

Research on Effective Positioning and Adjustment Scheme Based on Circular UAV Formation

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Abstract: This article investigates the effective positioning and adjustment of circular drone formations. Firstly, based on the passive reception signal positioning model, an effective positioning model was established, and the number of drones that meet the effective positioning requirements was solved through traversal algorithm. The results indicate that when there is no deviation in the position of the drone transmitting the signal, at least one drone is needed to transmit the signal in addition to the two known drones to achieve effective positioning. Furthermore, this article establishes a formation adjustment model for unmanned aerial vehicles, derives the overall adjustment model and multiple adjustment models through the polar coordinate system point distance formula and cosine theorem, and uses greedy algorithm to solve the adjustment plan. Finally, the effectiveness of the model was verified through Matlab simulation, and the adjusted drone position had a small error compared to the ideal position, indicating that the model can accurately achieve drone formation adjustment with high accuracy and stability.

Keywords: Drone Formation Positioning, Effective Positioning Area, Multiple Model Adjustments.

1. Introduction

The passive positioning technology of drone formations has important application value in military reconnaissance, civilian rescue and other fields due to its strong concealment and outstanding anti-interference ability. In recent years, numerous scholars have conducted extensive research on the passive positioning problem of drone formations. Deng Menglong [1] established a drift circle model to address the problem of pure azimuth passive positioning in drone formation flight, and used simulated annealing algorithm to optimize drone azimuth adjustment. However, this method is mainly applicable to two-dimensional planar circular formations and has insufficient adaptability to three-dimensional formations. Wu Tong [2] proposed a linearized pseudo measurement model that achieves data fusion of asynchronous azimuth information from multiple platforms through Kalman filtering. Although it improves positioning accuracy, it has high computational complexity and is sensitive to initial conditions. Liu Gaofeng [3] established mathematical programming models for three-level and four level positioning, and used the least squares method to solve the passive positioning problem of unmanned aerial vehicle formations. However, this method still has room for improvement in positioning accuracy and robustness. Zhao Guangyong [4] established a polar coordinate system and used geometric shapes and cosine theorem to calculate the position of the drone. Although the positioning was relatively accurate, they did not fully consider the problem of error control in complex environments. Cheng Yinsong [5] reviewed the passive positioning technology of unmanned aerial vehicle formations, analyzed the advantages and disadvantages of various positioning methods, but did not propose specific improvement plans.

In response to the shortcomings of existing research, this

paper proposes a passive positioning and adjustment method for unmanned aerial vehicle formations based on polar coordinate system and traversal algorithm. By establishing an effective positioning model and combining traversal algorithms to solve the number of unmanned aerial vehicles that meet the positioning conditions, multiple adjustment models were designed to achieve high-precision formation adjustment. Compared with existing research, the method proposed in this paper is not only applicable to various formation forms, but also significantly improves positioning accuracy and formation adjustment stability through multiple adjustments of the model. In addition, this article verified the effectiveness of the model through simulation, and the results showed that the method can achieve high-precision positioning with a small number of unmanned aerial vehicles transmitting signals, with high accuracy and stability, providing new ideas and methods for passive positioning of unmanned aerial vehicle formations.

2. Effective positioning model for circular drone formation

2.1. Establishment of an effective positioning model

A drone with slight deviation at a certain position received signals from drones numbered FY00 and FY01, as well as signals from several unknown numbered drones in the formation, as shown in Figure 1. If there is no deviation in the position of the drone transmitting the signal, except for FY00 and FY01, how many drones are needed to transmit the signal in order to achieve effective positioning of the drone.

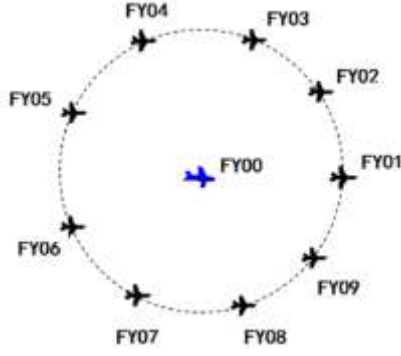


Figure 1. Schematic diagram of circular drone formation

Based on the passive signal receiving unmanned aerial vehicle positioning model established in the previous text:

$$\begin{cases} \rho'_t = \rho_t + \Delta\rho_t \\ \delta'_t = \delta_t + \Delta\delta_t \end{cases} \quad (1)$$

$$\left\{ \begin{array}{l} \Delta\rho_t = \rho \left(\frac{\sin(M_{ti} + P_{ri}\Delta\delta)}{\sin\alpha_{ti}'} - 1 \right) \\ \Delta\delta_t = \arctan G \\ G = \frac{\sin\alpha_{ti}' \sin M_{tj} - \sin\alpha_{tj}' \sin M_{ti}}{P_{ri}(\sin\alpha_{tj}' \cdot \cos M_{ti} - \sin\alpha_{ti}' \cdot \cos M_{tj})} \\ M_{ti} = 180^\circ - \alpha_{ti}' - \gamma_{ti}' \\ M_{tj} = 180^\circ - \alpha_{tj}' - \gamma_{tj}' \\ \gamma_{ti}' = \gamma_{ti0} - P_{\gamma i} \cdot \Delta\delta_t \\ P_{\gamma i} = \begin{cases} 1, & f_i > t \\ -1, & f_i < t \end{cases} \\ \Delta\delta_t = \frac{\theta[(10 - f_k) + P_{r1}(f_1 - t) + P_{rk}^2(f_k - t)] - 360^\circ}{P_{r1} + P_{rk}^2} \end{array} \right. \quad (2)$$

When there are $k+1$ unmanned aerial vehicles transmitting signals without positional deviation, the passive receiving unmanned aerial vehicle can extract k azimuth angles α_{ti}' , $i = 1, 2, \dots, k$, and further determine the position in the polar coordinate system to achieve positioning. When the extracted azimuth angle contains the receiving error angle α_ε and α_ε is within the range of $1/10$ of α_{ti}' , $\alpha_{ti}' = \alpha_{ti} \pm \alpha_\varepsilon$, where α_{ti} is the actual azimuth angle. When $\alpha_\varepsilon = 0$, $\alpha_{ti}' = \alpha_{ti}$. Obtain the most accurate coordinate positioning through the positioning model, which is the actual center of mass position of the drone $(\rho', \delta') = (\rho, \delta)$; As α_ε increases, the position coordinates obtained through the positioning model will deviate more from the actual center of mass of the drone. Due to the drone having a certain plane area S on the two-dimensional plane, when the coordinates (ρ', δ') obtained through the positioning model can fall within this plane area range, it can be considered as effective positioning, otherwise the positioning is invalid. Assuming that the drone body is circular and the body length is l , as shown in Figure 2, when (ρ', δ') falls in the S pearl point set area, it is considered effective positioning. Otherwise, it is considered ineffective positioning if (ρ', δ') deviates too far from the actual coordinates [6-7].

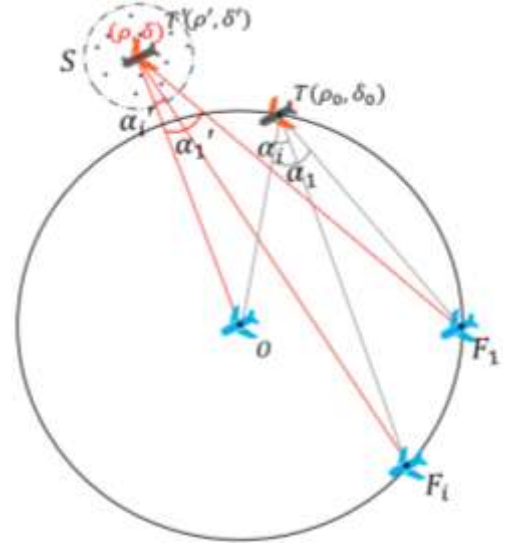


Figure 2. Effective positioning diagram

In order to determine the number of unknown numbered transmitters other than FY00 and FY01 that meet the requirement for effective positioning, it is first assumed that the actual position (ρ_0, δ_0) of the unmanned aerial vehicle to be located is the ideal position without deviation, and the ideal azimuth angle α_{ti0} is calculated. Then, since the positions of the unmanned aerial vehicles that emit signals are all unbiased, the passive signal receiving unmanned aerial vehicle positioning model mentioned earlier can be used to take $\alpha_{ti}' = \alpha_{ti} \pm \alpha_\varepsilon$, where α_ε gradually increases from zero. Calculate the position (ρ', δ') when there is a receiving error

angle, and then use the polar coordinate system distance formula to calculate and determine whether the unmanned aerial vehicle falls within the effective area range [8]. The distance formula in polar coordinate system is as follows (3), where (r_1, θ_1) takes (ρ', δ') , (r_2, θ_2) takes (ρ, δ) :

