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Research on Optimization of Crop Planting Plans Based on Simulated Annealing and Monte Carlo Simulation

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Abstract: In order to help rural agriculture achieve maximum economic benefits and controllable risks, this paper focuses on the optimization of crop planting strategies in a certain village from 2024 to 2030 and constructs a series of models. Firstly, simulated annealing and mixed-integer linear programming algorithms are used to optimize different scenarios of crop overproduction under stable assumptions, and the optimal planting plan is determined. It is found that the strategy of selling overproduced crops at a low price is more beneficial. Then, Monte Carlo simulation and sequential least-squares programming are introduced. By considering the uncertainty of multiple factors and improving the objective function, the planting plan is further optimized. The research results provide a scientific basis for rural agricultural planting decisions, demonstrate the effectiveness of models and algorithms in agricultural planning, and are of great significance for promoting the sustainable development of rural economy.

Keywords: Simulated Annealing, Monte Carlo Simulation, Mixed-Integer Linear Programming.

1. Introduction

In the development of rural economy, rationally planning crop planting strategies is crucial for enhancing economic benefits and ensuring the supply of agricultural products [1]. Traditional planting decisions often lack a scientific basis and are difficult to cope with complex and changeable market and environmental factors. This paper aims to use advanced models and algorithms to provide precise decision-making support for rural agricultural planting. Through methods such as the simulated annealing algorithm, mixed-integer linear programming, Monte Carlo simulation, and sequential least squares programming, an in-depth study is conducted on the crop planting problem. These methods can effectively handle the constraint conditions and uncertainty factors in the planting process, comprehensively consider market fluctuations, land resource limitations, and other situations, and precisely optimize the planting plan, thus providing a scientific and reasonable solution for the development of rural agriculture and contributing to the sustainable development of the rural economy.

2. Optimization of crop planting scheme based on simulated annealing and mixed integer linear programming

In this paper, simulated annealing algorithm is used to determine the crop types and the area of each plot type. After determining crop allocation for each plot type, mixed integer linear programming (MILP) is used to further allocate crops to specific plots. Through the combination of the above methods, the optimal planting plan of 2024~2030 crops in this village can be obtained.

2.1. Simulated annealing algorithm

Simulated annealing was proposed by Metropolis et al. in 1953 and was successfully applied to combinatorial optimization by Kirk et al. in 1983 [2]. Its inner loop uses the Metropolis rule, and the outer layer is a cooling process. Its idea is derived from the principle of solid annealing [3]. Simulated annealing has advantages that other algorithms do not have when dealing with optimization problems: (1) It provides an effective approximate solution for NP - hard problems; (2) It avoids the optimization process from falling into local optimal solutions; (3) It overcomes the dependence on the initial value.

The basic steps of the simulated annealing algorithm are as follows [4].

1) Initialize the temperature T_0, the initial solution state x_0, and the number of iterations L at each temperature T;
2) For $k=1, 2, \cdots, L$, execute steps (3) to (6);
3) Generate a new solution x';
4) Calculate the increment $\Delta f = f(x') - f(x_0)$, where $f(x)$ is the objective function;
5) Judge whether to accept the new solution according to the Metropolis criterion: If $\Delta f < 0$, then accept the new solution x' as the new solution and use it as the initial point for the next simulated annealing; otherwise, calculate the acceptance probability $e^{-\frac{\Delta f}{T}}$ of the new solution, and generate a pseudo- random number q uniformly distributed in the interval [0,1].

If $e^{-\frac{\Delta f}{T}} > q$, then accept x' as the new solution and use it as the initial point for the next simulated annealing;

6) If the termination condition is satisfied, output the current solution as the optimal solution and end the iteration;

7) The temperature is cooled by the cooling function $T_{k+1} = \alpha T_k$, where α is the cooling coefficient. Go to step (2) until the conditions are met.

2.1.1. Establishment of objective function

The optimization objective of the model is to maximize profits, and its mathematical expressions are as follows:

Scenario 1: The excess part is unsalable and wasted.

$$\max_{x_{i,j,k,t,s}} \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{i=1}^{N}\left[\min\left(\sum_{j=1}^{M} x_{i,j,k,t,s}\cdot y_{i,j,s}, e_{i,s}\right)\cdot p_{i,t,s} - \sum_{j=1}^{M} x_{i,j,k,t,s}\cdot c_{i,j,t,s}\right] \tag{1}$$

Scenario 2: The excess part is sold at a 50% discount.

