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Optimization Design Method for Wellbore Trajectory of Cluster Well Infill Drilling Based on Multi-Objective Genetic Algorithm

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Abstract: Cluster well infill drilling faces challenges such as limited space for wellbore passage, high collision risks with existing underground production wells, and difficulties in wellbore trajectory design. To improve the efficiency of wellbore trajectory design, reduce collision risks, and enhance control, this study proposes an intelligent optimization method for wellbore trajectory design parameters using a multi-objective genetic algorithm. Based on the analysis of geometric parameter relationships in wellbore trajectories, a multi-parameter model for wellbore trajectory design and a distance calculation model between the designed trajectory and adjacent well trajectories were established. With the objectives of minimizing well depth and reducing friction, a fitness function was constructed, an individual selection strategy was formulated, and a hierarchical iterative calculation process for optimizing wellbore trajectory design parameters was designed. A case study of infill drilling on an artificial island compared single-objective and multi-objective optimizations, demonstrating that the multi-objective optimized trajectory outperformed the single-objective optimized trajectory.

Keywords: Track Design; Genetic Algorithm; Multi-Objective Optimization; Anti-Collision

1. Introduction

Cluster well infill drilling is an important method to enhance oil production. However, cluster well sites often feature high development intensity, dense wellheads, complex underground spaces, numerous constraints on wellbore trajectory design, and stringent requirements for collision avoidance calculations. Conventional design methods require repeated iterations with different parameters to achieve the desired trajectory, resulting in low efficiency and difficulty in obtaining optimal solutions [1,2].

For wellbore trajectory design, Yi Hu et al.[3] proposed that early deviation is beneficial for collision avoidance in shale gas cluster wells. Gu Yue et al.[4] used a genetic algorithm for multi-objective optimization of the outermost well's depth, trajectory potential energy, and lateral displacement, but the weighted approach made it challenging to determine optimal weight coefficients. Vahid Mansouri et al. [5] applied a genetic algorithm to optimize the length and torque of extended-reach horizontal wells. Sha et al.[6] employed a quantum genetic algorithm based on the Fibonacci sequence for complex wellbore trajectories, achieving high-quality global optimal solutions through parallel computation. Liu Luo [7] used genetic and particle swarm algorithms for multi-objective optimization of wellbore trajectories, reducing trajectory length, drilling costs, and friction torque compared to the original design, validating the superiority of multi-objective optimization. While progress has been made in collision avoidance and multi-objective optimization, comprehensive studies integrating both aspects are limited, and further research is needed for top-down optimization methods in infill wells.

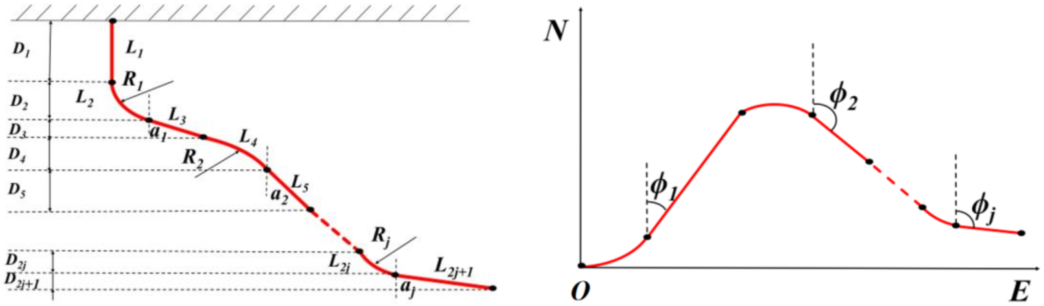
To address these issues, this study adopts a multi-objective genetic algorithm to optimize wellbore trajectories for infill wells, considering well depth and friction. The top-down minimum distance scanning method is used to calculate the closest distance between the designed trajectory and adjacent wells. A case study of infill drilling on an artificial island in China compared single-objective and multi-objective designs, showing that multi-objective optimization generally yields better results when no specific requirements are imposed.

2. Well hole track and friction

To optimize for smaller well depth and lower friction, a wellbore trajectory design model and friction-torque calculation equations were established, with explicit objective functions defined below.

