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Application of AHPEW-GT Model Incorporating AHP-Entropy Weights-Game Theory in the Regulation of Sustainable Tourism

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Abstract: This paper presents the AHPEW - GT fusion weight planning model, which combines Analytic Hierarchy Process (AHP), Entropy Weight Method, and Game Theory Optimization, to address the issues caused by over - tourism. The model is applied to Juneau's tourism industry. By collecting and preprocessing relevant data, objective functions and constraints are determined. The results show that the optimal number of annual tourists in Juneau is 191,600, but the current number exceeds this limit, causing environmental damage. After considering additional income and adjusting the model, it is found that 93.2% of the additional income should be used for environmental protection. The model is also sensitive to environmental factors, especially carbon footprint. Additionally, the model is extended to Venice. This research provides a scientific approach for sustainable tourism development.

Keywords: Analytic Hierarchy Process, Entropy Weight, AHPEW-GT

1. Introduction

The global tourism industry has been booming, but it has brought about a series of environmental and social problems, such as glacier degradation, increased carbon emissions, and conflicts between locals and tourists [1]. To tackle these challenges, a scientific and effective model is urgently needed. This study proposes the AHPEW - GT fusion weight planning model. It integrates AHP to determine subjective weights, the Entropy Weight Method for objective weights, and Game Theory Optimization to find the optimal combination of weights. By establishing relevant objective functions and constraints, this model can comprehensively consider economic, environmental, and social factors in tourism development. Through data collection, preprocessing, and model solving, it aims to provide a practical solution for local governments to balance tourism development and environmental protection, and to promote the sustainable development of the tourism industry.

2. AHPEW-GT fusion weight planning model of tourism industry

2.1. Determination of objective function

In order to build an optimization model for sustainable tourism in Juneau, factors in three aspects, namely economic, social and environmental aspects, need to be considered. We have three main objectives: (1) Maximize tourism income. (2) Minimize the pressure on the ecological environment. (3) Improve the overall satisfaction of residents and visitors.

We need to define some relevant variables and functions. Let's say that the number of tourists received annually is x, the total tourism income is $R(x)$ dollars, the carbon emission related to the number of tourists is $C(x)$ tons, the volume loss rate of glacier is $Y(x)$ km2/yr, the load cost of infrastructure is $P(x)$ dollars, and the satisfaction of residents and tourists is $S(x)$.

These functions are explained and illustrated below:

(1) Total revenue function

The total tourism revenue is proportional to the number of tourists, and the calculation formula is as follows:

$$R(x) = x \cdot F \tag{1}$$

In the formula, F is the amount spent per tourist. After statistical analysis, we get F =1500 dollars.

(2) Carbon footprint function

Carbon footprint is usually expressed in terms of carbon emissions. There is a nonlinear relationship between carbon emission and the number of tourists, here we consider it to be a quadratic function:

$$C(x) = \alpha x + \beta x^2 \tag{2}$$

In the formula, α and β are environmental sensitivity coefficients. After consulting the data, we think the most appropriate value is α =0.002 tons/person, β =1×10-6 tons/person^2.

(3) Glacier volume loss rate function

The regression equation in Section 4.2.3 is used directly as a function expression here:

$$Y(x) = -3.8737\times10^{-19}x^3 - 1.9136\times10^{-13}x^2 + 3.4159\times10^{-6}x + 2.786 \tag{3}$$

(4) Infrastructure load cost function

The pressure on infrastructure increases non-linear with the increase in visitor numbers, defined as:

$$P(x) = \gamma \cdot x + \delta \cdot x^2 \tag{4}$$

In the formula, γ and δ represent the number of infrastructure loads. Consult the literature [2] and analyze it to determine that γ =10 \$/ person and δ =0.0001 \$/ person ^{2}.

(5) Residents and tourists' satisfaction function

The satisfaction of residents and tourists should consider both positive and negative effects comprehensively, and its value should first increase and then decrease with the increase of the number of tourists. Therefore, we define:

$$S(x) = S_0 + \varepsilon \cdot \ln(1+x) - \theta x \tag{5}$$

In the formula, S_0 represents basic satisfaction, ε represents positive effects (such as job opportunities), and θ represents negative effects (such as price increases and overcrowding). By analogy with other studies, take $S_0 = 70$, $\varepsilon = 12$, and $\theta = 0.00011$.

