

Study on Coverage Width and Overlap Rate Based on Multibeam Bathymetry Model

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Abstract: Deep-water multibeam bathymetric systems can be used to detect seafloor topography and geomorphology in the whole ocean area, which is a manifestation of a country's comprehensive marine technology level and capability. In order to improve the measurement efficiency and quality, this paper studies the coverage width and overlap rate of multibeam bathymetry. A mathematical model based on the coverage width of multibeam bathymetry and the overlap rate of adjacent strips is established, and the coverage width and the overlap rate with the previous line when sailing in a straight line and sailing in a rectangular shape to be measured when the angle of the multibeam transducer is 120° , the slope is 1.5° , and the depth of seawater in the center of the sea area is 70m are found out respectively. This study not only improves the efficiency of the survey operation by optimizing the route planning, but also enhances the data quality and application value, which provides theoretical guidance for the research work of ocean exploration.

Keywords: Multibeam bathymetry, Coverage width, Overlap rate, Geometry

1. Introduction

Multibeam bathymetry is one of the important means of marine exploration, and it is widely used in seabed topographic surveys, marine resource exploration, channel mapping, and other fields. Compared with the traditional single-beam bathymetry technology, multibeam bathymetry can obtain a larger range of seabed topographic data in one scan, thus significantly improving the measurement efficiency and data accuracy[1-3]. However, due to the complexity and variability of seafloor topography, especially under special topographic conditions such as seafloor slopes, the coverage width of the multibeam and the overlap rate between the survey lines are affected by a variety of factors, such as the bathymetric angle, the depth of seawater, and the slope gradient.

Currently, there have been studies that have achieved significant results in multibeam bathymetry. However, there are still fewer systematic studies on the coverage performance under complex terrain conditions on the seafloor. Specifically, this study takes the seabed slope as the research object, according to the known conditions and data, it can be known that the coverage width of multibeam bathymetry should be the projection of the width covered by the multibeam on the slope, and the overlap rate is obtained by subtracting the spacing between two adjacent lines from the coverage width of the measured multibeam bathymetry and then dividing by

the coverage width of the measured multibeam bathymetry. First, the actual problem is transformed into a mathematical model, and the expression for the coverage width is obtained according to its intrinsic trigonometric relationship[4]. Then the formula of seawater depth of different seabed terrain is calculated according to the case of sailing on a straight line to derive the general formula of overlap rate. On this basis, the sailing direction of the survey vessel is changed. In this way, the mathematical relationship between the coverage width and overlap rate of multibeam bathymetry in a rectangular sea area is to be measured and the depth of seawater with the opening angle of the transducer and the overlap rate is explored, and the results of the calculation of the depth of seawater, the coverage and the overlap rate are obtained[5].

2. Modeling and Analysis of Multibeam Coverage Width and Overlap Rate

The data for this study were obtained from <https://cumcm.cnki.net/>.

2.1. Modeling of the coverage width W for multibeam bathymetry

To deeply investigate the relationship between the coverage width W and the slope gradient α , the transducer opening angle θ and the seawater depth D (the center seawater depth is D_n), this paper makes Figure 1 as follows:

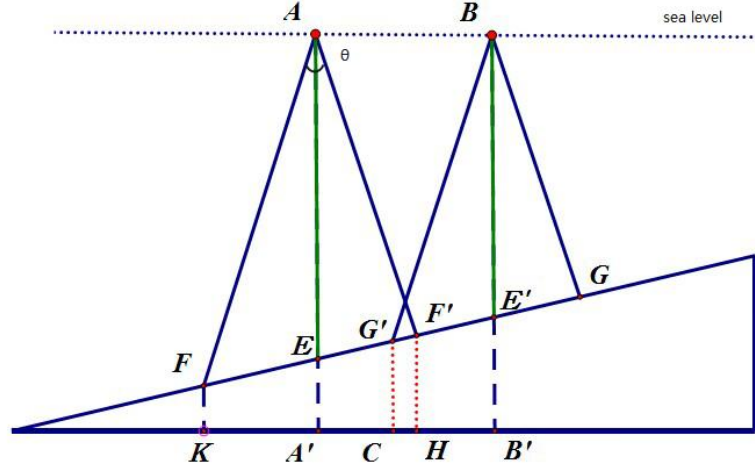


Figure 1. Geometrical modeling of the coverage width

Extend the measurement line of the multibeam to intersect the horizontal plane at points A', B', and make a vertical line to the points F, G', and F' of the multibeam intersecting the slope and the horizontal plane to intersect at points K, and C, H respectively, and then define the line segment FF' in Fig.1 to be S, the line segment FE to be S1, the line segment EF' to be S2, the line segment KA' to be W1, and the line segment A'H to be W2.

