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Path Optimization Based on Ant Colony Algorithm and Simulated Annealing Algorithm

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Abstract: In this paper, a collaborative distribution model based on intelligent optimization algorithms is proposed, focusing on how to optimize the distribution paths and costs through K-means algorithm, genetic algorithm, ant colony algorithm and simulated annealing algorithm. First, the K-means algorithm is utilized to cluster the demand points and delineate the regions to provide support for the subsequent path planning. Second, in optimizing the delivery path, genetic algorithm and ant colony algorithm are used to solve the TSP and the optimal flight path of the UAV, respectively, which in turn calculates the shortest path and the minimum delivery cost. Finally, in order to reduce the total cost, a simulated annealing algorithm is used to solve the number of drones and the scheduling scheme to ensure that the fixed cost is minimized, and the type of drones is rationally selected according to different task requirements. The model realizes the improvement of distribution efficiency and cost reduction through the cooperative work of effective multiple delivery tools. The model achieves path optimization and reasonable scheduling of resources in complex distribution tasks through the combined application of intelligent algorithms, and at the same time ensures the optimal use of different types of delivery tools in the distribution process, which makes the model not only have strong practical applicability, but also high computational efficiency and scalability.

Keywords: K-means, Simulated Annealing Algorithm, Ant Colony Algorithm, TSP.

1. Introduction

This paper focuses on the optimization of cooperative distribution system with different transportation carriers, aiming to optimize the path planning and resource scheduling in distribution tasks [1]. Firstly, by introducing the K-means clustering algorithm, the reasonable division of distribution demand points lays the foundation for the subsequent path optimization [2]. Subsequently, combining the genetic algorithm and ant colony algorithm, the TSP and the optimal selection of UAV flight paths are solved respectively, ensuring the shortest distribution route and the lowest transportation cost [3]. Meanwhile, the simulated annealing algorithm was used to optimize the scheduling and cost control of the UAVs, thus effectively reducing the overall cost. The model ensures the efficient completion of distribution tasks while rationally deploying the resources of different transportation carriers, which significantly improves the overall effectiveness of the distribution system [4]. The experimental results show that the proposed algorithm can significantly reduce the distribution cost, improve the efficiency of multiple carriers working together, and provide an effective solution for the optimization of similar distribution systems.

2. Delivery path and UAV scheduling optimization based on greedy algorithm

2.1. Problem splitting based on greedy algorithm

The greedy algorithm is to decompose the large problem into small problems and record the optimal solution for each subproblem. Here, it is split into solving the “total travel cost model” and the “UAV fixed cost model” separately.

2.2. K-means clustering algorithm-based region delineation

The K-means clustering algorithm is an unsupervised learning method that is mainly used to divide the samples in a dataset into a number of usually disjoint subsets. The specific steps are as follows:

1. Import the customer coordinate points transformed into Cartesian coordinates.
2. Input the desired number of clustering centers.
3. Select the coordinates of the clustering centers.
4. Calculate the Euclidean distance from each customer point to the 9 clustering centers, respectively, and compare them.

$$d(x,y)=\sqrt{(x_1-y_1)^2+(x_2-y_2)^2+\cdots(x_n-y_n)^2}=\sqrt{\sum_{i=1}^{n}x_i-y_i)^2} \tag{1}$$

Where: x_i denotes the horizontal coordinate of the i demand point, y_i denotes the horizontal coordinate of the i demand point.

5. According to the principle of proximity, the customer points are categorized into 9 clusters, after which the images and results are output. After K-means clustering algorithm, the result is shown in Figure 1:

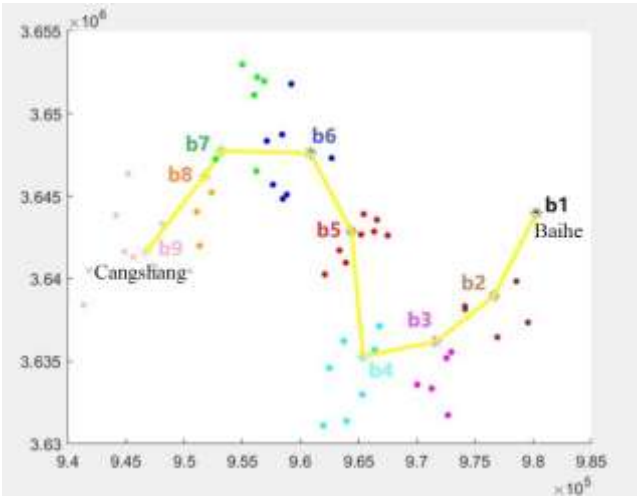


Figure 1 Coordinate map based on K-means aggregation algorithm

Since the starting point, Baihe b1, is far away from all customer points, there are no sample points in the cluster with b1 as the center of clustering, which need not be considered subsequently.