Finally, considering the three aspects of economy, environment and society comprehensively, and processing the five functions are treated dimensionless, the objective function of comprehensive benefit is obtained as follows:

$$\begin{aligned}\max\ Z = {} & \omega_1 \cdot \frac{R(x) - R_{\min}}{R_{\max} - R_{\min}} - \omega_2 \cdot \frac{C(x) - C_{\min}}{C_{\max} - C_{\min}} - \omega_3 \cdot \frac{Y(x) - Y_{\min}}{Y_{\max} - Y_{\min}} \\ & - \omega_4 \cdot \frac{P(x) - P_{\min}}{P_{\max} - P_{\min}} + \omega_5 \cdot \frac{S(x) - S_{\min}}{S_{\max} - S_{\min}}\end{aligned} \tag{6}$$

Where, $\omega_1, \omega_2, \omega_3, \omega_4, \omega_5$ are the weight coefficients, satisfying that $\omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 = 1$ .The subscript plus $\min$ represents the minimum value of each function, and the subscript plus $\max$ represents the maximum value.

2.2. Determine the index weight of the objective function

2.2.1. Subjective weights were determined using analytic hierarchy process（AHP）

First, we constructed the judgment matrix based on experience, as shown in Table. 1.

Table 1. Judgment matrix

Combined effect	$R(x)$	$C(x)$	$Y(x)$	$P(x)$	$S(x)$
$R(x)$	1	2	2	5	4
$C(x)$	1/2	1	1	4	3
$Y(x)$	1/2	1	1	4	3
$P(x)$	1/5	1/4	1/4	1	1/2
$S(x)$	1/4	1/3	1/3	2	1

Next, we need to conduct a consistency check. The calculation formula is:

$$CR = \frac{CI}{RI} \tag{7}$$

Among them, the value of RI (Random Consistency Index) is a constant related to the order n of the matrix, which is used to measure the consistency degree of a random judgment matrix [3]. The calculation of CI (Consistency Index): CI is used to measure the consistency degree of the judgment matrix [4], and the calculation method is as follows:

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{8}$$

Where, $\lambda_{\max}$ is the maximum eigenvalue of the judgment matrix, and n is the index number.

If the CR value is less than 0.1, it is considered that the consistency of the judgment matrix is within the allowable range, and the weights obtained by AHP calculation are credible. If the CR value is greater than or equal to 0.1, it is necessary to reconstruct the judgment matrix or make corrections.

It can be calculated that: $\lambda_{\max} = 5.2004, n = 5, CI = 0.0501$.

$$CR = \frac{CI}{RI} = \frac{0.0501}{1.12} = 0.045 < 0.1 \tag{9}$$

It can be considered that the judgment matrix has little difference with the consistent matrix and passes the consistency test. Then the weight is obtained by arithmetic average method, and the results are shown in Table. 2.

Table 2. Subjective weights

Weight coefficient	ω_1'	ω_2'	ω_3'	ω_4'	ω_5'
Weight value	0.3932	0.2325	0.2325	0.0601	0.0817

2.2.2. The entropy weight method is used to determine the objective weight

The entropy weight method is an objective weighting approach [5]. In the specific application process, based on the degree of variation of each indicator, the entropy weight method calculates the entropy weight of each indicator using information entropy, thus obtaining relatively objective weights. Using this method, we calculated the objective weights as shown in Table. 3.

Table 3. Objective weight

Weight coefficient	ω_1^*	ω_2^*	ω_3^*	ω_4^*	ω_5^*
Weight value	0.2092	0.0505	0.6133	0.0512	0.0759

2.2.3. Construct the game theory optimization model and determine the comprehensive weight

The purpose of game theory combinatorial weighting is to find the optimal linear combination of subjective and objective weights through mathematical optimization, so that the combinatorial weights can reflect the subjective experience and retain the objectivity of the data [6].

Let the vector $L_1=[\omega_1',\omega_2',\omega_3',\omega_4',\omega_5'],L_2=[\omega_1^*,\omega_2^*,\omega_3^*,\omega_4^*,\omega_5^*]$,The combined weight L is the linear combination of each weight:

$$L=[\omega_1,\omega_2,\omega_3,\omega_4,\omega_5],\ L=\eta_1L_1+\eta_2L_2.(\eta_1,\eta_2>0) \tag{10}$$

The goal is to minimize the deviation between the combined weight and each original weight:

$$\min\ [\|L-L_1\|+\|L-L_2\|] \tag{11}$$

By solving this optimization problem, the final combined weights can be obtained, as shown in the following Table. 4.