2.1. Wellbore Trajectory Design Model

The borehole track is a complex curve from the wellhead to the target according to the design requirements, which is usually composed of straight well section, inclined section and stabilized section. As shown in Figure 1((a) is a vertical profile, and (b) is a horizontal projection diagram). There are two types of two-dimensional and three-dimensional, so it is necessary to comprehensively analyze the constraints of anti-collision requirements, the distribution of wellhead targets and other conditions, and analyze the specific design methods of orbit. For the curve part of the orbit, there are many hypothesis calculation models, this paper uses the minimum curvature method, for the curvature, curvature radius, well depth and vertical depth of the orbit as shown in equation (1) - (5):



(a)Vertical profile (b) Horizontal projection diagram

Figure 1. A Schematic diagram of the wellbore track

$$K_j = \frac{\arccos(\cos a_j \cos a_{j-1} + \sin a_j \sin a_{j-1} \cos(\phi_j - \phi_{j-1}))}{L_{2j}} \tag{1}$$

$$R_j = \frac{1}{K_j} \tag{2}$$

$$K_j = R_j(\arccos(\cos a_j \cos a_{j-1} + \sin a_j \sin a_{j-1} \cos(\phi_j - \phi_{j-1}))) \tag{3}$$

$$TVD = D_1 + D_2 + \cdots + D_{2j} + D_{2j+1} \tag{4}$$

$$TMD = L_1 + L_2 + \cdots + L_{2j} + L_{2j+1} \tag{5}$$

In equation (1) ~ (5),D2j,L2j are the vertical depth and length of the corresponding well respectively, m, aj ,ϕj ,Rj and Kj are the angle of the well, (°), azimuth, (°), curvature radius, m, hole curvature 1 / m; TVD is the total vertical depth of the well, m; TMD is the total length of the well, m.

2.2. Friction-Torque Calculation

At present, there has been a lot of research on the calculation model of friction torque and is relatively mature. At present, according to the whether to consider the bending moment of the drilling column, the calculation model is divided into two models of soft rod and rigid rod. This paper uses the classical soft rod model for calculation, assuming: ignoring the shear force and the stiffness of the drilling column; the pipe column is in continuous contact with the shaft shaft, and ignoring the influence of drilling fluid flow. The diagram of the column is shown in 2, and the difference equation for calculating the model is shown in equation (6) - (9):

$$F_i = F_{i+1} + (-q_i \cos\bar{\alpha}_i \mp \mu_1 n_i)\Delta s_i \tag{6}$$

$$M_i = M_{i+1} + \frac{1}{2}\mu_2 n_i D_b \Delta s_i \tag{7}$$

$$\bar{\alpha}_i = \frac{\alpha_{i+1} + \alpha_i}{2} \tag{8}$$

$$n_i = \sqrt{(F_{i+1}(\phi_{i+1} - \phi_i)\sin\bar{\alpha}_i)^2} + (F_{i+1}(\alpha_{i+1} - \alpha_i) + q\sin\bar{\alpha}_i)^2 \tag{9}$$

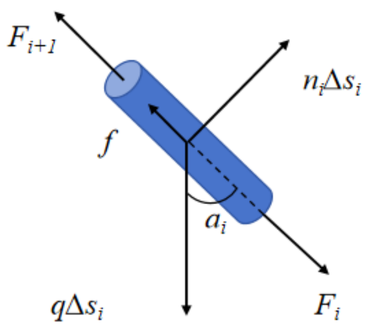


Figure 2. Schematic diagram of microelement of drill column

The above formula represents the calculation of the friction torque in the microelement of the drilling column, where Fi and Fi+1 is the axial force up and down of the segment, N；q for drilling column line weight, N/m; $\bar{a_i}$ is the average well Angle, °; μ1 and μ2 for the axial and circumferential friction coefficients; ni is the contact force between the drill column and the well wall, N/m; The Δsi is the length of the drill column, m; Mi and Mi+1 are the torque at the upper and lower sections, N·m; Db is the outer diameter of the drill column; m; ai and ai+1 are the upper and lower well slope, and ϕ and ϕi+1 are the upper and lower azimuth.

