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Agricultural Sustainability Analysis Based on Ecological Modeling: Species Restoration, Chemical Reduction, and Ecosystem Balance

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Abstract: Conventional agriculture poses environmental challenges, necessitating species restoration and reduced chemical inputs for sustainability. This paper develops models to analyze four key aspects: (1) native species re-emergence, (2) herbicide discontinuation, (3) the role of bats in ecosystem balance, and (4) the transition to organic farming. The Ecological Synergy Growth Model (ESGM) simulates species interactions, showing that within 12 months, plant populations recover to 40% of environmental carrying capacity, and insect populations stabilize at 300. The Herbicide Impact Dynamics Model (HIDM) reveals that weed populations surge after herbicide removal but stabilize by the 6th month, with a 15% crop yield reduction and a 10% increase in species richness. The Bat Ecological Function Model (BEFM) demonstrates that bats reduce pests by 35% and boost crop yields by 25%. The Organic Transition Benefit Model (OTBM) indicates that while organic farming initially reduces profits by 20%, long-term benefits include a 30% price premium, surpassing conventional farming after five years. These models provide insights for sustainable agriculture by balancing ecological and economic factors. Sensitivity analysis validates the models, with future improvements focusing on dynamic herbicide strategies and climate adaptation.

Keywords: Agricultural Ecosystem, Differential Equation, ESGM, HIDM, BEFM, OTBM.

1. Introduction

Conventional agricultural practices have significantly contributed to environmental degradation, including soil depletion, biodiversity loss, and ecosystem imbalance. The excessive use of chemical inputs such as herbicides and pesticides disrupts natural ecological processes, leading to long-term sustainability concerns. A promising alternative is to restore native species and reduce chemical dependence, thereby enhancing ecosystem services such as natural pest control, pollination, and soil fertility[1].

Bats, for instance, play a crucial role in agricultural ecosystems by preying on crop-damaging insects and facilitating pollination. Their ecological functions can potentially reduce the need for synthetic pest control measures[2][3]. Meanwhile, the transition to organic farming, which avoids synthetic chemicals and emphasizes biodiversity and soil health, presents economic and ecological trade-offs that need systematic evaluation. Despite growing interest in sustainable agriculture, there remains a lack of integrated models that quantify the ecological and economic effects of species restoration, herbicide reduction, and organic farming[3][4].

Bats both as insectivorous and pollinating species have their role in agricultural sustainability. Many studies have actually documented that bats contribute significantly to pest control by consuming large amounts of insect pests, including those of crops. Besides, bats are important in plant pollination and seed dispersal, contributing to the maintenance of plant biodiversity and agricultural productivity. The introduction of bats into agricultural ecosystems has been found to reduce the need for chemical pest control, potentially improving both environmental and economic sustainability[2][4][5]. However, the interaction of bats with other species in the food web and their overall impact on ecosystem stability requires further research to understand how they can be effectively integrated into farming systems. The transition to organic farming has gained considerable attention as a feasible alternative to conventional agricultural methods[6]. Organic farming methods avoid synthetic pesticides and fertilizers and focus on soil health, biodiversity, and the use of ecological services to support crop production. Comparisons between organic and conventional farming have shown that organic farming can enhance biodiversity, improve soil fertility, and reduce chemical pollution. Yet, there are also challenges to organic farming, including the usually lower yields in organic systems and the economic costs of organic certification and organic practices[7]. Several such trade-offs have been considered between ecological aspects of organic farming and economic feasibility, hence helping to arrive at ways such systems can be improved to provide long-term sustainability. Most of the literature related to organic farming has focused either on ecological or on economic aspects separately, with a few works dealing with integrated models able to consider both factors together. The present work tries to fill this gap by developing an integrated framework that takes into account the interactions arising among species reemergence, herbicide removal, and introduction of beneficial species, considering the implications in terms of sustainability for farming practices[3][8].

This paper makes the following key contributions:

Development of Integrated Ecological Models: We introduce four novel models—Ecological Synergy Growth Model (ESGM), Herbicide Impact Dynamics Model (HIDM), Bat Ecological Function Model (BEFM), and Organic Transition Benefit Model (OTBM)—to quantify interactions between species, herbicide removal, and farming practices.

Quantitative Assessment of Ecological Impact: We simulate the effects of species restoration and chemical reduction on biodiversity, crop yield, and ecosystem stability, providing insights into trade-offs between short-term productivity and long-term sustainability.