Since the moment when the bus arrives at each stop is related to the specific scenario, the coordinates of the bus stops are first used to find the distance between the two points, and then, based on the speed of the bus, the time interval between the buses traveling to and from the two points is found.

$$s = \sqrt{(Xg_{i+1} - Xg_i)^2 + (Yg_{i+1} - Yg_i)^2} \tag{2}$$

$$t = \frac{s}{vg} \tag{3}$$

2.3. Modeling based on the screening method

As the solution needs to satisfy the need for drones to meet the needs of each demand point and return to the starting point with the least number of drones used. Using the screening method, two routes are analyzed from Baihe-Cangshang and Cangshang-Baihe respectively. The final total fixed cost is obtained from the product of the number of drones and the unit fixed cost:

$$Wa_{\text{total fixed cost}} = n * w_{a\ \text{fixed cost}} \tag{4}$$

2.4. Solving the total cost of travel model

(1) The TSP issue

In TSP, given a set of cities and the distances between them, the goal is to find a shortest path that passes through all the cities and visits each city only once. In each region, a genetic algorithm is applied to solve for the shortest distance that traverses all the points in the region, starting with a bus stop.

(2) Genetic algorithm-based simplification of shortest distance model for TSP problem

Genetic algorithm is an algorithm for solving the optimal solution of an objective function based on the principle of heredity in biology [5]. The main steps of genetic algorithm:

1. Select the initial population. 2. Decoding. 3. Adaptation calculation. 4. Selection. 5. Reproduce offspring. 6. Repeat evolution.

The optimal distance routes for some of the clustered clusters are solved in MATLAB based on genetic algorithm as shown in Figure 2:

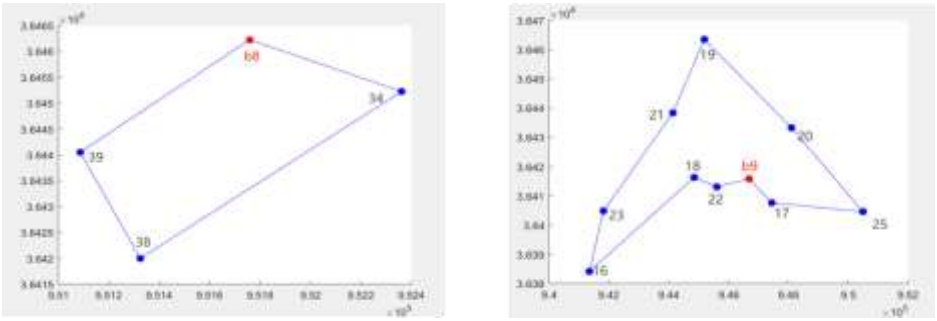


Figure 2 Optimal path diagram for each cluster without considering the loads

Where the UAV flight range of the cluster point centered on b9 exceeds its maximum range. For simplicity of calculation. The case for b9 can be discussed separately.

(3) Final solution process based on ant colony algorithm

Ant colony algorithm is a probabilistic algorithm used to solve optimization. The search process of each ant is independent of each other, and it starts searching independently at multiple points in the space at the same time, which has a strong search capability [6]. Main steps:

1. Initialization: build the model, set the parameters such as the number of ants, path nodes, pheromone concentration. In this problem set the number of ants $m = 1$.

2. Random walk: each ant chooses the path to take next according to a certain probability, which is usually related to the pheromone concentration on the path.

3. Update pheromone: after each ant walks its own path, it will leave pheromone on the path it has traveled. The concentration of pheromone represents the quality or superiority of the path, the better the path chosen by the ant, the more pheromone is left behind. Based on the above analysis, only the constraint of the UAV load is considered.

4. Iterative Calculation: Repeat the above steps until the preset number of iterations is reached or a satisfactory solution is found. As the iteration proceeds, the pheromone concentration on the high-quality paths will gradually increase, attracting more ants to choose these paths, thus gradually converging to the optimal solution.

5. Output result: finally, the optimal solution or better solution is derived based on the paths traveled by the ants. The optimal route for UAV flight is derived and the distance is found.

Several of the clusters of optimal paths are plotted in Figure 3.

