proof of Proposition 1:

Take the first and second order derivatives of $\Pi^D = (v_H - c)(1-\beta)(1+N)\alpha + (v-c)(\beta(1+N)\alpha + 1 - \alpha) - k\alpha^2$ w.r.t. α. The second order derivative $\frac{\partial^2 \Pi^D}{\partial \alpha^2} = -2k < 0$, indicating that Π^D is a concave function of α. From the first order condition $\frac{\partial \Pi^D}{\partial \alpha} = (1+N)v_H - (v + Nc) - (1+N)(v_H - v)\beta - 2k\alpha = 0$, it can be inferred that $\alpha^{D*} = \frac{(1+N)v_H - (v+Nc) - (1+N)(v_H - v)\beta}{2k}$. Substituting it into Π^D then generates $\Pi^{D*} = \frac{((1+N)v_H - (v+Nc) - (1+N)(v_H - v)\beta)^2 + 4(v-c)k}{4k}$.

Proof of Corollary 1:

Take the first order derivative of $\alpha^{D*} = \frac{(1+N)v_H - (v+Nc) - (1+N)(v_H - v)\beta}{2k}$ w.r.t. v, $\frac{\partial \alpha^{D*}}{\partial v} = \frac{(1+N)\beta - 1}{2k}$.

When $\beta > \frac{1}{1+N}$, $\frac{\partial \alpha^{D*}}{\partial v} = \frac{(1+N)\beta - 1}{2k} > 0$. Take the first order derivative of $\Pi^{D*} = \frac{((1+N)v_H - (v+Nc) - (1+N)(v_H - v)\beta)^2 + 4(v-c)k}{4k}$ w.r.t. v, it can be inferred that $\frac{\partial \Pi^{D*}}{\partial v} = \frac{((1+N)v_H - (v+Nc) - (1+N)(v_H - v)\beta)((1+N)\beta - 1) + 2k}{2k} > 0$.

Proof of Proposition 2:

Take the first and second order derivatives of $\Pi^{UH} = (v_H - c)(1+N)\alpha - k\alpha^2$ w.r.t. α. The second order derivative $\frac{\partial^2 \Pi^{UH}}{\partial \alpha^2} = -2k < 0$, indicating that Π^{UH} is a concave function of α. From the first order condition $\frac{\partial \Pi^{UH}}{\partial \alpha} = (1+N)(v_H - c) - 2k\alpha = 0$, it can be inferred that $\alpha^{UH*} = \frac{(1+N)(v_H - c)}{2k}$. Substituting it into Π^{UH} then generates $\Pi^{UH*} = \frac{(1+N)^2 (v_H - c)^2}{4k}$.

Proof of Proposition 3:

Take the first and second order derivatives of $\Pi^{UL} = (v-c)(1+N\alpha) - k\alpha^2$ w.r.t. α. The second order derivative $\frac{\partial^2 \Pi^{UL*}}{\partial \alpha^2} = -2k < 0$, indicating that Π^{UL} is a concave function of α. From the first order condition $\frac{\partial \Pi^{UL*}}{\partial \alpha} = N(v-c) - 2k\alpha = 0$, it can be inferred that $\alpha^{D*} = \frac{N(v-c)}{2k}$. Substituting it into Π^{UL} then generates $\Pi^{UL*} = \frac{(N^2(v-c) + 4k))(v-c)}{4k}$.

Proof of Proposition 4:

$\alpha^{UH*} - \alpha^{D*} = \frac{(1+N)(v_H - c)}{2k} - \frac{(1+N)v_H - (v+Nc) - (1+N)(v_H - v)\beta}{2k} = \frac{(1+N)\beta(v_H - v) + v - c}{2k} > 0$

$\alpha^{D*} - \alpha^{UL*} = \frac{(1+N)v_H - (v+Nc) - (1+N)(v_H - v)\beta}{2k} - \frac{N(v-c)}{2k} > 0$

Proposition 4 can be proved.

Proof of Proposition 5:

$\Pi^{UH*} - \Pi^{UL*} = \frac{(1+N)^2 (v_H - c)^2}{4k} - \frac{\left(N^2(v-c) + 4k\right)(v-c)}{4k} = \frac{((1+N)(v_H - c) + N(v-c))(N(v_H - v) + (v_H - c)) - 4k(v-c)}{4k}$

1. When $v_H \geq 2v - c$, it satisfies

$\frac{\left((1+N)(v_H-c)+N(v-c)\right)\left(N(v_H-v)+(v_H-c)\right)}{4(v-c)} > \frac{(1+N)(v_H-c)}{2}$.

(1) If $\frac{(1+N)(v_H-c)}{2} < k \leq \frac{\left((1+N)(v_H-c)+N(v-c)\right)\left(N(v_H-v)+(v_H-c)\right)}{4(v-c)}$, $\Pi^{UH*} - \Pi^{UL*} = \frac{\left((1+N)(v_H-c)+N(v-c)\right)\left(N(v_H-v)+(v_H-c)\right)-4k(v-c)}{4k} \geq 0$, hence uniform high pricing strategy dominates uniform low pricing strategy.

(2) If $k > \frac{\left((1+N)(v_H-c)+N(v-c)\right)\left(N(v_H-v)+(v_H-c)\right)}{4(v-c)}$, $\Pi^{UH*} - \Pi^{UL*} = \frac{\left((1+N)(v_H-c)+N(v-c)\right)\left(N(v_H-v)+(v_H-c)\right)-4k(v-c)}{4k} < 0$, hence uniform low pricing strategy dominates uniform high pricing strategy.