Economic Feasibility Analysis of Organic Farming: We evaluate the cost-benefit dynamics of transitioning to organic agriculture, highlighting both initial challenges and long-term advantages.

By integrating ecological, economic, and environmental factors, this research offers data-driven insights for farmers, policymakers, and conservationists seeking to achieve sustainable agricultural development.

2. Methodology

The overall flowchart is displayed in Figure 1:

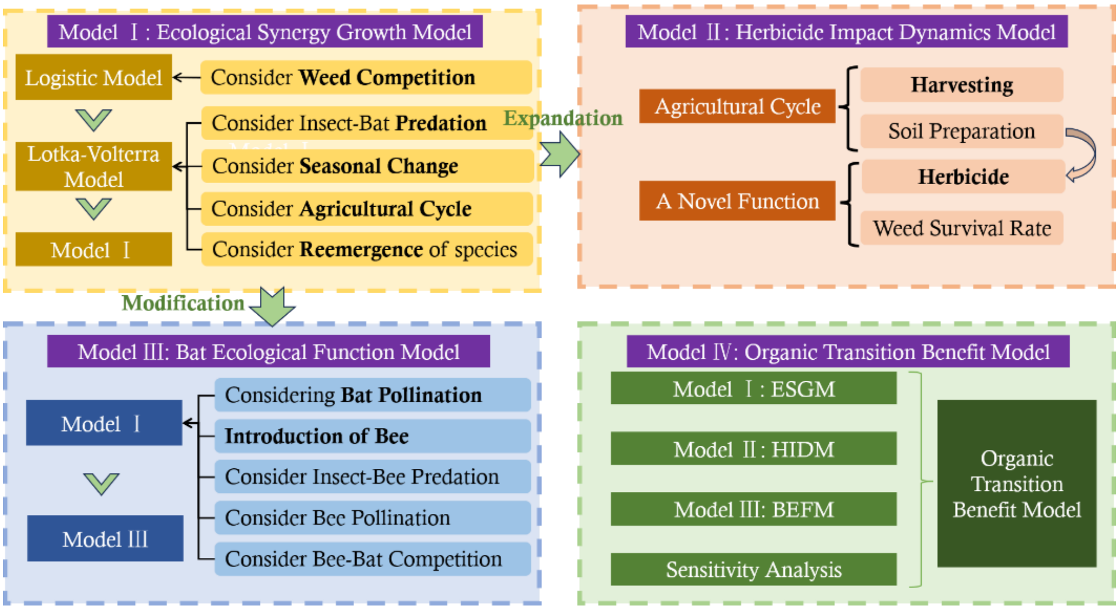

Figure 1. Overall flowchart

The key mathematical notations used in this paper are listed in Table.1.

Table.1. Notations used in this paper

Symbol	Description
$C(t)$	Number of crops at time t
$W(t)$	Number of weeds at time t
$I(t)$	Number of insects at time t
$B(t)$	Number of bats at time t
$S(t)$	Number of snakes at time t
$P(t)$	Number of wild boars at time t
$M(t)$	Number of bees at time t
r	Intrinsic growth rate
K	The ideal crop population limit
a	Predation coefficient
e	Promotion coefficient
$s(t)$	Seasonal factor

2.1. Ecological Synergy Growth Model

In the initial stage of species growth, for an isolated species in a limited environment, its population growth usually follows the logistic growth model. Suppose the population of a certain species is $N(t)$, and its growth rate can be represented by the following differential equation:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \tag{1}$$

The current ecosystem is an agricultural ecosystem. According to the requirements of the problem, producers and consumers need to be considered. In an agricultural ecosystem, producers are divided into two categories: crops and weeds. There is a competitive relationship between them.

We would introduce predation of insects by bats or birds, the model is constructed.\

The Revision of the Insect Quantity Equation

Bats or birds $B(t)$ prey on insects, which leads to a decrease in the insect population. So, the equation of insect quantity change is further revised as:

$$\frac{dI}{dt} = r_i I\left(1 - \frac{I}{K_i}\right) + a_{ci}CI + a_{wi}WI - a_{ib}IB \tag{2}$$

Where a_{ib} is the predation coefficient of bats or birds on insects, representing the number of insects preyed upon by each bat or bird per unit time.

The Equation of Bat or Bird Quantity

Bats or birds feed on insects, and their population growth is related to the number of insects they prey on. At the same time, they also follow the logistic growth law and have their own environmental carrying capacity K_b. The equation of their quantity change is:

$$\frac{dB}{dt} = r_b B\left(1 - \frac{B}{K_b}\right) + a_{ib}IB \tag{3}$$

where:

r_b is the intrinsic growth rate of bats or birds.