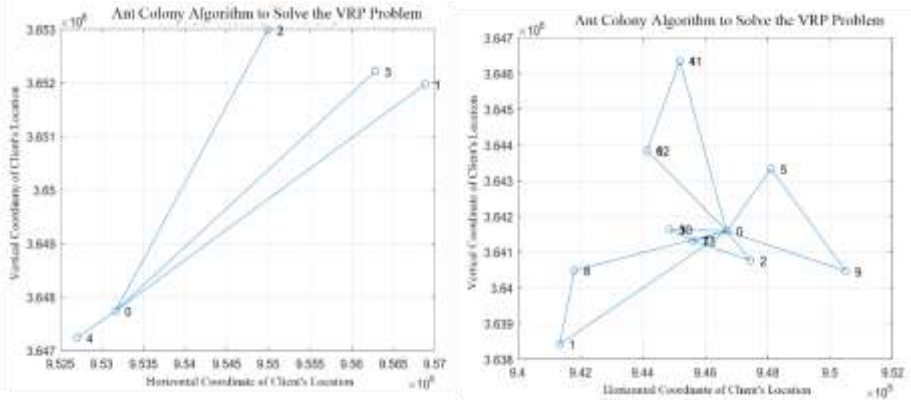


Figure 3 Optimized drone flight path

The total cost of the journey can be obtained by summing the distances of the optimal paths for each cluster and solving for the road cost per kilometer of the UAV type A.

$$Sa_{total\ cost}=\sum_{i=1}^{8} Sa_i \tag{5}$$

$$W_{a\ total\ cost\ of\ transport}=Sa_{\text{total distance}}*w_A \tag{6}$$

2.5. Optimal solution solving based on simulated annealing algorithm

The basic idea of the simulated annealing algorithm is to start from a higher initial temperature and perform state transformations according to certain probability rules, so that when the system reaches thermal equilibrium the corresponding result is the optimal solution.

The main steps are:

1. Initialization. First determine the parameter range and initial solution of the problem. Set the parameters and constraints on the parameters such as demand at the demand point, bus departure, loading goods and charging, and UAV delivery time in a cluster.

2. Calculate the objective function. Establish the objective function for the number of drone deliveries paired with the number of bus arrivals:

$$f(ZX)=\min\sum_{j=1}^{m}\left[1-\prod_{i=1}^{n} Zx_{ij}Zy_{ij}\right] \tag{7}$$

Where, ZX is the solution space for the case where the number of drone deliveries is paired with the number of bus arrivals:

$$ZX=\begin{pmatrix} Zx_{11} & \cdots & Zx_{1m} \\ \vdots & \ddots & \vdots \\ Zx_{m1} & \cdots & Zx_{nm} \end{pmatrix} \tag{8}$$

$i=1,2,\ldots,n$, n is the number of drone deliveries; $j=1,2,\ldots,m$, m is the number of bus arrivals.

Zx_{ij} denotes whether the number of drone deliveries. Zi is paired with the number of bus arrivals. Kj, which in this case is considered to be the ability of the drone to meet the delivery demand:

$$ZX=\begin{cases}1, & \text{Drones can meet delivery needs}\\ 0, & \text{Drones can not meet delivery needs}\end{cases} \tag{9}$$

Zy_{ij} denotes the predicted value of the effect of the number of drone deliveries versus the number of bus arrivals:

$$Zy_{ij}=\frac{\min_{i=1}^{n} t_{ij}}{t_{ij}}\quad ,0\le Zv_{ij}\le 1, t_{ij}=T_{\text{delivering goods}}+T_{\text{loading goods}} \tag{10}$$

3. Generate new solution. Generate a new solution by randomly changing some eigenvalues of the solution. May change the accusation path, and the time-sensitive goal, thus changing the time, etc.

4. Judge whether the new solution is better than the old one. First calculate the difference of the objective function:

$$\Delta f=f\left(Zx^{'}\right)-f(Zx) \tag{11}$$

A probabilistic model is used to determine whether the new solution is better than the old solution according to the Metropolis criterion.

$$r=\begin{cases}1, \Delta f<0\\ \exp\left(-\dfrac{\Delta f}{t}\right), \Delta f\ge 0\end{cases} \tag{12}$$

When $\Delta f<0$, the new solution is accepted, and when $\Delta f\ge 0$, the new solution is accepted with probability $r=\exp\left(-\dfrac{\Delta f}{t}\right)$, which is directly considered in this problem for $r=0$.

5. Update the system state. If the new solution is accepted, then the current solution is the new solution and the objective function value is recalculated; otherwise, the current solution remains unchanged.

