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Distributed Cooperative Tracking Algorithm Based on Finite Time Neurodynamics

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Abstract: In the rapidly growing field of multi-robot system applications, this study focuses on exploring the challenges associated with multi-robot cooperative tracking of time-varying targets. The core objective of the research is to achieve efficient cooperative target tracking by minimizing the distance between the robots and the target, facilitated by the exchange of local information among the robots. To address this challenge, a novel distributed finite-time algorithm is proposed, integrating predictive correction methods and sliding mode control techniques. On a theoretical level, a carefully constructed Lyapunov function is developed, and a comprehensive convergence analysis is conducted to ensure that the proposed algorithm can successfully solve the time-varying optimization problem within a finite time. To validate the practical effectiveness of the algorithm, multiple rounds of robot-target tracking experiments are conducted. The experimental results vividly demonstrate the efficiency of multi-robot cooperative tracking, showing that the performance remains unaffected by the number of obstacles or the time-varying target's movement trajectory, thereby confirming the algorithm's outstanding effectiveness and reliability in real-world applications.

Keywords: Time-Varying, Convex Optimization, Distributed, Finite Time, Neurodynamics.

1. Introduction

With the rapid development of technology, multi-robot cooperative tracking has become a significant research direction in the field of robotics, attracting widespread attention and in-depth investigation. In modern society, robotic systems are becoming increasingly prevalent in various application scenarios, such as industrial manufacturing, logistics distribution, military missions, and disaster rescue. In these applications, multi-robot cooperative tracking technology holds tremendous potential to enhance the efficiency, reliability, and flexibility of task execution.

Real-time monitoring of moving targets is a key objective in target tracking. To improve tracking performance, many researchers have focused on formulating optimization problems. In reference [1], an innovative multi-UAV formation surveillance cooperative system architecture was designed to overcome issues such as the limited field of view of a single UAV. In dealing with uncertain motion target trajectories, references [5] and [6] introduced the Kalman trajectory prediction algorithm, providing probabilistic guarantees for tracking dynamic targets. To address the trajectory planning problem of multiple UAVs monitoring ground vehicles over uneven terrain, reference [7] employed a model predictive control-based multi-UAV path planning algorithm. Reference [8] used sliding mode control methods to propose continuous-time and discrete-time layered tracking control protocols. For a multi-UAV cooperative surveillance network platform, reference [9] introduced a new system architecture and conducted experiments on multi-target recognition and tracking. In solving the problem of single UAV tracking independent targets, another study[10] combined the Lyapunov vector field method with the velocity obstacle method to ensure the safety of UAVs during tracking tasks. Finally, reference [11] proposed a bias-free target localization estimator and a multi-UAV formation controller, improving the orientation-based control law through coarse estimators and Kalman filtering to achieve accurate target state estimation.

This study focuses on time-varying problems, which differ from methods for handling static issues. In time-varying problems, finding the optimal solution requires considering the entire time-varying trajectory rather than just a single time point, making the problem more complex. Time-varying neural dynamics optimization algorithms obtain the optimal solution by accounting for temporal variations, causing the solution to evolve over time. This endows the algorithm with powerful real-time capabilities, effectively addressing the limitations of traditional optimization algorithms when handling highly dynamic systems. Furthermore, distributed neural dynamics algorithms feature parallel computation, rapid convergence, and do not require global information—only information exchange among neighbors, which helps to protect privacy. Numerous excellent time-varying algorithms have been proposed in past research [12]-[13]. In reference [12], a distributed fixed-time algorithm based on a prediction-correction method is proposed. This algorithm can solve time-varying optimization problems within a specified time frame and has been successfully applied to battery group problem solving. Reference [13] presents a distributed inertial projection neural dynamics algorithm based on Nesterov accelerated gradient descent to solve energy management issues in integrated energy networks for energy internet systems.

This paper innovatively proposes a distributed finite-time neural dynamics algorithm for multi-robot cooperative tracking of time-varying targets, integrating neural dynamics methods. First, facing a convex optimization problem with time-varying inequality constraints, a distributed finite-time neural dynamics algorithm is carefully designed using prediction-correction methods and sliding mode control techniques. Next, the stability and convergence of the proposed algorithm are systematically verified by constructing a Lyapunov function. Finally, the effectiveness of the proposed algorithm is validated through simulation experiments, ensuring that each robot can successfully track the time-varying target while skillfully avoiding obstacles.