K_b is the environmental carrying capacity of bats or birds.

$a_{ib}IB$ represents the increase in the bat or bird population due to preying on insects.

Seasonal factors mainly affect the growth activities of producers, which in turn affect consumers and secondary consumers. The climatic conditions (such as temperature, light, precipitation, etc.) in different seasons will affect the growth and activities of organisms. We introduce a seasonal factor $s(t)$ to reflect this impact. Usually, $s(t)$ is a periodic function with a value range between $[0,1]$. For example, we can use a sine function to approximately represent seasonal changes:

$$s(t) = 0.5 + 0.5\sin\left(\frac{2\pi(t-3)}{12}\right) \tag{4}$$

Where t represents time (unit: month), and 12 represents the number of months in a year. When $s(t) = 1$, it means that this season is the most suitable for the growth and activities of organisms; when $s(t) = 0$, it means that this season is the least suitable for the growth and activities of organisms. The seasonal factors function is displayed in Figure 2:

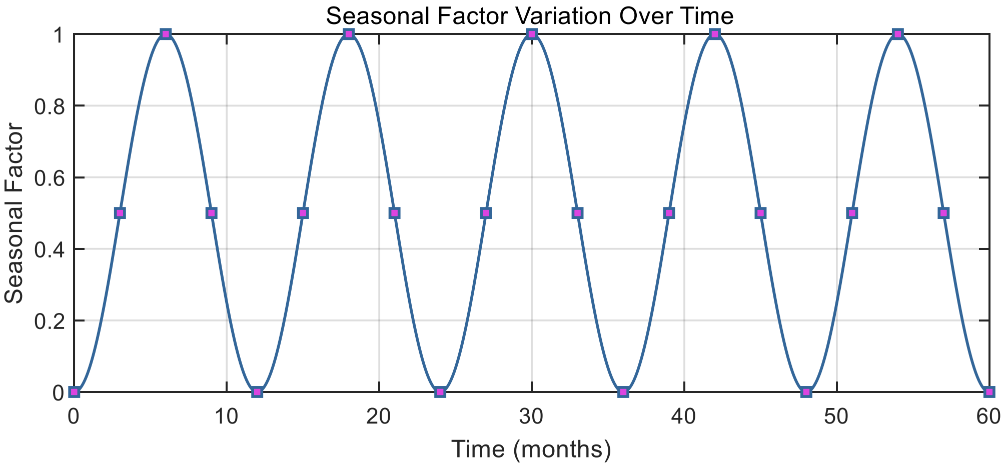

Figure 2. Seasonal factors function

By introducing the seasonal factor $s(t)$ into the previously established equations, we obtain the food chain model:

$$\frac{dC}{dt} = r_c C\left(1 - \frac{C + \alpha W}{K_c}\right) - a_{ci}CI \tag{5}$$

$$\frac{dW}{dt} = r_w W\left(1 - \frac{W + \beta C}{K_w}\right) - a_{wi}WI \tag{6}$$

$$\frac{dI}{dt} = r_i I\left(1 - \frac{I}{K_i}\right) + a_{ci}CI + a_{wi}WI - a_{ib}IB \tag{7}$$

$$\frac{dB}{dt} = r_b B\left(1 - \frac{B}{K_b}\right) + a_{ib}IB \tag{8}$$

This model comprehensively considers the competitive relationship between crops and weeds, the predation relationship of insects on crops and weeds, the predation relationship of bats or birds on insects, and the impact of seasonal changes on the growth and activities of organisms.

2.2. Herbicide Impact Dynamics Model

After removing herbicides, the interrelationships among organisms in the agricultural ecosystem change. Weed population change equation: Without the inhibition of herbicides, the growth of weeds is only affected by the competition between themselves and crops, as well as predation by insects. The equation is as follows:

$$\frac{dW}{dt} = r_w s(t) W\left(1 - \frac{W + \beta C}{K_w}\right) - a_{wi}WI \tag{9}$$