$$\max_{x_{i,j,k,t,s}} \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{i=1}^{N}\left[\min\left(\sum_{j=1}^{M} x_{i,j,k,t,s}\cdot y_{i.j.s}, e_{i.s}\right)\cdot p_{i.t.s} + \max\left(0, \sum_{j=1}^{M} x_{i.j.k.t.s}\cdot y_{i.j.s} - e_{i,s}\right)\cdot 0.5p_{i,2023,s} - \sum_{j=1}^{M} x_{i,j,k,t,s}\cdot c_{i.j.t.s}\right] \tag{2}$$

Where $x_{i,j,k,t,s}$ is the area of the i -th crop plant s ed on the j -th plot in the s -th season of the t -th year, $y_{i,j,s}$ is the unit-area yield of the i -th crop on the j -th plot in the -th season, $e_{i,s}$ is the expected sales volume of the i -th crop in the s -th season, $p_{i,t,s}$ is the unit price of the i -th crop in the s -th season of the t -th year, $p_{i,2023,s}$ is the unit price of the i -th crop in the s -th season of 2023, $c_{i,j,t,s}$ is the unit - area cost of planting the i -th crop on the i -th plot in the s -th season of the t -th year, T is the total number of years, S is the number of seasons per year, N is the number of crop types, M is the number of plots.

2.1.2. Integration and presentation of the optimization model

Under the consideration of the planting area and various planting constraints, the overall optimization model is obtained.

$$S1:\ \max_{x_{i,j,k,t,s}} \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{i=1}^{N}\left[\min\left(\sum_{j=1}^{M} x_{i,j,k,t,s}\cdot y_{i,j,s}, e_{i,s}\right)\cdot p_{i,t,s} - \sum_{j=1}^{M} x_{i,j,k,t,s}\cdot c_{i,j,t,s}\right]$$

$$S2:\ \max_{x_{i,j,k,t,s}} \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{i=1}^{N}\left[\min\left(\sum_{j=1}^{M} x_{i,j,k,t,s}\cdot y_{i.j.s}, e_{i.s}\right)\cdot p_{i.t.s} + \max\left(0, \sum_{j=1}^{M} x_{i.j.k.t.s}\cdot y_{i.j.s} - e_{i,s}\right)\cdot 0.5p_{i,2023,s} - \sum_{j=1}^{M} x_{i,j,k,t,s}\cdot c_{i.j.t.s}\right]$$

$$s.t.\begin{cases}
\sum_{c=1}^{C} x_{y,s,c,p} \le A_p, \quad \forall y,s,p \\
x_{y,2,cp} = 0, \forall y,c,p \in \{1,5,3\} \\
x_{y,1,c,p} = 0, \forall y,c \in \{C,D\}, p \in \{1,5,3\} \\
\sum_{c\in A} x_{y,1,c,p} \cdot \sum_{c\in C} x_{y,1,c,p} = 0, \quad \forall y, p=4 \\
\sum_{c\in A} x_{y,1,c,p} \cdot \sum_{c\in C} x_{y,2,c,p} = 0, \quad \forall y, p=4 \\
x_{y,s,c}, p = 0, \forall y,s,c \in D, p=4 \\
x_{y,1,c,p} = 0, \forall y,c \in \{A,D\}, p=6 \\
x_{y,2,c,p} = 0, \forall y,c \in \{A,C\}, p=6 \\
x_{y,s,c,p} = 0, \forall y,s,c \in \{A,D,E\}, p=6 \\
x_{y,s,c,p}\cdot x_{y+1,s,c,p} = 0, \quad \forall y,s,c,p \\
\sum_{y=k}^{k+2}\sum_{s=1}^{2}\sum_{c\in B} x_{y,s,c,p} > 0, \quad \forall k \in \{1,2,\ldots,Y-2\} \\
x_{y,s,c,p} = 0 \text{ 或 } x_{y,s,c,p} \ge A_{\min}, \quad \forall y,s,c,p \\
\sum_{c=1}^{C} [x_{y,s,c,p} > 0] \le N_{\max}, \quad \forall y,p,s
\end{cases} \tag{3}$$

2.1.3. The optimal planting plan based on the simulated annealing algorithm

The simulated annealing model was solved using Python to obtain the macroscopic allocation plan of crops to different plot types in the rural area. The results are as follows:

Scenario 1: The partial optimal planting plans for each plot type are shown in Table. 1, with the data in the table in mu.

Table 1. Optimal planting plans

Year	Season	Plot	Crop1	Crop2	Crop3	
2024	1	1	0	0	0	…
		2	0	0	0	
		3	0	0	52.7849	
		4	0	0	0	
		5	0	94.0952	0	
…						

The annual profits from 2024 to 2030 in Scenario 1 are shown in Table. 2.