Based on the above definitions this paper concludes that W

= W1 + W2, Since W is the projection of S in the horizontal plane, it is obtained that $W = S1 \cos \alpha + S2 \cos \alpha$. As shown in Figure 2, triangle AEF and triangle AEF' are perpendicular to each other, and according to their intrinsic trigonometric relationships, it is concluded that $S_1 = \frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} + \alpha)}$, $S_2 = \frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} - \alpha)}$.

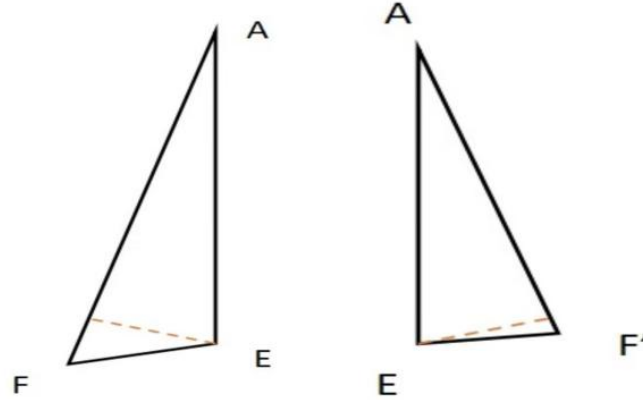


Figure 2. Two triangles formed by multiple waves

In summary, modeling the coverage width W of multibeam bathymetry.

$$W = \left(\frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} + \alpha)} + \frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} - \alpha)} \right) \times D_n \cdot \cos \alpha \quad (1)$$

2.2. Modeling of the overlap rate η

According to the definition in this paper, the overlap rate $\eta = 1 - \frac{d}{W}$, Coverage width of multibeam bathymetry with

overlap rate $W = S1 \cos \alpha + S2 \cos \alpha$. In summary, modeling the overlap rate.

$$\eta = 1 - \frac{d}{\left(\frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} + \alpha)} \times D_n \cdot \cos \alpha + \frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} - \alpha)} \times D_n \cdot \cos \alpha \right)} \quad (2)$$

2.3. Solving the model

According to the given data, the distance d between two adjacent lines is 200 m. This problem requires the depth of the seawater at different distances from the center of the line,

the coverage width, and the overlap rate with the previous line. The formulas for the coverage width and overlap rate are given in the modeling above. The given data and the formulas for coverage width and overlap ratio are brought into MATLAB and the required results are shown in Table 1.

Table 1. Calculation results of indicators

The distance between the measuring line and the center point /m	Depth of seawater /m	Coverage width /m	Overlap rate with the previous survey line /%
-800	90.9487	314.9479	—
-600	85.7115	296.8120	34.61
-400	80.474	278.6760	30.49
-200	75.2371	260.5401	25.81
0	70.0000	242.4041	20.46
200	64.7628	224.2682	14.28
400	59.5256	206.1323	7.06
600	54.2884	187.9963	-1.49
800	49.0512	169.8604	-11.78

3. Multibeam Coverage Models Under Various Scenarios

3.1. Re-establishment of the model

According to the analysis, the model of the coverage width W of the multibeam bathymetry sailing on a straight line is

also suitable for sailing in a rectangular sea area to be measured.

In this problem the water depth D_n is the water depth of the survey vessel at the center of the rectangular sea area to be measured, ΔD is the change in depth of the survey vessel at different distances from the center of the sea area, and D_0 is the actual depth of the seawater.

$$D_0 = D_n + \Delta D \quad (3)$$

Because D_0 and α' are the variables in the measurement formula of the coverage width W required in this paper, if D_0 and α' are required, this paper needs to explore the mathematical relationship with β . To be more intuitive, this paper will β , α' , α , that is, the line segments related to it into

the rectangle for solving, and according to the actual situation, is divided into six cases are discussed as follows:

3.1.1. When the range of β is $0^\circ < \beta < 90^\circ$

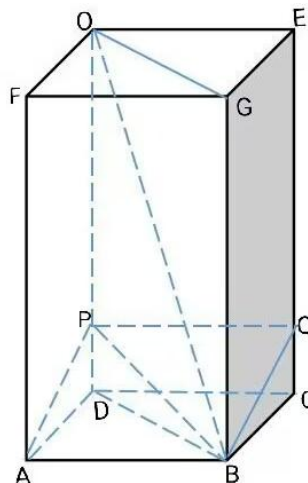


Figure 3. Stereoscopic map of the sea area in the first scenario

As shown in Figure 3, the center point O is the position of the center point, the point G is the distance of the measuring line from the center point (the distance of the ship from the center of the sea area), the surface $FGEO$ is the sea level, the surface $ABCD$ is the horizontal surface, the surface $ABQP$ is the slope of the seabed, the point P is the intersection of the slope of the seabed and the rectangular, the point D is the projection point of the point P in the horizontal surface, the

line OG is the direction of the ship's movement, the angle $\angle ADB$ is the angle β , the angle $\angle PAD$ is the angle α , and the angle $\angle PBD$ is the angle α' . $\angle ADB$ is angle β , $\angle PAD$ is angle α , and $\angle PBD$ is angle α' .