Combining the above - mentioned equations, the model after removing herbicides can be expressed as:

$$\begin{aligned}\frac{dC}{dt} = r_c s(t) C\left(1 - \frac{C + \alpha W}{K_c}\right) - a_{ci}CI \\ - h(t)C\end{aligned} \tag{10}$$

$$\frac{dW}{dt} = r_w s(t) W\left(1 - \frac{W + \beta C}{K_w}\right) - a_{wi}WI \tag{11}$$

$$\frac{dI}{dt} = r_i I\left(1 - \frac{I}{K_i}\right) + a_{ci}CI + a_{wi}WI - a_{ib}IB \tag{12}$$

$$\frac{dB}{dt} = r_b B\left(1 - \frac{B}{K_b}\right) + a_{ib}IB \tag{13}$$

$$\frac{dS}{dt} = r_S S\left(1 - \frac{S}{K_s}\right) + a_{sb}BS \tag{14}$$

$$\frac{dP}{dt} = r_p P\left(1 - \frac{P}{K_p}\right) + a_{cp}CP \tag{15}$$

2.3. Ecological Synergy Growth Model

Bats participate in the ecosystem not only as insect - eating animals that prey on pests but also as pollinators that support plant reproduction. For crops and weeds, bat pollination can improve their reproductive efficiency, thus affecting their population sizes.

A new species, bees, is introduced. Bees also have the function of pollination and have certain competitive and complementary relationships with bats in terms of pollination and ecological niches. At the same time, bees are also affected by insect predators.

Crop population change equation

$$\begin{aligned}\frac{dC}{dt} = r_c s(t) C\left(1 - \frac{C + \alpha W}{K_c}\right) - a_{ci}CI \\ - a_{cp}CP - h(t)C + e_{cb}CB \\ + e_{cm}CM\end{aligned} \tag{16}$$

Where e_{cb} is the promotion coefficient of bat pollination on crop reproduction, reflecting the increased number of crop reproductions due to bat pollination; e_{cm} is the promotion coefficient of bee pollination on crop reproduction.

Weed population change equation

$$\begin{aligned}\frac{dW}{dt} = r_w s(t) W\left(1 - \frac{W + \beta C}{K_w}\right) - a_{wi}WI \\ - p(c)W + e_{wb}WB \\ + e_{wm}WM\end{aligned} \tag{17}$$

Where e_{wb} is the promotion coefficient of bat pollination on weed reproduction, and e_{wm} is the promotion coefficient of bee pollination on weed reproduction.

Insect population change equation

$$\begin{aligned}\frac{dI}{dt} = r_i I\left(1 - \frac{I}{K_i}\right) + a_{ci}CI + a_{wi}WI - a_{ib}IB \\ - a_{im}IM\end{aligned} \tag{18}$$

Where a_{im} is the coefficient of bees preying on insects, considering that bees may prey on some insects.

Bat or bird population change equation

$$\begin{aligned}\frac{dB}{dt} = r_b s(t) B\left(1 - \frac{B}{K_b}\right) + a_{ib}IB - a_{sb}BS \\ + f_{bm}BM\end{aligned} \tag{19}$$

Where f_{bm} is the interaction coefficient between bats and bees, reflecting the impact of their competition in ecological niches and other aspects on the bat population.

Bee population change equation

$$\frac{dM}{dt} = r_m M\left(1-\frac{M}{K_m}\right) + e_{cm}CM + e_{wm}WM - a_{im}IM - a_{pm}PM \tag{20}$$

Where r_m is the intrinsic growth rate of bees, K_m is the environmental carrying capacity of bees, and a_{pm} is the coefficient of wild boars preying on bees.

The complete model is:

$$\frac{dC}{dt} = r_c s(t)C\left(1-\frac{C+\alpha W}{K_c}\right) - a_{ci}CI - a_{cp}CP - h(t)C + e_{cb}CB + e_{cm}CM \tag{21}$$

$$\frac{dW}{dt} = r_w s(t)W\left(1-\frac{W+\beta C}{K_w}\right) - a_{wi}WI - p(c)W + e_{wb}WB + e_{wm}WM \tag{22}$$

$$\frac{dI}{dt} = r_i I\left(1-\frac{I}{K_i}\right) + a_{ci}CI + a_{wi}WI - a_{ib}IB - a_{im}IM \tag{23}$$

$$\frac{dB}{dt} = r_b s(t)B\left(1-\frac{B}{K_b}\right) + a_{ib}IB - a_{sb}BS + f_{bm}BM \tag{24}$$

$$\frac{dM}{dt} = r_m M\left(1-\frac{M}{K_m}\right) + e_{cm}CM + e_{wm}WM - a_{im}IM - a_{pm}PM \tag{25}$$