Table 4. Comprehensive weight

Weight coefficient	ω_1	ω_2	ω_3	ω_4	ω_5
Weight value	0.3011	0.1415	0.4229	0.0557	0.0788

We plotted the values of subjective weights, objective weights, and combined weights in a line chart, as shown in Figure 1.

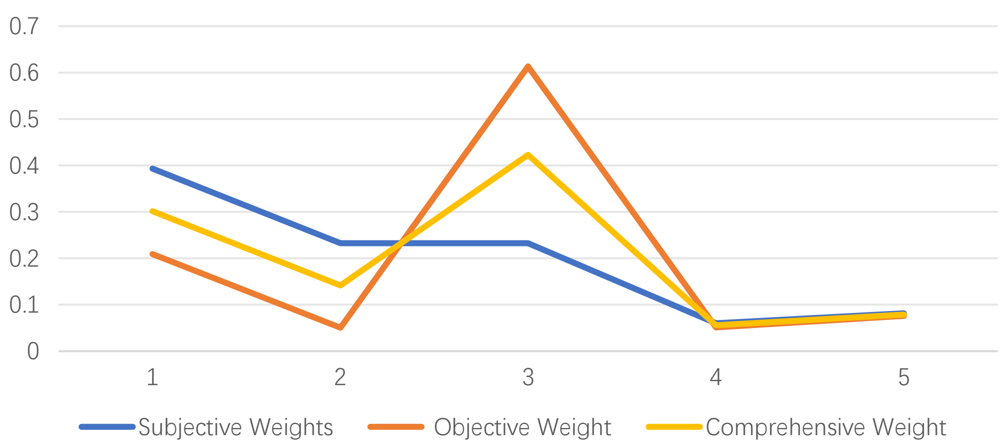

Figure 1. Weight comparison

By observing Figure 1, it can be found that the combined weight takes into account both the subjective weight and the objective weight. This indicates that our method of determining weights is reasonable.

When calculating the weights, we used three methods: the Analytic Hierarchy Process, the Entropy Weight Method, and Game Theory. This is a planning model originally created by our team. We named this planning model the AHPEW-GT Fusion Weight Planning Model.

2.3. The determination of constraints

Tourist load constraint: The number of tourists must not exceed the local limit, as specified by the following equation:

$$0 \leqslant x \leqslant x_{\max} \tag{12}$$

Where, $x_{\max} = 1700000$ persons.

Carbon footprint constraint:

$$C(x) \leqslant C_{\lim it} \tag{13}$$

Where, $C_{\lim it} = 3 \times 10^7$ tons.

Resident and tourist satisfaction constraints:

$$S(x) \geqslant S_{\min} \tag{14}$$

Where, $S_{\min} = S_0$.

2.4. Programming problem solving

Finally, we get the following planning problem:

$$\max\ Z = \omega_1 \cdot \frac{R(x) - R_{\min}}{R_{\max} - R_{\min}} - \omega_2 \cdot \frac{C(x) - C_{\min}}{C_{\max} - C_{\min}} - \omega_3 \cdot \frac{Y(x) - Y_{\min}}{Y_{\max} - Y_{\min}} - \omega_4 \cdot \frac{P(x) - P_{\min}}{P_{\max} - P_{\min}} + \omega_5 \cdot \frac{S(x) - S_{\min}}{S_{\max} - S_{\min}} \tag{15}$$

$$s.t. \begin{cases} 0 \leqslant x \leqslant x_{\max},\ and\ x\ can\ only\ be\ integer \\ C(x) \leqslant C_{\lim it} \\ S_x \geqslant S_{\min} \end{cases} \tag{16}$$

The optimal results obtained by solving this programming problem using MATLAB are shown in Table. 5.

Table 5. Optimization results

Z	x (person)	R ($)	C (tons)	Y (km2/yr)	P ($)	S
0.86517	191608	287412000	37097	3.4334	5587400	313.0726

As can be seen from the results, the current number of tourists in Juneau has far exceeded the environmental carrying capacity, causing serious damage to the local ecosystem. Therefore, Juneau may need to limit the number of tourists further.