Table 2. Annual profit from 2024 to 2030

Year	Profit
2024	4226026.895
2025	3571954.126
2026	3591546.796
2027	4160652.805
2028	3549034.997
2029	3654841.516
2030	4088295.827
Total	26842352.96

Scenario 2: The partial optimal planting plans for each plot type are shown in Table. 3, with the data in the table in mu.

Table 3. Optimal planting plans

Year	Season	Plot	Crop1	Crop2	Crop3	
2024	1	1	0	78.5239	0	…
		2	0	0	0	
		3	0	0	0	
		4	0	0	0	
		5	0	0	252.8743	
…						

The annual profits from 2024 to 2030 in Scenario 2 are shown in Table. 4.

Table 4. Annual profit from 2024 to 2030

Year	Profit
2024	7766807.391
2025	10144821.13
2026	9085061.945
2027	7510958.23
2028	10057057.32
2029	7046624.9
2030	9682855.812
Total	61294186.73

The profit data visualization for the two scenarios is shown in Figure 1.

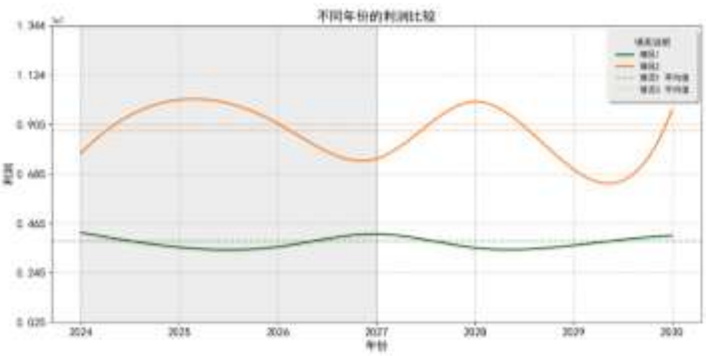

Figure 1. Profit comparison between scenario 1 and scenario 2

The profit of Scenario 2 is significantly higher than that of Scenario 1 (about 2.28 times that of Scenario 1). This indicates that even if the over - produced part is sold at a low price, it is still more profitable than complete waste. In addition, considering the requirements of crop rotation and legume planting, the profits of both scenarios fluctuate greatly. The profit of Scenario 1 fluctuates less, with the annual profit ranging from 3.5 million to 4.2 million. The profit of Scenario 2 fluctuates more, with the annual profit ranging from 7.04 million to 10.14 million. Part of the reason for this fluctuation is the impact of the crop rotation system. In some years, a large amount of land needs to be planted with leguminous plants, resulting in a decrease in profits. At the same time, in high-profit years, the yield is increased due to leguminous crops improving the soil environment.

The above data shows that although agricultural practices face many limitations, the results of Scenario 2 show that overall profits can still be increased through flexible marketing. The key lies in abiding by agricultural practice regulations, such as crop rotation and regular legume planting, optimizing the crop planting structure and market strategies to maximize the overall benefits of each crop rotation cycle.

2.2. Mixed-integer linear programming

The MINL algorithm combines the characteristics of integer programming and linear programming [5]. It contains both continuous variables and integer variables and can find the optimal solution when both the objective function and the constraint conditions are linear.

2.2.1. Establishment of objective function

Minimize planting dispersity and small area planting:

$$\min \sum_{y,s,p,c}\left(\sum_{i} z_{y,s,p,i,c}-1\right)+\alpha \sum_{y,s,p,i,c}\left(a_{\min }\cdot z_{y,s,p,p,c}\right) \tag{4}$$

Where, α is a weight parameter used to balance the two objectives. The first term minimizes the number of plots on which each crop is planted, and the second term penalizes small-area planting.

2.2.2. Integration the optimization model

Under the consideration of main constraint conditions: Total area constraint: ensure that the area allocated to specific plots is equal to the total area determined in the first step. Plot capacity constraint: the total planting area of each specific plot does not exceed its available area. Minimum allocation area constraint: if a certain crop is planted on a certain plot, its area is not less than the preset minimum value.