2. When $v_H < \frac{(\sqrt{2}-1)\left((3+2\sqrt{2})v-c\right)}{2}$, $k > \frac{\left((1+N)(v_H-c)+N(v-c)\right)\left(N(v_H-v)+(v_H-c)\right)}{4(v-c)}$ always holds, hence uniform low pricing strategy dominates uniform high pricing strategy.

3. When $\frac{(\sqrt{2}-1)\left((3+2\sqrt{2})v-c\right)}{2} \leq v_H < 2v-c$,

(1) If $N \leq \frac{(v-c)\sqrt{2(v_H-c)(v_H-v)}-(v_H-c)(v_H-v)}{2}$, then $k > \frac{\left((1+N)(v_H-c)+N(v-c)\right)\left(N(v_H-v)+(v_H-c)\right)}{4(v-c)}$ always holds, thus uniform low pricing strategy dominates uniform high pricing strategy.

(2) If $N > \frac{(v-c)\sqrt{2(v_H-c)(v_H-v)}-(v_H-c)(v_H-v)}{2}$, $k \leq \frac{\left((1+N)(v_H-c)+N(v-c)\right)\left(N(v_H-v)+(v_H-c)\right)}{4(v-c)}$ implicates uniform high pricing strategy dominates; or else uniform low pricing strategy dominates.

Proof of Proposition 6:

$\Pi^{D*} - \Pi^{UL*} = \frac{\left((1+N)v_H-(v+Nc)-(1+N)(v_H-v)\beta\right)^2+4(v-c)k}{4k} - \frac{\left(N^2(v-c)+4k\right)(v-c)}{4k} = \frac{(1+N)(1-\beta)(v_H-v)\left((1+N)(1-\beta)(v_H-v)+2N(v-c)\right)}{4k} > 0$. Proposition 6 can be proved.

Proof of Proposition 7:

$\Pi^{D*} - \Pi^{UH*} = \frac{\left((1+N)v_H-(v+Nc)-(1+N)(v_H-v)\beta\right)^2+4(v-c)k}{4k} - \frac{(1+N)^2(v_H-c)^2}{4k} = \frac{(1+N)^2(v_H-v)^2\beta^2-2(1+N)(v_H-v)\left((1+N)v_H-(v+Nc)\right)\beta+(v-c)(4k+v+(1+2N)c-2(1+N$

1. When $v_H \geq 2v-c$, $\frac{(1+N)^2(v_H-c)^2}{4(v-c)} > \frac{(1+N)(v_H-c)}{2}$ always holds.

(1) If $k \geq \frac{(1+N)^2(v_H-c)^2}{4(v-c)}$, $\Pi^{D*} - \Pi^{UH*} \geq 0$, dynamic pricing strategy always dominates uniform high pricing strategy.

(2) If $k < \frac{(1+N)^2(v_H-c)^2}{4(v-c)}$ and $\beta \leq \frac{(1+N)v_H-(v+Nc)-\sqrt{(1+N)^2(v_H-c)^2-4(v-c)k}}{(1+N)(v_H-v)}$, $\Pi^{D*} - \Pi^{UH*} \geq 0$, dynamic pricing strategy dominates.

(3) If $k < \frac{(1+N)^2(v_H-c)^2}{4(v-c)}$ and $\beta > \frac{(1+N)v_H-(v+Nc)-\sqrt{(1+N)^2(v_H-c)^2-4(v-c)k}}{(1+N)(v_H-v)}$, $\Pi^{D*} - \Pi^{UH*} < 0$, uniform high pricing strategy dominates.

2. When $v < v_H < 2v-c$,

(1) If $N \leq \frac{2v-c-v_H}{v_H-c}$, then $\frac{(1+N)^2(v_H-c)^2}{4(v-c)} \leq \frac{(1+N)(v_H-c)}{2}$, hence $k \geq \frac{(1+N)^2(v_H-c)^2}{4(v-c)}$ always holds, $\Pi^{D*} - \Pi^{UH*} \geq 0$, and dynamic pricing strategy dominates.

(2) If $N > \frac{2v-c-v_H}{v_H-c}$ and $k \geq \frac{(1+N)^2(v_H-c)^2}{4(v-c)}$, then $\Pi^{D*} - \Pi^{UH*} \geq 0$, dynamic pricing strategy dominates.

(3) If $N > \frac{2v-c-v_H}{v_H-c}$, $k < \frac{(1+N)^2(v_H-c)^2}{4(v-c)}$, and $\beta \leq \frac{(1+N)v_H-(v+Nc)-\sqrt{(1+N)^2(v_H-c)^2-4(v-c)k}}{(1+N)(v_H-v)}$, then $\Pi^{D*} - \Pi^{UH*} \geq 0$, dynamic pricing strategy dominates.

(4) If $N > \frac{2v-c-v_H}{v_H-c}$, $k < \frac{(1+N)^2(v_H-c)^2}{4(v-c)}$, and $\beta > \frac{(1+N)v_H-(v+Nc)-\sqrt{(1+N)^2(v_H-c)^2-4(v-c)k}}{(1+N)(v_H-v)}$, then $\Pi^{D*} - \Pi^{UH*} < 0$, uniform high pricing strategy dominates.