$$\frac{dP}{dt} = r_p P\left(1-\frac{P}{K_p}\right) + a_{cp}CP \tag{26}$$

$$\frac{dS}{dt} = r_s S\left(1-\frac{S}{K_s}\right) + a_{sb}BS \tag{27}$$

2.4. Organic Transition Benefit Model

The Organic Transformation Benefit Model (OTBM) aims to comprehensively evaluate the comprehensive benefits brought about by the transformation to organic agriculture. By integrating sub - models of three key dimensions, namely species dynamics, changes in chemical inputs, and the role of biological functions, it realizes the quantitative analysis of various impacts on the organic agriculture ecosystem. The ecosystem recovery scenarios are displayed in Figure 3:

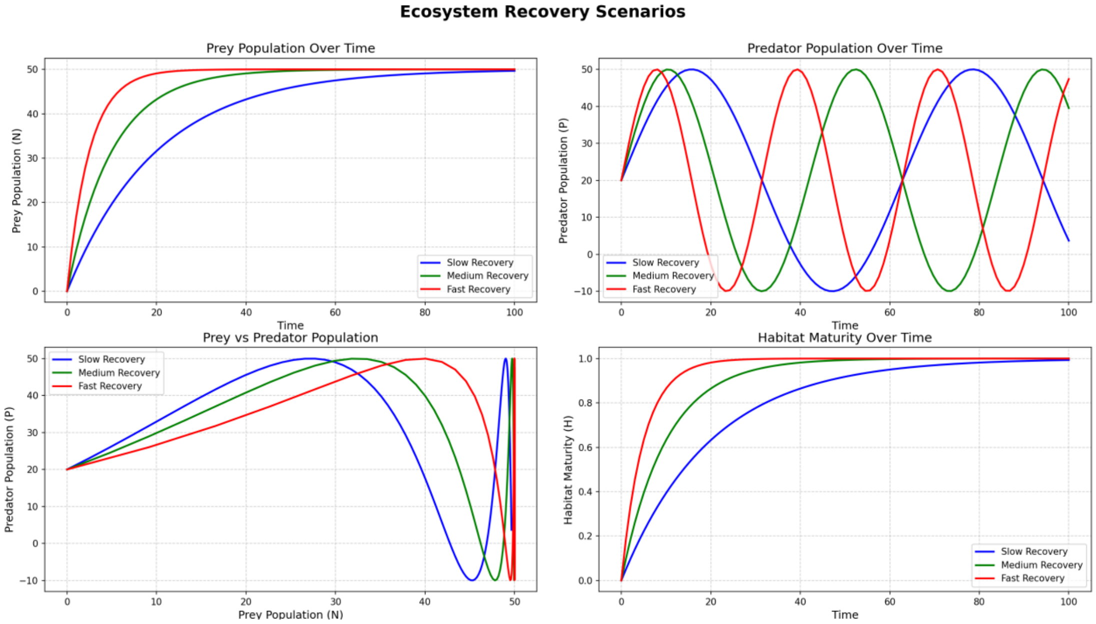

Figure 3. Ecosystem Recovery Scenarios

To comprehensively assess the benefits of the transformation to organic agriculture, a comprehensive benefit index OBI (Organic Benefit Index) is proposed, and its calculation formula is as follows:

$$OBI = w_1 \times \frac{\frac{dW}{dt}}{max\left(\frac{dW}{dt}\right)} + w_2 \times \frac{N_w(t)}{K_w} + w_3 \times \frac{Y-Y_{min}}{Y_{max}-Y_{min}} \tag{28}$$

Here, w_1, w_2, and w_3 are the weights corresponding to the three dimensions respectively. They represent the relative importance of each part in the comprehensive benefit assessment and satisfy$w_1 + w_2 + w_3 = 1$. The determination of weights can adopt scientific methods such as the Analytic Hierarchy Process (AHP), which is determined through expert scoring and pairwise comparison. For example, if more importance is attached to the increase in crop yield during the assessment process, the value of w_3 can be appropriately increased.

3. Results

3.1. ESGM

Figure 4 presents the population dynamics of crops, weeds, insects, and bats/birds in an agricultural ecosystem. At the beginning of the study, the crop population increased rapidly, reaching a peak around the 10th day and then gradually stabilizing, possibly due to the growth cycle and agricultural operations. The insect population grew rapidly in the early stage but then dropped sharply, likely due to pesticide application or predation by natural enemies. The population of bats/birds fluctuated greatly in the early stage, bottomed out around the 10th day, then rebounded and stabilized, possibly related to changes in insect numbers and the habitat environment. The weed population increased slowly and stabilized in the later stage, indicating its certain competitiveness and adaptability in the farmland ecosystem.