$$\min \sum_{y,s,p,c}\left(\sum_{i} z_{y,s,p,i,c}-1\right)+\alpha \sum_{y,s,p,i,c}\left(a_{\min }\cdot z_{y,s,p,p,c}\right)$$

$$s.t.\begin{cases}\sum_{i\in I_p} x_{y,s,p,i,c}=A_{y,\beta,p}\\ \sum_{a\in C} x_{y,s,p,i,c}\leq B_{p,i}\\ x_{y,s,p,i,c}\geq a_{\min}\cdot z_{y,s,p,i,c}\\ x_{y,s,p,i,c}\leq M\cdot z_{y,s,p,i,c}\\ x_{y,s,p,i,c}\geq 0\\ z_{y,s,p,i,c}\in\{0,1\}\end{cases} \tag{5}$$

2.2.3. The optimal planting plan based on MINL

The macroscopic allocation data obtained from the simulated annealing model using Python is input into the mixed - integer linear programming model. The optimal planting plans for the crops in this rural area from 2024 to 2030 under the two scenarios are obtained respectively. Part of the data is shown in Table. 5.

Table 5. Optimal planting plan

Year	Season	Plot name	Soybeans	Black beans	Red beans	Mung beans	Cowpeas	Wheat	
2024	First Season	A1	0	70.85	0	0	0	9.155	…
		A2	0	0	0	0	0	55	
		A3	0	0	0	0	0	35	
		A4	0	0	65.33	0	0	6.673	
		A5	0	0	0	0	0	68	
		A6	0	0	0	0	0	55	
…									

The combination of these two algorithms demonstrates the global search ability of the simulated annealing algorithm and the precise optimization characteristics of mixed-integer linear programming. Through this approach, we can obtain a crop allocation plan that not only meets the macroscopic economic and ecological goals but also takes into account the microscopic practical operation requirements.

3. Optimization of crop planting scheme based on Monte Carlo simulation and sequential least squares programming

In this paper, a model is established by combining the Monte Carlo simulation algorithm and the sequential least squares programming (SLSQP) algorithm [6,7]. Optimization is carried out in multiple simulated scenarios. The Monte Carlo simulation generates samples, and then the SLSQP is used to optimize each sample. This approach helps to better understand and manage potential risks. Moreover, through continuous parameter adjustment and correction, a solution that takes risks into account and pursues optimality will be finally provided.

3.1. Model establishment

3.1.1. Establishment of the objective function

In the previous section, through data analysis, it was found that the profit of Scenario 2 is significantly higher than that of Scenario 1. To obtain higher returns and be more in line with the actual market environment, this section makes an important improvement to the objective function: if the total output of a certain crop in each season exceeds the corresponding expected sales volume, the excess part will be sold at a 50% discount based on the current - year sales price, instead of fixedly using the 2023 price.

3.1.2. Constraint conditions

Since the constraint conditions in this section remain unchanged compared with those in the first section, the constraint conditions of the first section are adopted as the constraint conditions for this problem and are omitted here.

3.1.3. Parameter updates

The key to the Monte Carlo simulation is to identify the uncertainty of random variable parameters and determine the probability distributions and variation ranges of these parameters. We can obtain the variables for the next few years as follows,

Expected sales volume update, for wheat and corn,

$$Q_{i,t}=Q_{i,0}\cdot(1+r_i)^t,\ \ r_i\sim U(0.05,0.10) \tag{6}$$

For other crops,

$$Q_{i,t}=Q_{i,0}\cdot(1+r_i)^t,\ \ r_i\sim U(-0.05,0.05) \tag{7}$$

Yield per mu update,

$$Y_{i,t}=Y_{i,0}\cdot(1+\epsilon_{i,t}),\ \ \epsilon_{i,t}\sim U(-0.10,0.10) \tag{8}$$

Planting cost update,

$$C_{i,t}=C_{i,0}\cdot(1.05)^t \tag{9}$$

Sales price update, for vegetables,

$$P_{i,t}=P_{i,0}\cdot(1.05)^t \tag{10}$$

For other crops,

$$P_{i,t}=P_{i,0} \tag{11}$$

Expected sales volume update, for wheat and corn,

$$e_{i,s,t}=e_{i,s,0}\cdot(1+r_i)^t,\ \ r_i\sim U(0.05,0.10) \tag{12}$$

For other crops,

$$e_{i,s,t}=e_{i,s,0}\cdot(1+r_i)^t,\ \ r_i\sim U(-0.05,0.05) \tag{13}$$

Sales price update, for vegetables,

$$p_{i,t,s}=p_{i,0,s}\cdot(1.05)^t \tag{14}$$

For other crops,

$$p_{i,t,s}=p_{i,0,s} \tag{15}$$

When sufficient information is available in 2023, the evaluation process can obtain more valuable information. Such an evaluation system is more reliable, which will make the generated multiple future scenarios more in line with the actual situation and be responsible for the final results.