6. Determine if the end condition is met. If it is satisfied, keep the result and repeat the steps until the optimal solution is obtained after calculating all the cases, if not, the solution can be discarded directly. Finally, the optimal solution is obtained.

2.6. Optimal distribution scheme based on greedy algorithm

This can be obtained by solving the model for the total cost of the distance traveled

$$Sa_{total\ cost}=305694.64\text{m} \approx 305.69\text{km} \tag{13}$$

$$Wa_{total\ cost\ of\ transport} \approx 244.56(\text{yuan}) \tag{14}$$

Ten drones were used and the total cost of the program:

$$W_{total\ cost} = Wa_{total\ of\ cost} + Wa_{total\ cost\ of\ transport} = 244.56+800=1044.56\ (\text{yuan}) \tag{15}$$

3. Optimal path and cost analysis of collaborative distribution with three types of drones

3.1. Modeling based on dynamic programming

Dynamic programming is the decomposition of the original problem into a number of overlapping subproblems, each of which constitutes a stage in the solution process. The computation of one stage is completed before the computation of the next stage is executed.

(1) Objective function: minimize the total cost, including the fixed cost of the UAV and the transportation cost.

$$MinZ = \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k\in\{A,B,C\}}\left(F_k \times x_{ijk} + C_k \times d_{ijk} \times y_{ijk}\right) \tag{16}$$

(2) Constraints:

1. Drone Type and Number Limit: The type and number of drones assigned to each station must meet the bus capacity limit (assuming a maximum of two drones).

$$\sum_{j=1}^{M}\sum_{k\in\{A,B,C\}} x_{ijk} \le 2, \forall_i \tag{17}$$

2. Drone takeoff and return: the drone must take off from the station and return to the same station.

$$\sum_{i=1}^{N} x_{ijk} = \sum_{i=1}^{N} x_{ijk}, \forall_{j,k} \tag{18}$$

3. Time window constraints: the bus arrival time, drone takeoff and return time must meet the time window requirements.

4. UAV flight distance constraints: the UAV must fly within its maximum flight distance.

$$d_{ijk} \le D_k, \forall_{i,j,k} \tag{19}$$

5. Tasks cannot be split: the delivery task of a demand point needs to be completed by one UAV independently.

6. Vehicle capacity limit: The loading capacity of the bus needs to meet the total weight limit of the drone and cargo.

7. Start and end site: the drone needs to return to the start site after the task is completed.

8. A and B or C time size constraints: to determine whether it is necessary to use type B, C drones to replace the type A drones to achieve the optimization purpose, respectively, the joint type A and B, C drones, the total cost of the equation, and plotting, when $S_a = S_b$ ($S_a = S_c$) to find the point of intersection, the solution of the X_0 that is, for the cut-off point, when the $S_b(S_c) > X_0$ when, the replacement.

$$\begin{cases} W_{a\ total\ cost} = 0.8 \times S_a + 80 \times 2 \\ W_{b\ total\ cost} = 1.5 \times S_b + 150 \end{cases} \tag{20}$$

$$\begin{cases} W_{a\ total\ cost} = 0.8 \times S_a + 80 \times 3 \\ W_{c\ total\ cost} = 2.5 \times S_c + 200 \end{cases} \tag{21}$$

(3) Decision Variable:

1. $x_{ijk} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$: 0-1 variable indicating whether a class k UAV is used to fly from site i to demand point j.

2. $y_{ijk} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$: 0-1 variable indicating the flight distance if a class k UAV is used to fly from station i to demand point j.

3. Other auxiliary variables, such as bus arrival time at the station, UAV take-off and return time, etc.

3.2. Optimal delivery solution with multiple drones working together

By comparing the data with the type A drones and considering whether the number of drones taking off at the same time at the same bus station is greater than or equal to 2 in combination with the timetable, it can be seen that when $S_b < 14.29km$, i.e., at stations b2, b3, and b8, the type B drones can be used as an alternative to the type A. When considering whether the number of drones is greater than or equal to 3 and $S_c < 23.53km$, it can be obtained that in b5, b6, b7 stations can be used type C drones instead of type A.

Transportation by 1 type A, 1 type B and 1 type C UAV, plus their fixed costs get:

$$W_{total\ cost} = 20+17+27+36+49+46+13+15+80+150+200 = 688(\text{yuan}) \tag{22}$$

4. An optimization model for collaborative UAV delivery considering pickup tasks

4.1. Added modeling analysis of the need for pickup

If there is an increased demand for pickups and the pickups can generate some revenue. This means that both delivery and pickup tasks, as well as the revenue generated by pickups, need to be considered when planning the delivery scheme.