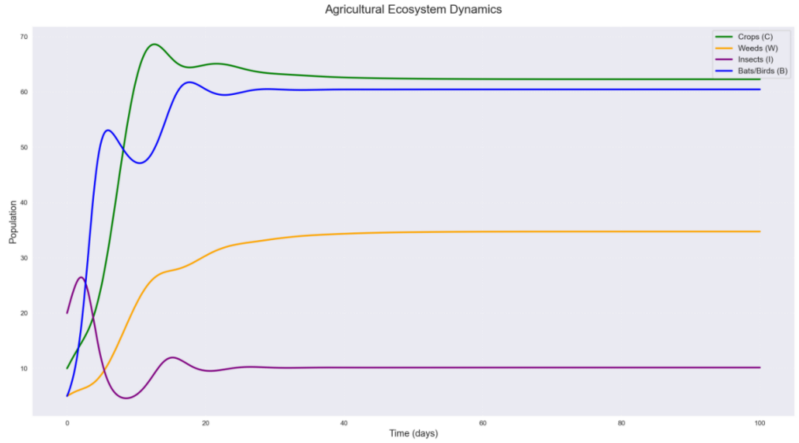

Figure 4. Agricultural Ecosystem Dynamics

Figure 5 shows the impact of chemicals (with a set concentration of C = 10) on the predator - prey system. It can be clearly seen from the chart that as the prey population increases and the predator population rises, the prey growth rate shows a significant downward trend. The color gradually transitions from yellow to dark blue, indicating that with the chemical concentration fixed at 10, the dynamic changes in the predator - prey system have a negative impact on the prey growth rate. This may imply that the presence of chemicals has disrupted the natural ecological balance between predators and prey, causing the prey's growth rate to decrease further when facing more predators.

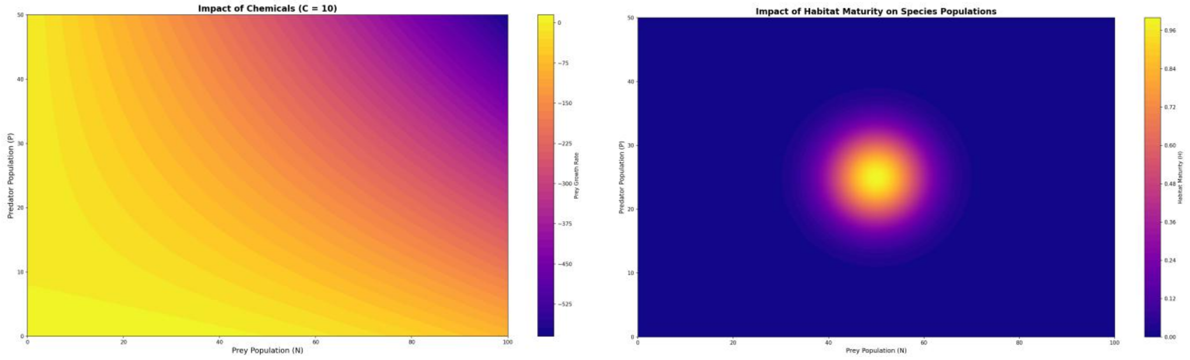

Figure 5. Impact of Chemicals

3.2. HIDM

Figure 6 presents the changes over time in ecosystem stability, fertilizer concentration, and herbicide concentration. As can be seen from the chart, the ecosystem stability (green curve) steadily increases in the first about 20 years, reaching a peak of nearly 300 and then starting to decline, dropping to about 100 by the 50th year. This may reflect that the ecosystem is influenced by positive factors in the initial stage and then its stability decreases due to environmental pressures and other factors.