2.3. Multi-Objective Optimization

Wellbore trajectory design involves numerous constraints, and for cluster wells, additional consideration must be given to the distance relationships with adjacent completed wells. These requirements significantly increase the complexity of the problem. By employing multi-objective optimization, it becomes possible to simultaneously address these nonlinear and even conflicting objectives, enabling all target functions to achieve relatively optimal conditions and thereby enhancing the drillability of the designed trajectory. The improved multi-objective solutions obtained through computational models are commonly referred to as "Pareto optimal solutions."

For a minimized function F(x) multi objective solution, it has m objective functions (f1(x),f2(x)··fm(x)), including:

$$\begin{aligned} & minF(x)=\left[f_1(x), f_2(x),\ldots, f_m(x)\right]^T \\ & s.t.\ \ h_j(x)=0,\ j=1,2,\ldots,J \\ & \qquad g_k(x)\le 0, k=1,2,\ldots,K \end{aligned} \tag{10}$$

Multi-objective optimization is the optimization of all objectives. For a feasible solution vector x, if there is no feasible solution in the decision space X that can dominate it, then the solution vector x is a non-dominant solution, which can also be called a Pareto optimal solution. The corresponding map of the non-dominant solution set P in the target space is the Pareto front. As shown in Figure 3, to find the minimum value of double objective functions f1 and f2, graph A Pareto can dominate C and B Pareto dominates C, so A and B are superior to C, and there is no point point A or B, so A and B are the optimal solution of Pareto.

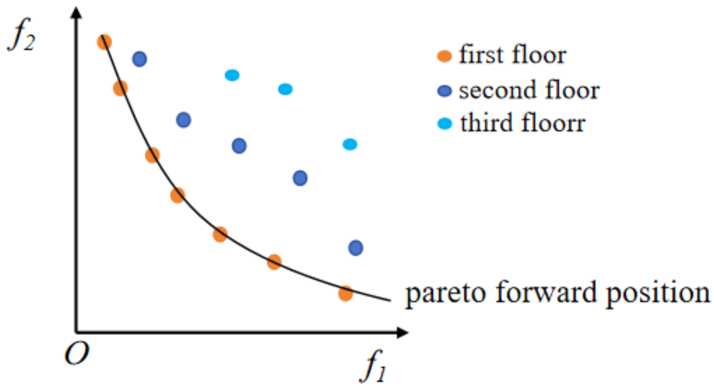


Figure 3. Pareto Schematic diagram of the front edge

3. Genetic algorithm

The difference between multi-objective genetic algorithm and single objective genetic algorithm lies mainly in the improvement of fitness function, instead of quantifying and optimizing one objective, but optimizing all objective functions. The main improvement is to calculate the non-dominant hierarchy of all solutions, rank the non-dominant hierarchy of all solutions, and select the population of the next generation based on the number of population and the selection mechanism. For the solutions of the same non-dominant layer are sorted according to the crowding distance for selection. As shown in Fig. 4, the crowding degree of the i th solution in the target space can be measured by the circumference of the rectangle formed by the two adjacent solutions i 1 and i+1, which is expressed as id. For the solution at both ends of the non-dominant layer, the id can be set to infinite size.

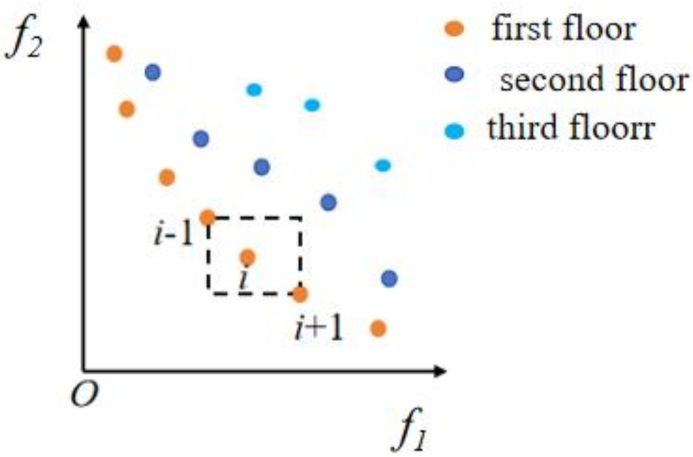


Figure 4. A Schematic diagram of the crowding degree calculation

In terms of coding, this paper conducts 12-bit binary coding for the oblique point, steady oblique Angle, azimuth, and borehole curvature. Roulette method is used to select the solution space after non-dominant ranking. The lower the ranking level, the greater the chance of being selected.