In $\triangle ABD$, the trigonometric formula shows that $AD = BD \cos \beta = \cos \beta$. Similarly, in $\triangle ADP$, $PD = AD \tan \alpha$, PD is ΔD . In $\triangle PBD$ $\tan \alpha' = \cos \beta \tan \alpha$, After passing the Lian $\text{Li}(\cos \alpha')^2 + (\sin \alpha')^2 = 1$ One can find $\cos \alpha'$.

The combination of the above equations gives the depth of the seawater in this case $D_0 = D_n + l \cos \beta \tan \alpha$.

In summary, the mathematical model of multibeam

bathymetric coverage width W in the range of β is $0^\circ < \beta < 90^\circ$.

$$W = \left(\frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} + \alpha)} + \frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} - \alpha)} \right) \times (D_n + l \cos \beta \tan \alpha) \cos \alpha' \quad (4)$$

3.1.2. When the range of β is $90^\circ < \beta < 180^\circ$

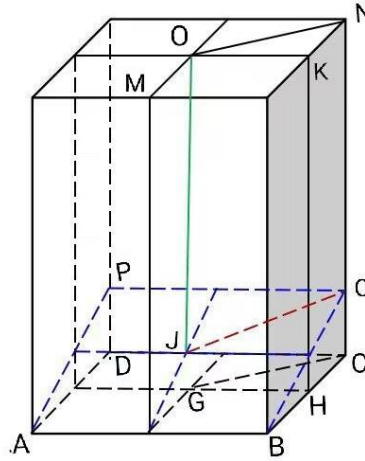


Figure 4. Stereoscopic map of the sea area in the second scenario

As shown in Figure 4, the center point O is the position at the center point, the point N is the position where the line of measurement is at a distance from the center point, the surface OMKN is the sea level, the surface ABCD is the horizontal surface, the surface ABQP is the seabed slope, the point J is an intersection of the seabed slope and the rectangular body, the point G is the projection point of the point J in the horizontal surface, the line ON is the direction of the ship's movement, the angle $\angle MON$ is the β -angle, the angle $\angle PAD$

is the α -angle, and the angle of the line JQ with the sea level is α' . The angle between line JQ and the sea level is α' .

The depth of the seawater in this case is derived by the same reasoning $D_0 = D_n + l \sin(\beta - \frac{\pi}{2}) \tan \alpha$, pass $\tan \alpha' = \cos \beta \tan \alpha$ and $(\cos \alpha')^2 + (\sin \alpha')^2 = 1$ can be found as $\cos \alpha'$.

In summary, the mathematical model of multibeam bathymetry coverage width W in the range of β is $90^\circ < \beta < 180^\circ$.

$$W = \left(\frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} + \alpha)} + \frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} - \alpha)} \right) \times \left(D_n + l \sin(\beta - \frac{\pi}{2}) \tan \alpha \right) \cos \alpha' \quad (5)$$

3.1.3. When the range of β is $180^\circ < \beta < 270^\circ$

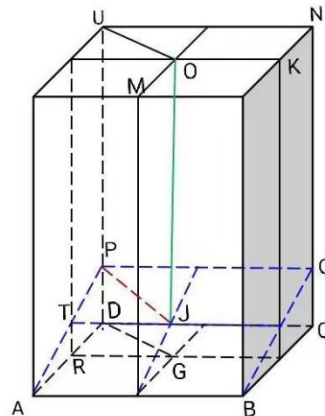


Figure 5. Stereoscopic map of the sea area in the third scenario

As shown in Figure 5, the depth of the seawater for this scenario is derived from the same reasoning. $D_0 = D_n + l \cos(\beta - \pi) \tan \alpha$. pass $\tan \alpha' = -\cos \beta \tan \alpha$ and

$(\cos \alpha')^2 + (\sin \alpha')^2 = 1$, One can find $\cos \alpha'$.

In summary, the mathematical model of multibeam bathymetric coverage width W in the range of β is $180^\circ < \beta < 270^\circ$.

$$W = \left(\frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} + \alpha)} + \frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} - \alpha)} \right) \times (D_n + \cos(\beta - \pi) \tan \alpha) \cos \alpha' \quad (6)$$

3.1.4. When the range of β is $270^\circ < \beta < 360^\circ$

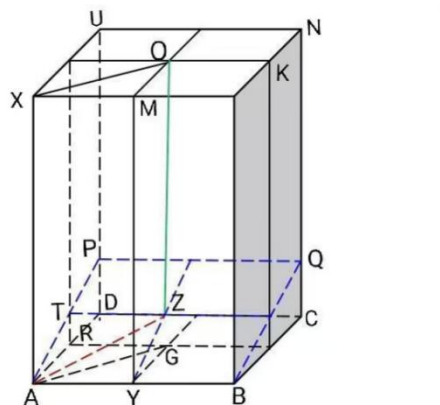


Figure 6. Stereoscopic map of the sea area in the fourth scenario

As shown in Figure 6, the depth of the seawater in this case is derived by the same reasoning $D_0 = D_n + l \sin(\beta - \frac{2}{3}\pi) \tan \alpha$, pass $\tan \alpha' = -\cos \beta \tan \alpha$ and $(\cos \alpha')^2 + (\sin \alpha')^2 =$

1, can be found as $\cos \alpha'$.