1. Task allocation: first, the pickup and delivery tasks of the drone need to be allocated reasonably according to the location of the demand point and the pickup and delivery demand, as well as the performance of the drone.

2. Path Optimization: After assigning the tasks, the flight path of the drone needs to be optimized to minimize the flight distance and total cost while maximizing the revenue from pickup and delivery.

3. Time synchronization: Since the drone needs to wait for the bus at the bus stop for battery replacement and cargo loading, it needs to ensure the time synchronization between the drone and the bus to avoid the drone waiting too long at the stop.

4.2. Modeling

(1) Objective function:

The goal is to minimize total costs (including UAV fixed costs, transportation costs, and pickup revenues, but pickup revenues are negative, so they actually increase revenues):

$$Z = c_d^{fix} \times \left(x_{ijd} + y_{ijd} \right) + c_d^{mile} \times d_{ij} \left(x_{ijd} \times w_j^{out} + y_{ijd} \times w_j^{in} \right) - \sum_{j \in M} \sum_{i \in N} \sum_{d \in D} r_{in} \times w_j^{in} \times y_{ijd} \tag{23}$$

Where: Part I calculates the fixed costs for all drones, whether delivering or picking up goods.

Part II calculates the transportation costs for all deliveries and pickups, taking into account the weight of the goods and the distance flown.

Part III calculates the revenue for all pickup missions, which is a negative value because it actually reduces the total cost.

(2) Restrictive conditions:

In addition to the objective function, a number of constraints need to be considered, such as:

1. Limit on the number of drones, the number of drones needs to be as small as possible.

2. Drone number limitation: each bus can carry at most two drones.

3. Drone usage constraints: drones must take off from a bus stop and return to the same stop.

4. Time constraints: drones must take off and return within a certain time window before or after the bus arrives at the station.

5. Demand point fulfillment: delivery and pickup tasks must be fulfilled once and only once for each demand point.

6. Other possible constraints, e.g., load limit of the UAV, flight time limit, etc.

$$s.t. \sum_{b \in \mathrm{B}} \mathrm{y}_{cb} = 1, \forall c \in C \tag{24}$$

$$\sum_{i \in U} \sum_{j \in C} z_{ijf} CQ_f \le FQ_f, \forall f \in F \tag{25}$$

$$\sum_{i \in U} \sum_{j \in U} z_{ijf} d_{ij} \le S_{total}, \forall f \in F \tag{26}$$

(3) Comparative Hierarchical Analysis to Analyze the Advantages of UAV Delivery Paths

1. Determine the importance. Assume that the pickup requirement is weighted more heavily than meeting the minimum spend.

2. Calculate the weight. Assume that meeting the pickup requirement and meeting the minimum spend have a weight of 2:1.

3. Normalization. Since they are of different orders of magnitude, it is convenient to find out the proportion of each scenario that meets the pickup requirement and meets the least amount of spending in all cases.

$$W_{ij} = \frac{w_{ij}}{\sqrt{\sum_{i=1}^{n} w_{ij}^2}}, i = 1, 2, \ldots, n; j = 1, 2, \ldots n \tag{27}$$

4. Result Analysis. The value S of the degree of merit of the distribution scheme is:

$$S = \omega_1 W_0 + \omega_2 W_1 \tag{28}$$

4.3. Optimal distribution scheme considering pickup tasks

$$W_{\text{total cost}} = W_{\text{cost of transport}} + W_{\text{fixed cost}} - W_{\text{cost of pickup}} \tag{29}$$

In order to achieve the purpose of picking up the goods, the drone distance traveled 22.47km more compared to the previous one. Solve for the minimum cost is to be $676.45.

5. Conclusions

This paper proposes a cooperative distribution model between rural public transportation and heterogeneous UAVs based on an intelligent optimization algorithm, which fully combines the advantages of K-means clustering, genetic algorithm, ant colony algorithm and simulated annealing algorithm to optimize the distribution paths and resource scheduling problems. First, the K-means clustering algorithm provides effective support for the regional division of demand

points and ensures the accurate distribution of distribution tasks. Second, the genetic algorithm and ant colony algorithm minimized the transportation cost and improved the efficiency of path planning by solving the traveler problem and optimizing the UAV flight path. Finally, the simulated annealing algorithm optimizes the number of UAV configurations and scheduling scheme, which further reduces the fixed costs and improves the overall economic efficiency of the system. Future research can further explore more complex distribution network scenarios and introduce more diversified optimization algorithms to improve the adaptability and computational efficiency in large-scale and dynamic environments.

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