2.5. Spending plans for additional revenue

Additional revenue could be allocated to environmental protection and infrastructure. Assuming the additional revenue are H dollars, where H_1 dollars are spent on reducing the carbon footprint and H_2 dollars are spent on infrastructure, there should be:

$$H = H_1 + H_2 \tag{17}$$

For every $1 million increase in environmental protection expenditure, α can reduce 1×10^{-4} tons/person and β can reduce 2×10^{-7} tons/person ^{2}, then the new carbon footprint function is:

$$C^*(x) = (\alpha - 1 \times 10^{-4} H_1) \cdot x + (\beta - 2 \times 10^{-7} H_1) \cdot x^2 \tag{18}$$

Investing in infrastructure can save load costs, and for every additional $1 million invested, δ decreases by 2×10^{-5} dollars / person ^{2}. So, the new infrastructure load cost function is:

$$P^*(x) = \gamma x + (\delta - 2 \times 10^{-5}) x^2 \tag{19}$$

Thus, the adjusted planning model is:

$$\max\ Z = \omega_1 \cdot \frac{R(x) - R_{\min}}{R_{\max} - R_{\min}} - \omega_2 \cdot \frac{C^*(x) - C^*{}_{\min}}{C^*{}_{\max} - C^*{}_{\min}} - \omega_3 \cdot \frac{Y(x) - Y_{\min}}{Y_{\max} - Y_{\min}} - \omega_4 \cdot \frac{P^*(x) - P^*{}_{\min}}{P^*{}_{\max} - P^*{}_{\min}} + \omega_5 \cdot \frac{S(x) - S_{\min}}{S_{\max} - S_{\min}} \tag{20}$$

$$s.t. \begin{cases} 0 \leqslant x \leqslant x_{\max},\ and\ x\ can\ only\ be\ integer \\ H_1 + H_2 = H,\ H_1, H_2 \geqslant 0 \\ C^*(x) \leqslant C_{\lim it} \\ S_x \geqslant S_{\min} \end{cases} \tag{21}$$

Let H =6000000 dollars. Use MATLAB to solve the adjusted programming model. Then, compare the obtained optimization results with those before adjusting the model, as shown in Figure 2.