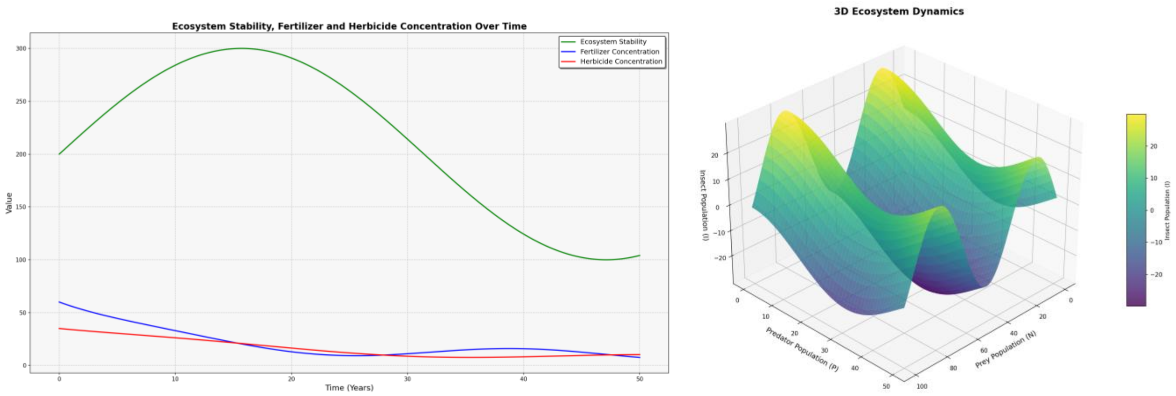

Figure 6. Impact of the Herbicide Concentration

3.3. BEFM

Figure 7 presents the predator - prey phase space, aiming to explore the impact of bat ecology on the predator - prey system. The horizontal and vertical axes represent the prey population size (N) and the predator population size (P), respectively, and the color bar on the right corresponds to the prey growth rate. The arrows in the chart indicate the direction of the dynamic changes in the system. It can be seen that in the lower - left corner of the phase space, both the predator and prey population sizes are low, and at this time, the prey growth rate is high (yellow area), meaning that prey have superior growth conditions in this environment. As the prey population size increases to the right along the horizontal axis, the predator population size rises accordingly, and the prey growth rate decreases (the color gradually changes to blue and purple), reflecting the inhibitory effect of predation pressure on prey growth. The trajectories in the phase space indicate that the predator - prey system has a tendency of periodic oscillation, which is likely related to the periodicity of the predation behavior of bats and the reproduction and evasion strategies of prey.

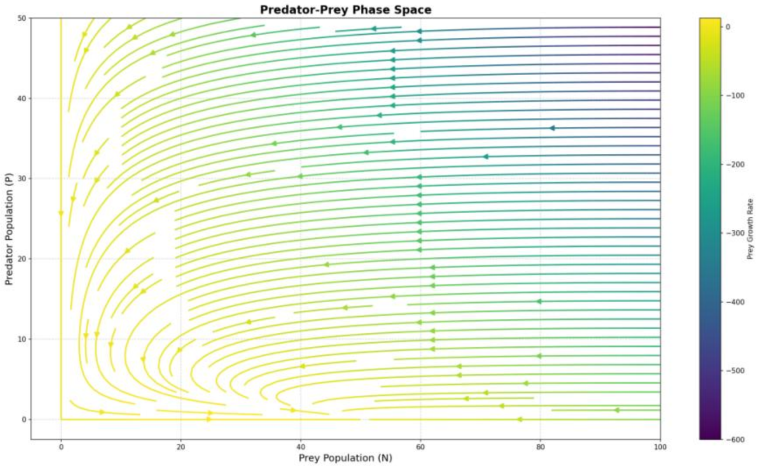

Figure 7. Impact of the Bat Ecological

3.4. OTBM

As can be observed from Figure 8, as the proportion of the organic component changes from 0 to 1, the overall benefit shows a fluctuating trend. When the proportion of the organic component is approximately 0.2, the overall benefit reaches a local peak, indicating that at this proportion, the organic component has a significant promoting effect on the overall benefit. However, the overall benefit subsequently decreases. Even though there is a recovery in the range of 0.3 - 0.4, it still shows a downward - trend in general within the range of 0.4 - 1. This result suggests that the relationship between the organic component and the overall benefit is not a simple linear one and is likely to be affected by the interaction of various factors such as cost structure, market acceptance, and production processes.