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_1 - \theta_2)} \quad (3)$$

In this way, the maximum allowable receiving error angle $\alpha_{\varepsilon_{kmax}}$ for effective positioning can be found under different numbers of drones transmitting signals. When the maximum receiving error angle $\alpha_{\varepsilon_{kmax}}$ stabilizes with the number of drones transmitting signals, the corresponding number of drones transmitting signals is the number of transmitters v that can achieve effective positioning of drones.

$$v = k + 1, \alpha_{\varepsilon_{kmax}} \text{ is within the stable region} \quad (4)$$

2.2. Traversal algorithm for solving effective models of unmanned aerial vehicles

The effective positioning model for unmanned aerial vehicles established above is used to solve the number of unmanned aerial vehicles that meet the requirements for effective positioning using Matlab traversal algorithm. The algorithm steps are as follows:

(1) Firstly, select any transmitter and obtain the unbiased

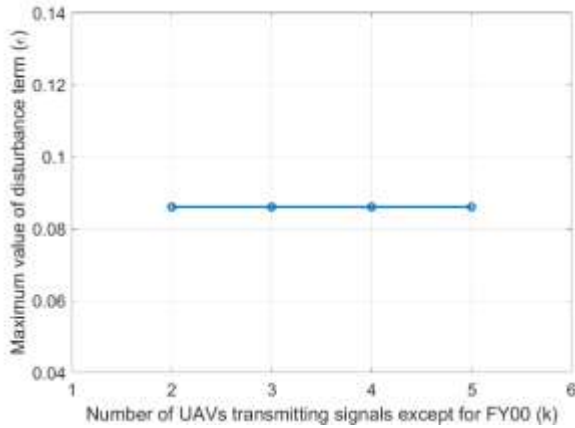


Figure 3. The influence of the number of drones on the maximum allowable error angle

2.3. Result analysis

As the number of drones transmitting signals increases, the maximum allowable error disturbance term $\alpha_{\varepsilon_{kmax}}$ that satisfies effective positioning remains stable in the range of $k = 2 \sim 5$, as shown in Figure 3. This indicates that an effective positioning effect can be achieved with 3 drones transmitting signals. If the number of transmitters that can achieve effective positioning of drones is $v \geq 3$, then when there is no deviation in the position of the drones transmitting signals, at least 1 drone needs to transmit signals in addition to FY00 and FY01 to achieve effective positioning of the drones.

Figure 4 shows the superposition of effective areas at different k values. The effective area simulated by the traversal algorithm is a circular area, which conforms to the effective localization model. At the same time, it can be intuitively seen that the value of k has little impact on the effective area. The effect of effective positioning when $k =$

azimuth of FY02 based on the equation $\gamma_i = (f_i - t) \times \theta$, $\alpha_i = [90^\circ - \frac{\gamma_i}{2}] \times P_{ai}$;

(2) Add a perturbation term ε to the unbiased azimuth, where ε takes a random number of $(-0.01, 0.01)$ for the first time, to obtain the biased azimuth angle α_i' . According to the equation $\Delta\delta = \arctan G$, $\Delta\rho = \rho \left(\frac{\sin(M_i + P_{ri}\Delta\delta)}{\sin \alpha_i'} - 1 \right)$, loop through 100000 times, and calculate different biased positions $(\rho + \Delta\rho, \delta + \Delta\delta)$ and deviation distances d . Find the maximum value of d and its corresponding ε , denoted as d_{max} and ε_{max} . Draw the maximum deviation position point of the current ε , and finally draw the effective positioning area.

(3) Determine whether d_{max} is greater than half the length of the drone body. If it is true, it indicates that the drone is currently on the boundary of effective positioning. Otherwise, expand ε by 0.001 on the original interval and repeat (2) until the condition is met.

(4) Increase the number of transmitters, traverse all combinations of transmitters, repeat (1) to (3), record the $\alpha_{\varepsilon_{kmax}}$ that exactly satisfies effective positioning for different numbers of transmitters k , and plot the variation curve between the two.