The crossover rate and the variation rate are adaptive in terms of crossover and variation, which will also change as the population evolves. The crossover rate Pc, the change range is [0.3,0.9], gradually decreases with the increase of iteration, and the greater the fitness in the current evolutionary generation, the smaller the variation rate Pm, the change range is [0.01,0.3], as the increase of iteration, and the greater the fitness in the current evolutionary generation, the smaller. The specific formula[8] is shown in equation (11):

$$P_c = 0.9 - 0.3 \cdot \frac{F' - F_{\min}}{F_{\max} - F_{\min}} \cdot \frac{1 + tanh(2n/N)}{2}$$
$$p_m = 0.3 - 0.09 \cdot \frac{F - F_{\min}}{F_{\max} - F_{\min}} \cdot \frac{1 + tanh(2(1 - n/N))}{2} \quad (11)$$

Specifically, Fmax and Fmin are the maximum and minimum fitness obtained by the individuals of the current population, respectively, F' is the parent of the parent and the parent, F is the fitness of the variable individual, n and N are the current number of population evolution and the maximum evolutionary number of the population, respectively, and tanh () is the hyperbolic tangent function.

4. Example analysis

For an artificial island in China, the encrypted drilling track design of cluster Wells is carried out. By 2024, more than 100 of the artificial islands have been drilled, with dense wellhead and underground space complex. The track design requires not only anti-collision analysis, but also the complexity, well depth and friction torque of the designed track, so as to design a track conducive to actual drilling. In this regard, the present method is used to design the track. The relative coordinates of the well wellhead are (210, -30,0), and the design parameters of the target site relative to the wellhead are shown in Table 1.

Table 1. Design of well target parameters and intermediate target requirements

Target spot	Down deep(m)	Target Point Design					Target requirements
		coordinate		azi(°)	horizontal shift (m)	Inc(°)	
		+E/-W	+N/-S				
A	3000.00	-1445.00	579.00	284.00	1969.12	—	Steady falls deep through the target≤2m horizontal shift ≤10m
B	3087.16	-2411.62	820.00	284.00	2964.57	84.95	

In this paper, we comprehensively determine the GA parameters through the[8,9] of numerous experiments and the influence of related domain GA parameters on the results, and Table 2 shows the GA parameter settings.

Table 2. Genetic algorithm parameter setting

Parameter	Set the value
Gene length for each parameter	12
Population quantity	200
Cross probability	[0.6,0.9]
Mutation rate	[0.01,0.3]
Iterations	150

According to the general value range of drilling parameters of the artificial island and the requirements of[10], the value range of drilling track design parameters is shown in Table 3.

Table 3. Value range of borehole track design parameters

Constraint parameters	Span
KOP(m)	[100,300]
Stable Angle(°)	[10,85]
DLS(°/30m)	[1,5]
The nearest distance to the adjacent well(m)	≥MD/100(MD for the oblique depth of the bore track，m)

For the friction torque calculation, the parameters are set as shown in Table 4:

Table 4. Friction torque calculation parameters

Parameter	Set the value
Drilling rod outer diameter(mm)	139.70
Drilling rod inner diameter(mm)	89.00
Floating force coefficient	0.7
Friction coefficient	0.25
Unit weight (kg/m)	30

After multi-objective optimization of the orbit, the obtained optimization results of well depth and upper tipper resistance are shown in Figure 5, and the design orbit parameters with the shortest well depth, the least upper tipper resistance and the largest crowding in the pareto front are shown in Table 5.