2. Preparatory Work

2.1. Time varying convex optimization

Consider the following time-varying convex optimization problem with inequality constraints:

$$\min \quad f(x,t)=\sum_{i=1}^{n} f_i(x,t),$$
$$s.t. \quad g_j(x,t)\le 0, \quad j=1,\cdots,p,\ p\le n$$

Here, x represents the optimization variable, $f_i(x,t)$ is the objective function, and $g_j(x,t)$ represents the constraint conditions. The values of $f_i(x,t)$ 、 $g_j(x,t)$ are only known to the corresponding nodes, and each node shares local information only with its neighboring nodes. Since both the objective function $f(x,t)$ and the constraint function $g_j(x,t)$ are time-varying, the optimal solution to this problem is a dynamic solution that depends on time. The primary objective is to design an optimization algorithm that allows each node to optimize (minimize) the sum of local time-varying objective functions while satisfying the time-varying constraint conditions. Below, important lemmas and assumptions regarding the objective function and constraint conditions will be introduced. These lemmas and assumptions play a crucial role in the design and analysis of the algorithm.

Lemma 1[14]: If $f(x)$: $\mathbb{R}^n \to \mathbb{R}$ is a differentiable convex function, if and only if $\nabla f(x^*)=0$, x^* is the optimal solution of the function $f(x)$.

Lemma 2[15] : For a system:

$$\dot{x}=f(x) \tag{1}$$

Where $x\in\mathbb{R}^n$, $f:\mathbb{R}^n\to\mathbb{R}^n$, and $f(0)=0$, for all $t\ge 0$, assume that the solution $x(0)\in\mathbb{R}^n$ exists and is continuous for any initial condition.

Suppose there exists a non-negative scalar function $V:\mathbb{R}^n\to\mathbb{R}$ that is differentiable. If V is satisfied:

$$\dot{V}\le -\mu V^{\varpi} \tag{2}$$

Where, $\mu>0$ and $\varpi\in(0,1)$, then the origin is the finite-time stable equilibrium point of (1), and the settling time T satisfies $T(x(0))\le \dfrac{V(x(0))^{1-\varpi}}{\mu(1-\varpi)}$.

3. Problem Description

This paper considers the navigation problem of driving a disk robot with radius $r>0$ to a given target x_d without colliding with an obstacle in the environment. More precisely, let's consider the $W\subset R^n$ of an enclosed convex workspace. Assume that the working area is filled with a m disjoint spherical obstacle, where the center and radius of the i th obstacle are represented by $x_i\in W$ and $r_i>0$, respectively. In addition, free space $\mathrm{H}=\{x\in W:\bar{B}(x,r)\subseteq W\setminus\cup_{i=1}^{m}B(x_i,r_i)\}$ is defined to represent a feasible set of robots that do not collide with any obstacles in the workspace, where $B(x,r)$ is centered on x, a n dimensional kickoff with a radius of r, and $\bar{B}(x,r)$ represents its closure.

Let x_c represent the centroid of the robot. Given a desired final path $x_d\in\mathrm{H}$, the navigation problem can be summarized as finding a trajectory x_c such that $x_c(t)\in\mathrm{H}$, where $t\ge 0$, and $\lim_{t\to\infty} x_c(t)=x_d$. This problem can be interpreted as solving a time-varying convex optimization problem. To clarify this issue, we need to describe some definitions. First, based on the power distance $P(x,B(x_i,r_i))=\|x-x_i\|_2^2-r_i^2$ between a point x and a disk $B(x_i,r_i)$, we define the local workspace of xc as $L\mathrm{H}(x_c)=\{x\in\mathrm{H}:P(x,B(x_c,r))\le P(x,B(x_i,r_i))\forall i\}$. In other words, the set of points in H are closer to the robot (in terms of power distance) than to any obstacles. The local workspace is defined by a boundary, which is a hyperplane, and its boundary is also a hyperplane.

In addition, define the collision-free local workspace around x_c as

$$L\mathrm{H}(x_c)=\{x\in\mathrm{H}:a_i(x_c)^T x-b_i(x_c)\le 0, i,\cdots,m\}$$

Among them,

$$a_i(x_c)=x_i-x_c,\theta_i(x_c)\frac{1}{2}-\frac{r_i^2-r^2}{2\|x_i-x_c\|^2}$$

$$b_i(x_c)=(x_i-x_c)^T(\theta_i x_i+(1-\theta_i)x_c+r\frac{x_c-x_i}{\|x_c-x_i\|})$$

Assuming that the robot follows integrator dynamics, the controller proposed in [36] :

$$x_c=-K(x_c-x^*)$$

Where K > 0 is the gain of the controller and x^* is the orthogonal projection of the desired target x_d on the collision-free local working area $L\mathrm{H}(x_c)$.