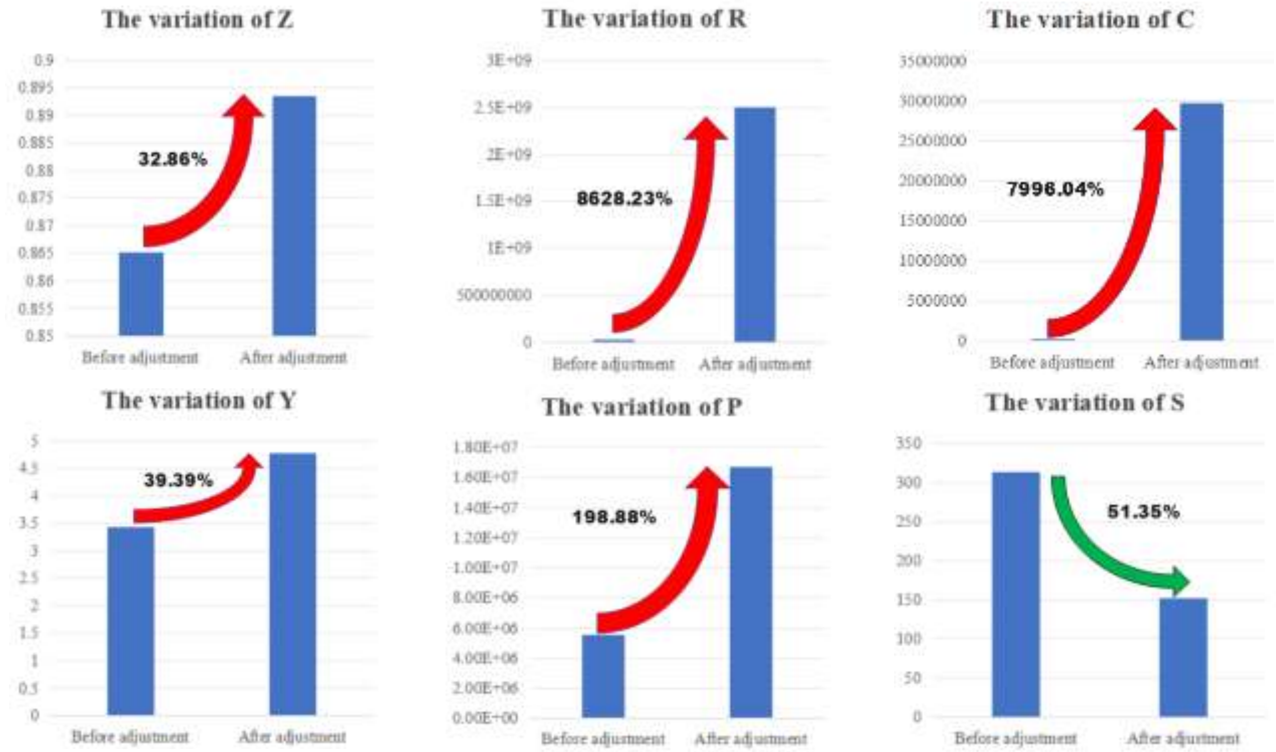

Figure 2. Comparison of optimization results

As can be seen from Figure 2, after considering the additional income, the comprehensive benefit has increased slightly. Among them, the total income brought by tourism has experienced explosive growth. The damage to the environment has increased significantly, but it has not exceeded Juneau's environmental carrying capacity. The infrastructure load cost has increased, but it is negligible compared with the increase in total income. Although the satisfaction of residents and tourists has decreased, it has not dropped below the basic satisfaction level.

Surprisingly, through calculation, the number of tourists after the model adjustment has increased from 191,000 to 1.67 million, which has greatly increased the upper limit of tourist carrying capacity.

Finally, the calculation results show that the local area should allocate 93.2% of the additional income to environmental protection and 6.8% to infrastructure construction. Therefore, we suggest that the local area should use the vast majority of the additional income for environmental protection to achieve greater comprehensive benefits.

2.6. Prediction of development without optimization

Using the least square method to fit the number of visitors [7], we get the following formula:

$$x(t) = 9t^3 + 280t^2 - 8120t + 81200\,(R^2 = 0.985) \tag{22}$$

Where x is the number of visitors function and t is the year ($t = 1$ for 1990 and $t = 35$ for 2024).

Draw a graph of the number of tourists and the comprehensive benefits in the next ten years, as shown in Figure 3.

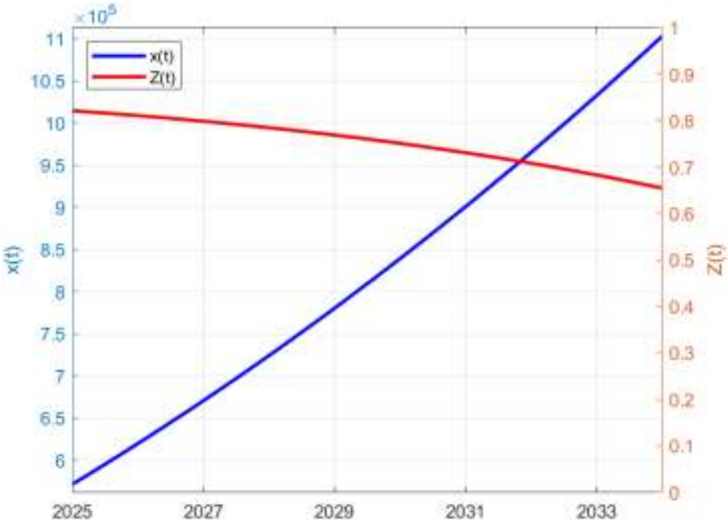

Figure 3. Predictions for the next 10 years

As can be seen from the Figure 3, if no optimization is carried out, the number of tourists will increase year by year, but the revenue will first reach a peak and then enter a continuous decline stage.

3. Sensitivity analysis

Change the coefficient of each function and observe the change in the comprehensive benefit as shown in Figure 4.