In summary, the mathematical model of multibeam bathymetry coverage width W in the range of β is $270^\circ < \beta < 360^\circ$.

$$W = \left(\frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} + \alpha)} + \frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} - \alpha)} \right) \times \left(D_n + l \sin \left(\beta - \frac{2}{3} \pi \right) \tan \alpha \right) \cos \alpha' \quad (7)$$

3.1.5. When the value of β is 0° or 180°

In this paper, it is found that when $\beta = 0^\circ$ or 180° , the cases are the same as in the case of sailing on a straight line, so the

mathematical model is as follows.

$$W = \left(\frac{\sin \frac{\theta}{2}}{\cos \left(\frac{\theta}{2} + \alpha \right)} + \frac{\sin \frac{\theta}{2}}{\cos \left(\frac{\theta}{2} - \alpha \right)} \right) \times D_n \cdot \cos \alpha \quad (8)$$

3.1.6. When the value of β is 90° or 270°

When $\beta = 90^\circ$ or 270° , the sea depth D_0 does not change,

at this time $D_0 = D_n$, the mathematical

model of the bathymetric coverage width is as follows.

$$W = \left(\frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} + \alpha)} + \frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} - \alpha)} \right) \times D_0 \cdot \cos \alpha \quad (9)$$

3.2. Solving the model

The given data are brought into MATLAB to solve the

problem, and the results are shown in Table 2.

Table 2 calculation results of the coverage width

Coverage width/m	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
0	415.6921	466.0910	516.4899	566.8887	617.2876	667.6864	718.0853	768.4842
45	415.5498	451.1750	486.8002	522.4254	558.0506	593.6757	629.3009	664.9261
90	415.5500	415.5500	415.5500	415.5500	415.5500	415.5500	415.55005	415.5500
135	415.5498	379.9247	344.2995	308.6743	273.0491	237.4240	201.79883	166.1736
180	415.6921	365.2933	314.8944	264.4956	214.0967	163.6978	113.2990	62.90016
225	415.5498	379.9247	344.2995	308.6743	273.04918	237.4240	201.79883	166.1736
270	415.5500	415.5500	415.5500	415.55005	415.5500	415.5500	415.5500	415.5500
315	415.5498	451.1750	486.8002	522.4254	558.0506	593.6757	629.3009	664.9261

From the geometric relationship, it can be seen that the change of the direction of the survey line of the survey ship

only affects the depth of the sea water D and the angle α' between the line segment covered by the multibeam sounder

on the seabed slope and the horizontal edge line when sailing, so the mathematical model of the coverage width is still applicable to the rectangular sea area to be measured. In this paper, the seawater depth D and $\cos\alpha'$ are solved by β and categorized into six types according to the actual situation. Under these six different mathematical models, the given data are brought into the solution. And the final results are shown in the table above.

4. Conclusions

In this paper, a mathematical model of the coverage width and overlap rate between adjacent strips of multibeam bathymetry is established. By setting a suitable coordinate system, the seawater depth D from the center of the sea area and the relationship between the coverage width W and the seawater depth D , the transducer opening angle θ , and the slope α can be calculated sequentially. Then, the analytical equation of the overlap rate when the slope of the seabed is α is given in combination with the intrinsic meaning of the overlap rate between neighboring strips, and the corresponding mathematical model is established. The given data are substituted into the mathematical model of coverage width and overlap rate between adjacent strips to solve the problem. This paper provides a research idea and framework applied to the field of seabed mapping. Through the mathematical model of multibeam bathymetry coverage width and overlap rate between adjacent strips established in this paper, the relationship between multibeam bathymetry coverage width and overlap rate between adjacent strips and seawater depth D and transducer θ opening angle in different

cases is deduced. On this basis, the measurement route of the survey vessel is planned. The feasibility of the method is proved. Looking ahead, future research will further deepen the practical application of multibeam bathymetry, including the optimization of existing models to adapt to more complex marine environments, the development of more efficient algorithms to reduce the time and cost of measurements, and the integration of multibeam bathymetry with other oceanographic exploration techniques to achieve the fusion of multi-source data and comprehensive analysis.

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