Next, we extend the above problem to a multi-robot cooperative target tracking problem:

$$
\begin{aligned}
\min \quad & \sum_{i}^{n} f_i = q\left\| x_i - U(\mathrm{t}) \right\|_2^2 \\
s.t. \quad & a_j(x_i)^T x_i - b_j(x_i) \le 0, \quad \forall i \in V, j \in [1,\cdots,n] \\
& L \otimes I_m x = 0_{mn}
\end{aligned} \tag{3}
$$

Where x_i is the coordinates of robot i in the (x, y) plane, and $U(t)$ is the position of the time-varying target.

By introducing a penalty function, the target problem (3) is converted to the following form:

$$
L_i(x_i,t) = f_i(x_i,t) - \frac{1}{c(t)} \sum_{i=1}^{n} \log\left\{ b_j(t) - a_j(t)^T x_i \right\} \tag{4}
$$

4. Algorithm Description

In order to solve the problem, this paper proposes a dynamic algorithm to solve it, and the dynamic equation is described as follows:

$$
\begin{aligned}
\dot{x}_i &= -\nabla^2 L_i(x_i,t)^{-1}[\xi_1 \sum_{\mathrm{j}=1}^{n} a_{ij} sign(x_i - x_j) + \eta_i + \frac{\partial}{\partial t} \nabla L_i(x_i,t)] \\
\eta_i &= \xi_2 \varphi_i(x_i) \tanh(\nabla L_i(x_i,t)) \left| \nabla L_i(x_i,t) \right|^{\kappa} \\
\varphi_i(x_i) &= \begin{cases} \dfrac{1}{\left\| \nabla L_i(x_i,t) \right\|_2^{1-\kappa}}, & \left\| \nabla L_i(x_i,t) \right\|_2 \ne 0 \\ 0 & \end{cases}
\end{aligned} \tag{5}
$$

Where ξ_1, ξ_2 are normal numbers, $\kappa \in (0,1)$.

5. Convergence analysis

Theorem 1: Let fixed graph G be undirectionally connected. The state x_i of the i-th agent defined for (4) converges in finite time to the corresponding optimal state.

Proof: By defining the Lyapunov function as $V = \frac{1}{2}[\sum_{i=1}^{n} \nabla L_i(x_i,t)]^T [\sum_{i=1}^{n} \nabla L_i(x_i,t)]$, we get the following:

$$
\begin{aligned}
\dot{V} &= [\sum_{i=1}^{n} \nabla L_i(x_i,t)]^T [\sum_{i=1}^{n} \nabla^2 L_i(x_i,t)\dot{x}_i + \frac{\partial}{\partial t} \nabla L_i(x_i,t)] \\
&= [\sum_{i=1}^{n} \nabla L_i(x_i,t)]^T [-\sum_{i=1}^{n} \xi_1 \sum_{\mathrm{j}=1}^{n} a_{ij} sign(x_i - x_j) - \sum_{i=1}^{n} \xi_2 \frac{\tanh(\nabla L_i(x_i,t)) \left| \nabla L_i(x_i,t) \right|^{\kappa}}{\left\| \nabla L_i(x_i,t) \right\|_2^{1-\kappa}} \\
&\quad - \frac{\partial}{\partial t} \nabla L_i(x_i,t) + \frac{\partial}{\partial t} \nabla L_i(x_i,t)] \\
&= [\sum_{i=1}^{n} \nabla L_i(x_i,t)]^T [-\sum_{i=1}^{n} \xi_1 \sum_{\mathrm{j}=1}^{n} a_{ij} sign(x_i - x_j) - \sum_{i=1}^{n} \xi_2 \frac{\tanh(\nabla L_i(x_i,t)) \left| \nabla L_i(x_i,t) \right|^{\kappa}}{\left\| \nabla L_i(x_i,t) \right\|_2^{1-\kappa}}]
\end{aligned} \tag{6}
$$

According to the undirected connection graph and $\xi_{1i} = \xi_{1j}$, for all $i, j \in N$, it is possible to obtain $\sum_{i=1}^{n} \xi_{1i} \sum_{j=1}^{n} a_{ij} sign(x_i - x_j) = 0_m$, which is obtained by plugging in the above formula

$$
\dot{V} = -[\sum_{i=1}^{n} \nabla L_i(x_i,t)]^T [\sum_{i=1}^{n} \xi_2 \frac{\tanh(\nabla L_i(x_i,t)) \left| \nabla L_i(x_i,t) \right|^{\kappa}}{\left\| \nabla L_i(x_i,t) \right\|_2^{1-\kappa}}] \le -\xi_2 (2V)^{\kappa} \tag{7}
$$

According to lemma 3, under dynamic control, the state x_i of the i-th agent will converge to the optimal state in a finite time. Proof complete.