Table 5. Design scheme table

Plan			The minimum depth of the well	The minimum drag	Multiobjective optimization
Section1	Kop(m)		144.25	130.48	270.35
Section2	DLS(°/30m)		4.77	1.01	1.93
	Stable oblique point1	inc(°)	28.55	16.11	25.10
		azi(°)	292.04	300.48	294.68
Section3	Long stable oblique section(m)		2580.66	1943.80	1952.72
Section4	DLS(°/30m)		1.8	4.12	2.22
	Stable oblique point2	inc(°)	49.11	74.35	63.32
		azi(°)	284.00	284.00	284.00
Section5	Long stable oblique section(m)		87.90	835.81	369.90
Section6	DLS(°/30m)		4.80	4.07	3.69
	Stable oblique point3	inc(°)	85.00	85.00	85.00
		azi(°)	284.00	284.00	284.00
Section7	Long stable oblique section(m)		1000.00	1000.00	1000.00
Total well depth(m)			4568.89	4896.53	4684.03
Friction (kN)			180.01	160.05	165.21

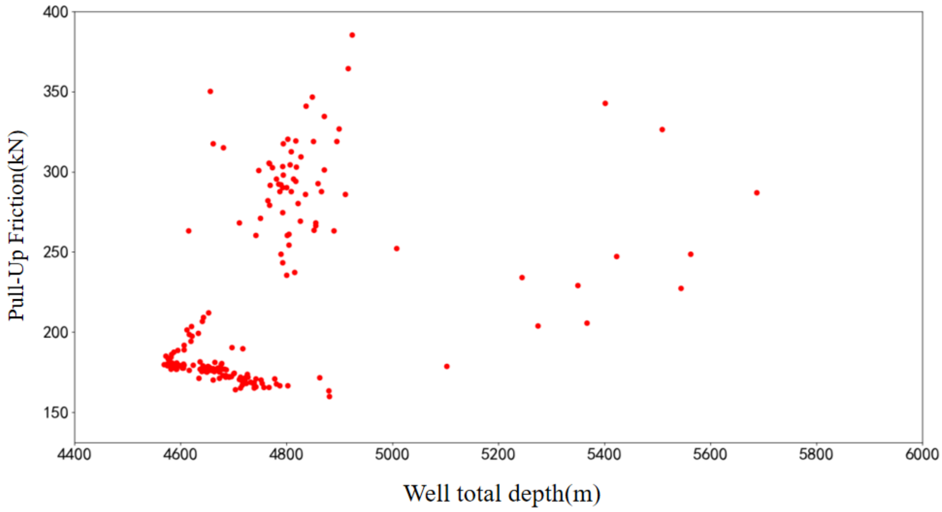
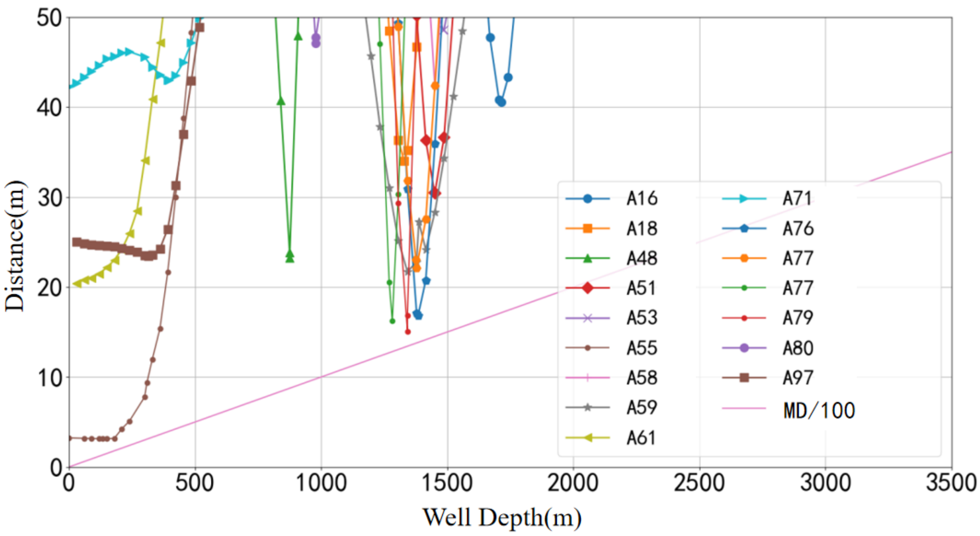
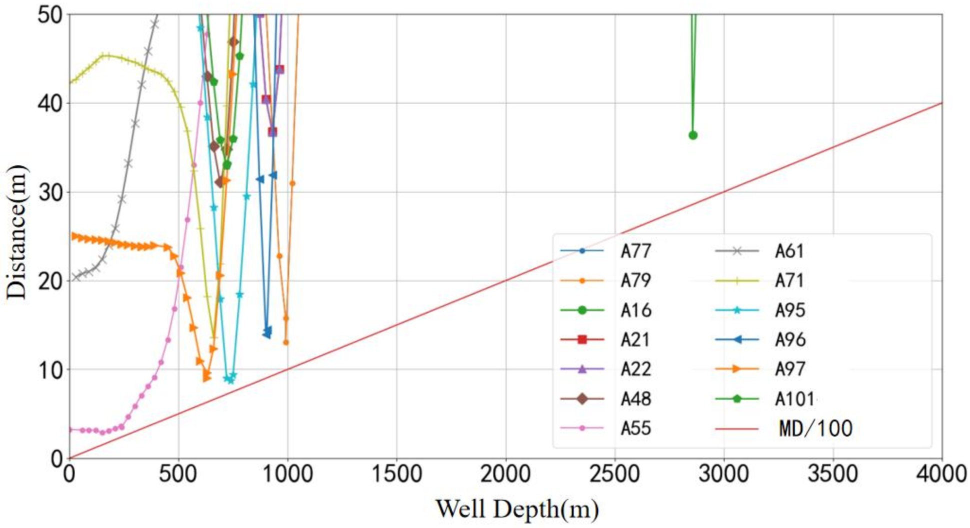