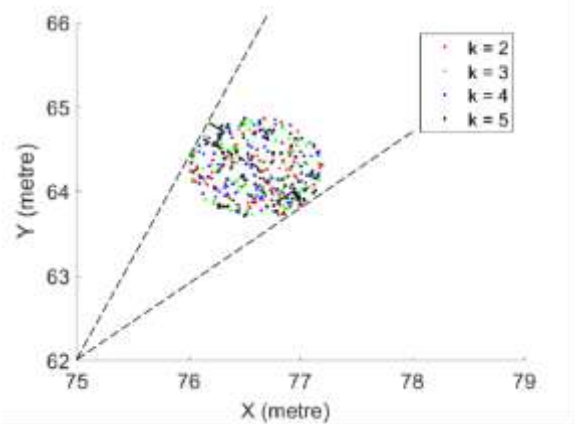


Figure 4. Overlay of effective areas at different k values

5 is the same as that when $k = 2$. For drone swarms flying in formation, electromagnetic silence should be maintained as much as possible and electromagnetic wave signals should be emitted less. Therefore, when selecting $k = 2$, which means a total of 3 drones transmitting signals, the best effect can be achieved.

2.4. Sensitivity analysis

Figure 5 intuitively shows that as the length of the drone body changes, the maximum allowable error disturbance term α_{ε} gradually increases under the premise of effective positioning: when the body length is $l = 0.5, 0.6, 0.7$, the maximum allowable error disturbance term is $\alpha_{\varepsilon} = 0.073, 0.086, 0.101$, respectively. Obviously, for unmanned aerial vehicles with a large area, the reception error of the received signal has a smaller impact on the effective positioning of the drone, which is consistent with the theoretical model of effective positioning.

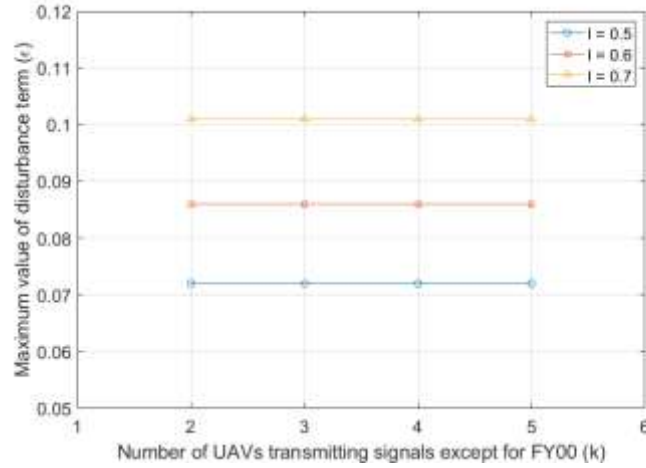


Figure 5. The max value of the error disturbance term α_ϵ varies with the length of the drone body

3. The drone then adjusts the model in formation

The initial positions of each drone in the formation are shown in Table 1, and it is required that the drones adjust only based on the received directional information. Under this premise, a drone formation adjustment model is established. Firstly, an overall adjustment model was established using the polar coordinate system point distance formula and cosine theorem. Based on this, a drone multiple adjustment model was further established. Then, the traversal algorithm was used to solve the multiple adjustment schemes of the drone at the given initial position, and finally a relative error test was conducted.

Based on the passive reception signal positioning model of the unmanned aerial vehicle mentioned earlier, the azimuth angle of the initial position can be obtained, and then the position of the unmanned aerial vehicle can be adjusted by adjusting the azimuth angle, so that the position of the

unmanned aerial vehicle is uniformly distributed on a certain circumference [9].