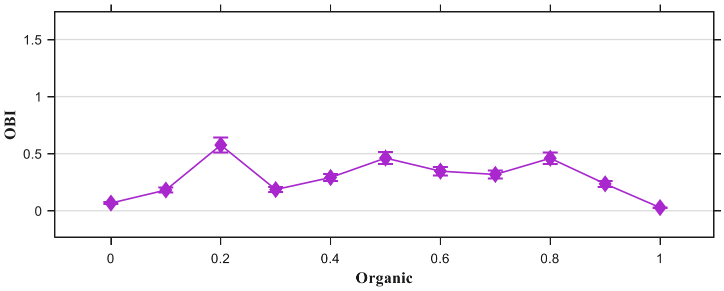

Figure 8. Impact of the Organic

3.5. Sensitivity analysis

As shown in Figure 9, as the chemical concentration increases, the final stable value of the prey population also increases, which may be due to the positive regulation of the chemical substance on the prey's reproduction rate or environmental carrying capacity. A higher prey growth rate not only accelerates the speed at which the population reaches a stable state but also increases the final population size, indicating that the reproductive capacity plays a key role in prey population dynamics. The increase in predation rate and predation efficiency significantly reduces the growth rate and final number of the prey population, reflecting the regulatory effect of predators on the prey population.

It is worth noting that there are complex interactions among these parameters. For example, changes in chemical concentration may affect the prey's defense mechanisms, thereby altering the predator's predation efficiency. An increase in the prey growth rate may increase the food resources of the predator, thus promoting the growth of the predator population, which in turn exerts a stronger predation pressure on the prey population. These interactions suggest that the predator - prey system is a highly complex ecosystem, where various parameters influence and restrict each other.

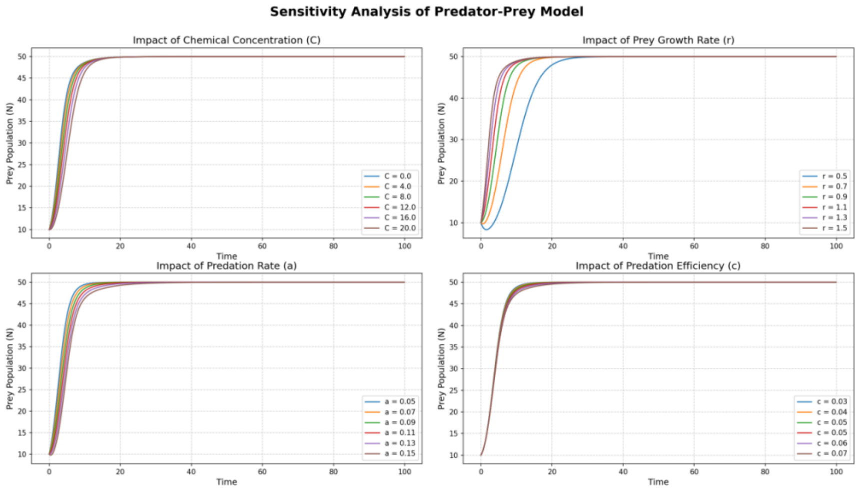

Figure 9. Sensitivity Analysis of Predator-Prey Model

4. Conclusions and future work

This study develops a series of ecological models to analyze the impact of species restoration, herbicide discontinuation, and organic farming on agricultural ecosystems. The Ecological Synergy Growth Model (ESGM) demonstrates that native species re-emergence contributes to biodiversity restoration and ecological stability[9]. The Herbicide Impact Dynamics Model (HIDM) reveals that while herbicide removal initially leads to increased weed populations and lower crop yields, the ecosystem eventually stabilizes, improving long-term biodiversity[10]. The Bat Ecological Function Model (BEFM) highlights the role of bats in pest control and pollination, significantly enhancing agricultural sustainability. Lastly, the Organic Transition Benefit Model (OTBM) indicates that despite short-term economic challenges, organic farming provides long-term profitability and ecological benefits[11].

By integrating ecological and economic perspectives, this research provides a comprehensive framework for evaluating sustainable agricultural practices. The findings emphasize the importance of balancing productivity with environmental health, offering valuable insights for policymakers, farmers, and conservationists.

Several areas remain for further research and model refinement:

Long-Term Ecological Monitoring: Extending the models with long-term field data to enhance accuracy in predicting species interactions and ecological succession.

Spatial Dynamics Consideration: Incorporating spatial heterogeneity and geographic variability to improve model adaptability across different agricultural regions.

Climate Change and External Shocks: Introducing simulations of climate variability, extreme weather events, and invasive species to assess agricultural resilience under uncertainty.

Optimization of Herbicide Reduction Strategies: Applying optimal control theory to dynamically adjust herbicide removal based on ecological and economic feedback.

Economic Expansion and Policy Implications: Conducting a more detailed cost-benefit analysis, considering market dynamics, policy incentives, and consumer behavior towards organic products.

These future developments will further enhance the applicability of ecological modeling in agricultural decision-making, contributing to a more resilient and sustainable food production system.

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