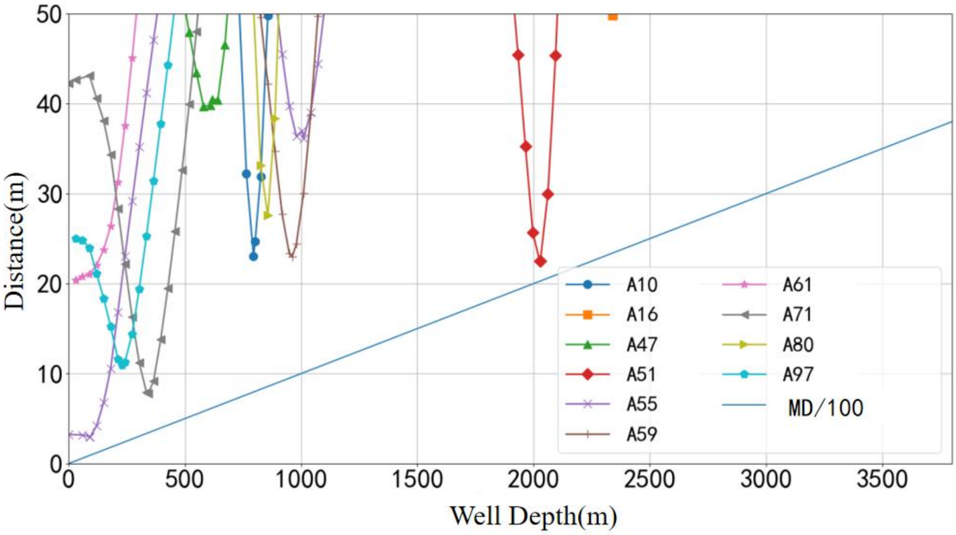
Figure 5. Multi-objective optimization result Fig



(a)



(b)



(c)

Figure 6. Design the track anti-collision safety distance map((a)-(c))

From Table.5 and Figure.6, all three design schemes can meet the collision prevention requirements. With the minimum well depth, the well depth is 4568.89m, the upper friction depth is 180.01kN, and the well depth is larger; the well depth is 4684.03m, and the upper friction resistance is 165.21kN, although the well depth and friction resistance are worse than the results of single target optimization. Therefore, the results of multi-objective optimization are often better than the single-objective optimization results without special requirements.

5. Summary

(1) for cluster well encryption drilling borehole track design requirements, in view of the depth and resistance, build multi-target genetic algorithm, on the basis of the establishment of hole spacing calculation model, the design from top to bottom layer optimization design parameters, implements the intelligent optimization of hole track design parameters, can improve the efficiency and reliability of complex conditions hole track design.

(2) This paper realizes various optimization of the track, establishes the single-objective optimization model and multi-objective optimization model, can be selected according to the different needs of the site, the different optimization goals, providing theoretical guidance and technical support for practical engineering applications.

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