6. Experimental simulation

In our experiment, experiments with different number of obstacle environments and different time-varying target trajectories were carried out for verification. Among them, the communication network of multiple robots is a ring figure: $1 \leftrightarrow 2 \leftrightarrow 3 \leftrightarrow 4 \leftrightarrow 5 \leftrightarrow 1$, as shown in Figure 1:

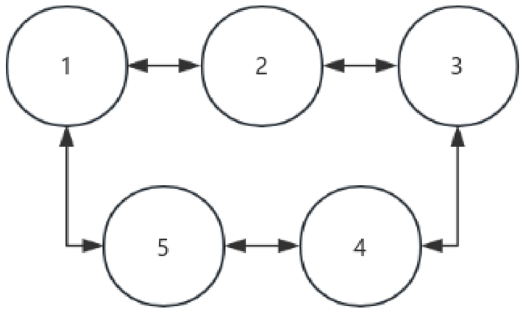

Figure 1 Connected graph

The positions and target positions of the robots are randomly given. In this experiment, the initial positions of the five robots are set as $x_1(0) = [-0.4, -2.4]^T$, $x_2(0) = [-1.1, -2.4]^T$, $x_3(0) = [-1.1, -2.1]^T$, $x_4(0) = [-0.4, -2.1]^T$, $x_5(0) = [0.3, -2.1]^T$, parameters $K = 0.05$, $\xi_1 = 2$, $\xi_2 = 3$, $\kappa = 0.8$. The experimental results obtained are shown in the figure below:

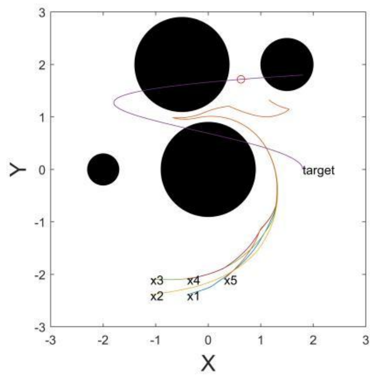

Figure 2 Tracking track diagram

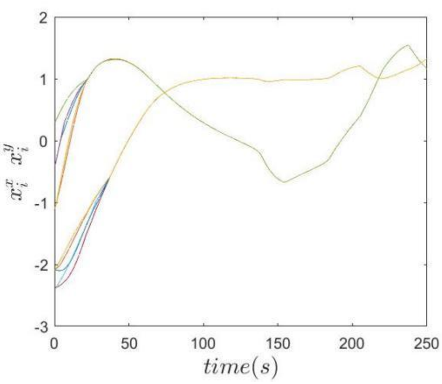

Figure 3 Location map

Figure 2 presents the successful obstacle avoidance and time-varying target tracking process of five robots in an environment with obstacles, guided by a distributed finite-time algorithm. By observing the motion trajectories in the figure, it is evident that each robot skillfully avoids obstacles and proactively adjusts its path to track the time-varying target. This clearly demonstrates that the distributed finite-time algorithm proposed in this paper not only enables collaborative actions among multiple robots, effectively preventing collisions between them, but also maximizes the use of space to achieve precise tracking of the time-varying target. Figure 3 shows the trajectory of the robots in the x and y directions, clearly indicating that the coordinates of the five robots converge, further confirming the effectiveness of the distributed finite-time algorithm.

7. Closing remarks

In practical robotic applications, large numbers of robots are often involved, and the communication between robots, as well as potential obstacles encountered during the tracking process, must be considered. In such cases, employing a distributed time-varying algorithm can effectively coordinate the movement of multiple robots, improving tracking efficiency and task completion capabilities. At the same time, in complex multi-obstacle environments, traditional centralized algorithms face significant implementation challenges, whereas distributed algorithms can more flexibly and quickly handle coordination issues. This study focuses on the robot time-varying tracking model and implements collaborative tracking among robots using a distributed finite-time algorithm. For obstacle avoidance in complex multi-obstacle environments, the application of distributed algorithms demonstrates higher scalability and flexibility. Through simulation experiments, this study also validates the effectiveness of the proposed algorithm in enabling multi-robot collaborative tracking.

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