3.1.4. Parameters of the sequential least squares programming algorithm

Suppose the results generated by the continuous iteration of the Monte Carlo simulation are Q_i, where i is a positive integer. Then for each result sample Q_i, in each iteration, the SLSQP algorithm calculates the gradients of the objective function and the constraint function based on the current point Q_i to determine how to update in the next step. The SLSQP algorithm updates the decision - variable Q_i according to the calculated gradients and step-sizes and checks whether Q_i meets the constraint conditions and the convergence conditions. The algorithm terminates and generates the optimal solution until the convergence criterion is satisfied (such as the change in the objective function being very small or the gradient approaching zero).

3.2. The optimal planting plan

The model was solved using Python. The results are shown in Table. 6 below, and the data in the table are in mu.

Table 6. Optimal planting plan

Year	Season	Plot name	Soybeans	Black beans	Red beans	Mung beans
2024	First Season	C1	0	15	0	0
		C2	0	1.23	0	0
		C3	0	15	0	0
		C4	0	0	0	0
		C5	0	0.77	0	0
		C6	0	20	0	0
…						

The annual profits from 2024 to 2030 are shown in Table.7.

Table 7. Annual profit from 2024 to 2030

Year	Profit
2024	9773231.683
2025	7834996.116
2026	10729145.53
2027	6804977.812
2028	11836182.24
2029	13322816.97
2030	8078604.81
Total	68379955.17

Visualize the profit data, as shown in Figure 2.

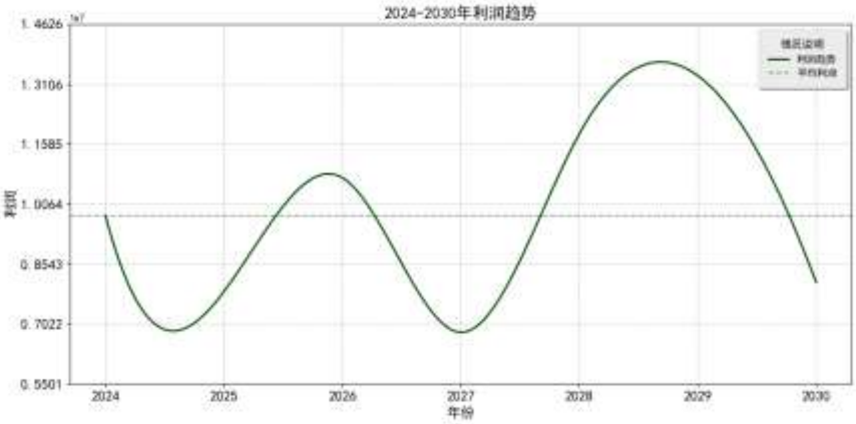

Figure 2. Profit trends for 2024-2030

(1) Analysis of the overall trend: The profit shows obvious annual fluctuations, but the overall trend is upward. The profit in the highest year (2029) is approximately twice that in the lowest year (2027), which reflects the uncertainties in crop yields, crop cultivation structures, sales volumes, and prices.

(2) Analysis of fluctuation information: The upward trend of the overall profit is contributed by the growth trend of the sales volumes of wheat and corn. At the same time, the ±5% changes in other crops account for part of the annual fluctuations. The ±10% changes in the yield per mu are an important factor causing significant fluctuations, especially in 2027 (the lowest point) and 2029 (the highest point).

(3) Analysis of the data cycle: By observing the line chart, there is a profit pattern with a cycle of three years under this allocation method: 2024 - 2026, 2027 - 2029, and a new cycle starts in 2030. The main reason for this is the periodic cultivation of leguminous crops.

4. Conclusions

Through constructing multiple models and applying a series of algorithms, this study has optimized the rural crop planting strategies. Firstly, the combination of the simulated annealing algorithm and the mixed-integer linear programming algorithm provides an effective optimization approach for the planting plan under stable conditions. It is clarified that selling the overproduced crops at a low price can increase the overall profit, and at the same time, it reveals the impact of the crop rotation system on profit fluctuations. Then, the application of the Monte Carlo simulation and the sequential least squares programming fully considers the actual uncertain factors, and the improved objective function is more in line with the actual market situation. The results show that although there are annual fluctuations in profit, the overall trend is upward, and the impact of the periodic planting of leguminous crops on the profit pattern is discovered. These achievements provide a scientific reference for rural agricultural planting decision-making. In the future, the model can be further improved by incorporating more complex factors to enhance the practicality and accuracy of the model, so as to better serve the development of the rural economy.

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