Table 1. Initial position of drones

Drone Number	Polar Coordinates (m, °)
FY00	(0, 0)
FY01	(100, 0)
FY02	(98, 40.10)
FY03	(112, 80.21)
FY04	(105, 119.75)
FY05	(98, 159.86)
FY06	(112, 199.96)
FY07	(105, 240.07)
FY08	(98, 280.17)
FY09	(112, 320.28)

3.1. Establishment of the overall adjustment model for unmanned aerial vehicles

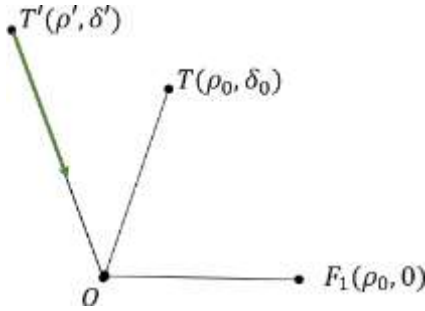


Figure 6. Overall adjustment model

OF_1 is the polar axis of the original polar coordinate system, as shown in Figure 6 and Figure 7. In this polar coordinate system, according to the distance formula between two points:

$$\begin{cases} T'F_1 = \sqrt{\rho^2 + (\rho')^2 - 2\rho\rho' \cos \delta'} \\ T'T = \sqrt{\rho^2 + (\rho')^2 - 2\rho\rho' \cos(\delta' - \delta)} \\ TF_1 = \sqrt{2\rho^2 - 2\rho^2 \cos \delta} \end{cases} \quad (5)$$

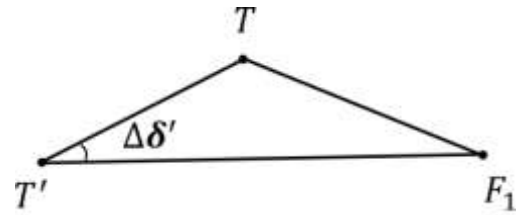


Figure 7. Model Details

According to the cosine theorem:

$$TF_1^2 = T'T^2 + T'F_1^2 - 2T'T \cdot T'F_1 \cos \Delta \delta' \quad (6)$$

According to equations (5) and (6), obtain:

$$\Delta\delta' = \cos^{-1} \left[\frac{\rho^2 + (\rho')^2 - \rho\rho' \cos \delta' - \rho\rho' \cos(\delta' - \delta)}{\sqrt{\rho^2 + (\rho')^2 - 2\rho\rho' \cos(\delta' - \delta)} \cdot \sqrt{\rho^2 + (\rho')^2 - 2\rho\rho' \cos \delta'}} \right] \quad (7)$$

$\Delta\delta'$ can be divided into the following two situations:

(1) When $\Delta\delta' = 0$, this means that there is no need to further adjust the direction from T' to F_1 counterclockwise, as it is already aligned with the direction from T' to T , indicating that the drone has been adjusted to the correct azimuth angle;

(2) When $\Delta\delta' = +\infty$, this means that the denominator in the calculation is zero. In this case, since the distance of $T'F_1$ is not zero, So this situation only occurs when $T'T$ completely overlaps, that is, $T'T = 0$.

Establish a relative polar coordinate system with $T'O$ as the polar axis, and define the overall adjustment angle ω_{t0} when rotating counterclockwise from the polar axis to $T'T$. The length of $T'T$ is the overall adjustment length λ_{t0} of the unmanned aerial vehicle receiving the signal numbered t , as shown in Figure 8.

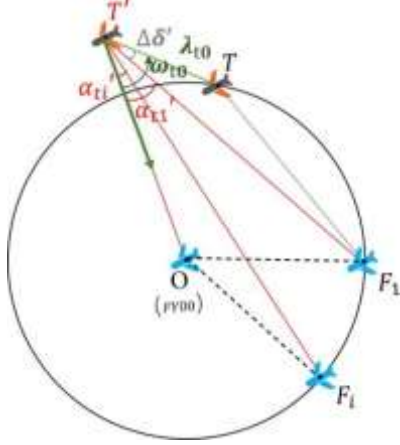


Figure 8. Establishment of Polar Coordinates for UAV Adjustment Model

The calculation formula for the overall adjustment angle ω_{t0} under different offset conditions is:

$$\omega_t = \begin{cases} \alpha_{ti}' + \Delta\delta', & \text{when } \Delta\rho_t > 0, \Delta\delta_t > 0 \\ 360^\circ + \alpha_{ti}' + \Delta\delta', & \text{when } \Delta\rho_t > 0, \Delta\delta_t < 0 \\ 360^\circ - \alpha_{ti}' + \Delta\delta', & \text{when } \Delta\rho_t < 0, \Delta\delta_t < 0 \\ 360^\circ - \alpha_{ti}' + \Delta\delta', & \text{when } \Delta\rho_t < 0, \Delta\delta_t > 0 \end{cases} \quad (8)$$