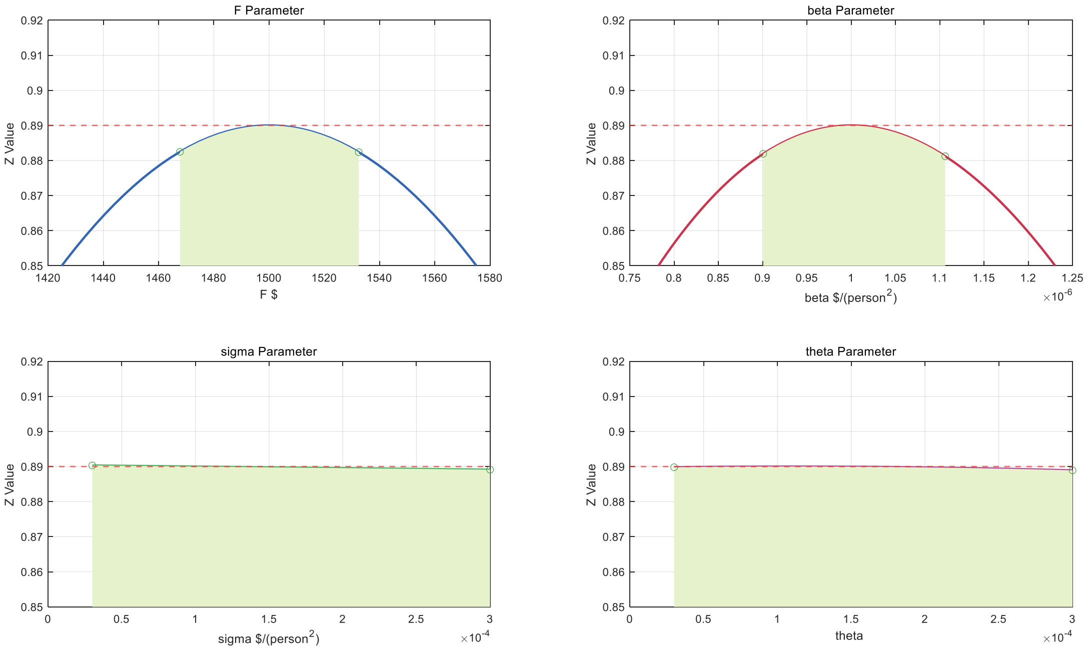

Figure 4. Variation of comprehensive benefits with parameters

In addition, we also calculated the tolerance intervals and sensitivities of these parameters as shown in Table. 6. The results show that the factor having the greatest impact on the comprehensive benefits is the environmental factor, especially the carbon footprint. Secondly, the second most influential factor is per capita consumption. Therefore, Juneau needs to increase its investment in environmental protection. Only by practicing the concept of green development can the local area achieve sustainable development. A small sensitivity value indicates that the model is robust.

Table 6. The tolerance intervals and sensitivities

Indicator	Tolerance intervals	Sensitivities
F	[1.32e+03, 1.68e+03]	0.0647
β	[8.14e-07, 1.19e-06]	0.0792
δ	[9.02e-05, 1.10e-04]	0.0012
θ	[9.87e-05, 1.21e-04]	0.0009

4. The practical application of the model

We show the application of the model in Venice. The reason for choosing Venice is that it is a world-famous tourist destination that has been greatly affected by over-tourism.

Considering that there are no glaciers in Venice, the function of glacier volume loss rate needs to be removed. Then, the weights are reassigned. In addition, the coefficient of each function should be adjusted accordingly, so as to obtain a relatively reasonable result.

After calculation, the optimization results of Venice's tourism are obtained, as shown in Table. 7.

Table 7. Optimization results

Indicator	Z	x (person)	R (euros)	C (tons)	P (euros)	S
Before optimization	0.53	16 million	70 billion	8 million	12 billion	95
After optimization	0.65	12 million	58 billion	5.8 million	8 billion	125

It can be seen from the results that the comprehensive benefits of Venice's tourism industry have been significantly improved through optimization. Not only that, local tourism revenue, residents and visitor satisfaction have increased. In addition, the optimization also reduces the pressure on the local ecological environment and infrastructure. Therefore, the local government should limit the number of tourists, protect the ecological environment, and realize sustainable development as soon as possible.

5. Conclusions

This research successfully applied the AHPEW-GT fusion weight planning model to analyze and solve tourism-related problems in Juneau and Venice. The model, which combines multiple methods, effectively quantifies the complex relationships among economic, environmental, and social factors in tourism. It clearly indicates that controlling the number of tourists is crucial for environmental protection and sustainable tourism development. For Juneau, restricting the number of visitors to no more than 1.7 million annually is necessary. Moreover, the model's sensitivity analysis highlights the importance of environmental protection, especially reducing carbon footprints. In the future, local governments should take the lead in formulating and implementing more sustainable tourism policies. This study provides a valuable reference for other over-tourism regions to achieve sustainable development through scientific planning and management.

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