For all cases, the overall adjustment length is λ_{t0} ,

$$\lambda_{t0} = T'T = \sqrt{\rho^2 + (\rho')^2 - 2\rho\rho' \cos(\delta' - \delta)} \quad (9)$$

For the drone receiving the drone signal, when it extracts the azimuth angle, the required angle ω_{t0} can be solved based on the above model, that is, the drone facing FY00

rotates counterclockwise ω_{t0} and adjusts the length λ_{t0} . When the overall adjustment angle and length of the drone are large, it is not possible to complete the adjustment at once, and multiple adjustments need to be made.

3.2. The establishment of multiple adjustment models for unmanned aerial vehicles

According to the analysis of the results of the effective positioning model in the previous text, the best effect can be achieved when selecting $k = 2$, which means a total of 3 drones can transmit signals. In multiple adjustments, without considering the cumulative impact of receiving error angles, it is assumed that the drone receives and adjusts within the error range each time. Since both FY00 and FY01 drones are already in their initial positions, selecting them as transmitters requires using an overall adjustment model to determine the angle and length that each drone needs to adjust. Select a drone on the circumference as the transmitter, and use the greedy algorithm to solve the adjustment plan for the n th adjustment. Set ω_{tn} as the adjustment angle for the n th adjustment of the drone receiving the signal with the number t , and λ_{tn} as the adjustment length for the n th adjustment of the drone receiving the signal with the number t [10]. Traverse multiple times until each drone reaches the ideal position, and the specific algorithm steps are as follows:

(1) Determine the transmitter number. Calculate the angle and length to be adjusted for each drone except for FY00 and FY01 numbers using the overall adjustment formula (7) to (9), and select the drone with the smallest length to be adjusted as the third transmitter;

(2) By using greedy algorithms, the maximum angle and length that each drone can adjust are specified. When the angle and length that need to be adjusted exceed the maximum angle and length, the adjustment plan is the maximum angle and length. Otherwise, it can be directly adjusted to the ideal position, and the coordinate position is updated after the adjustment is completed;

(3) Repeat steps (1) and (2) until each drone is adjusted to the desired position.

(4) Output the plan and final adjustment results for each drone during each adjustment.

3.3. Multiple adjustments to the model's solution

According to the above solving process using Matlab, it is necessary to make 4 adjustments to obtain a reasonable drone position, including 1 adjustment of δ to obtain δ_1 , 3 adjustments of ρ to obtain ρ_1, ρ_2, ρ_3 . The specific adjustment plan is shown in Table 2.

Table 2. Results of drone adjustment plan

Drone Number i	Adjusted position(m, °)	δ_i^1	ρ_i^1	ρ_i^2	ρ_i^3
3	(99, 80.01)	80.01	107	102	99
4	(99, 119.95)	119.95	100	99	99
5	(99, 160)	160	99	99	99
6	(99, 199.96)	199.96	107	102	99
7	(99, 240.07)	240.07	100	99	99
8	(99, 280)	280	99	99	99
9	(99, 320.08)	320.08	107	102	99

After multiple adjustments, there is a slight error between the position of the drone and the ideal position. The relative length error of all drones is 1%, and the relative angle error is within 0.2%. Some drones even achieve the ideal adjustment effect of zero error. This indicates that the drone can achieve accurate position adjustment by adjusting the model multiple times, and the overall accuracy is high and stable.

4. Conclusions

This article proposes a passive positioning and adjustment method for circular unmanned aerial vehicle formations based on polar coordinate system and traversal algorithm. The results indicate that effective positioning can be achieved with only 3 drones transmitting signals, and adjusting the model can accurately and stably adjust the drones to the ideal position. This method significantly improves the accuracy and stability of formation adjustment while reducing the number of signal transmissions, providing a new approach for passive positioning of drone formations. However, there are still limitations to the research. The current model assumes that the drone body is circular and the positioning environment is free from complex interference, which may be limited in practical applications. In addition, multiple adjustments to the model did not fully consider the real-time adjustment problem in dynamic environments. Future research can further optimize the positioning model to adapt to complex environments, combine machine learning algorithms to improve the adaptability and real-time performance of the adjustment model, and explore optimization strategies for multi formation collaborative positioning and adjustment.

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