

Analysis on the Development of the Pet Industry Based on Multi-Step Combined Model

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Abstract: The multi-step combined model integrates multiple sub-models in sequential steps based on the internal correlations among factors. This paper formulates sub-problems for relevant issues, establishes corresponding sub-models, and ultimately solves them through integration. Through multiple linear regression, equations were derived for each outcome variables. A multi-order differential regression equation was applied to historical total scores. Combining predictions from direct, first-order difference, second-order difference regression, a weighted average (0.6,0.3,0.1) was calculated. The global pet food demand scores were predicted to be 33445.32, 33445.32, and 34157.15, with annual growth rates of 0.757%, 0.941%, and 1.176%.

Keywords: Cubic spline interpolation, Multiple linear regression, Factor analysis, Multi-step difference equation, Production and export model.

1. Introduction

With the rapid development of the global economy and the significant increase in per capita income, the pet industry has rapidly risen globally as an emerging industry[1]. People's care and investment in pets are increasing, which promotes the vigorous development of pet industry. It is beneficial to clarify the development trend of the pet industry and market demand, thereby promoting the benign and long-term development of the national pet industry[2].

Advanced materials research, such as biodegradable PET alternatives, uses AI-driven hybrid modeling to predict material lifecycles and recyclability[3]. Similarly, the pet sector benefits from IoT-enabled devices, with smart feeders and health monitors gaining traction among tech-savvy pet owners[4]. These innovations are validated by multi-stage demand forecasts, where GRU-based models incorporating pinball loss functions achieve higher accuracy in predicting seasonal consumption trends[5]. Governments worldwide are standardizing pet-related services, such as veterinary care and insurance, while promoting sustainable practices[6]. Meanwhile, supply chain analyses using SCP (Structure-Conduct-Performance) and PEST models identify vulnerabilities, such as raw material shortages and trade barriers, which are critical for long-term strategic planning[7].

This paper analyzed the development of pet industry over the past five years by pet type, and analyze the factors affecting the development of pet industry and establish a mathematical model to predict the development of the industry for the next three years. Then we analyzed the development of the global pet industry and establish a mathematical model to predict the global demand for pet food for the next three years.

2. Models constructions

2.1. Data collection and analysis

By collecting data and searching related literature, this paper initially considers using 14 indicators to measure the development of Chinese pet industry over the past five years.

These indicators mainly describe the scale and penetration rate of the pet industry, pet consumption and expenditure, the development and competition of the pet industry, and the relationship between the macroeconomy and the pet industry. They cover many important aspects of studying the pet industry and can fully reflect the detailed development of the pet industry. From 2019 to 2023, the number of pets in China, the pet household breeding rate, and the amount of money spent per pet have been increasing year by year[8]. The pet industry market size has risen from 33.2 billion USD in 2019 to 45.5 billion USD in 2023, showing an expanding trend in the pet industry market size[9]. Since dogs and cats account for the vast majority of pets, this paper considers classifying other types of pets into one category for subsequent analysis. Based on these three types, the development of Chinese pet industry in the next five years is analyzed. According to the data of dogs, the number of pet dogs reached the highest in 2019 and then showed an alternating trend of increase and decrease; according to the data of cats, the number of pet cats in China has been increasing year by year, from 4412 million in 2019 to 6980 million in 2023[10], indicating that cats account for the largest share in Chinese pet market; other types of pets have a slightly increased share in Chinese pets, indicating that Chinese pets are becoming more diversified and personalized, and the number of families with multiple types of animals as pets is increasing, with aquatic animals as pets ranking second only to dogs and cats. With the rapid development of Chinese pet industry, related industries such as the pet food industry and pet health care have also developed rapidly. The pet food market size and the market share change of CR5 in the pet food industry have both been improved.

2.2. Construction of Forecasting Model Based on Cubic Spline Interpolation

This paper considers using the number of pets, pet market size, and pet household penetration rate as three indicators to represent the development of Chinese pet industry. Since these three indicators are affected by many aspects, this paper considers calculating the predicted values of the above three

indicators for the next three years through 11 indicators such as pet food market size, per pet consumption amount, and Chinese GDP per capita, to further analyze the development trend of Chinese pet industry in the next three years.

1. Use cubic spline interpolation to predict the data for the next three years (2024-2026) of pet household quantity, pet food market size, newly established pet business enterprises, Chinese GDP per capita, per capita disposable income of Chinese residents, per pet consumption amount, pet industry financing event quantity, number of pets per household, CR5 market share change in the pet food industry, growth rate of pet businesses, and growth rate of financing events, denoted as $x_i (i = 1, 2, \dots, 11)$.

2. Let the weight coefficients of the 11 indicators be $\beta_i (i = 1, 2, \dots, 11)$. Use these 11 indicators as independent variables, and use the number of pets, pet market size, and pet household penetration rate as dependent variables to construct a multiple linear regression equation system.

3. This paper considers calculating the predicted value of the dependent variable through the predicted value of each independent variable and its corresponding weight. The calculation formula is as follows.

$$Y = \sum_{n=1}^{11} \beta_n x_n \quad (1)$$

$$X = (A^T A)^{-1} A^T b \quad (2)$$

First, for each indicator, this paper takes the indicator as the dependent variable and time (2019-2023) as the independent variable, denoted as $t_j (j = 1, 2, 3, 4, 5)$, and encodes 2019、2020、2021、2022、2023 as 1, 2, 3, 4, 5 respectively. Secondly, divide the time into four sub-intervals and calculate the cubic spline interpolation polynomial for each indicator and sub-interval, the formula is as follows:

$$S_{ik}(x) = a_{i0} + a_{i1}x + a_{i2}x^2 + a_{i3}x^3 \quad (3)$$

$S_{ik}(x)$ represents the expression of the i indicator in the k interval.

The final cubic spline interpolation function formula is a piecewise cubic polynomial composed of segment functions in each sub-interval. Using MATLAB's algorithm can solve the function expression for each indicator and predict the value for the next three years, denoted as $x_{mi} (m = 1, 2, 3; i = 1, 2, \dots, 11)$

2.3. Solution of the Forecasting Model Based on Cubic Spline Interpolation and Multiple Linear Regression

In each sub-interval, $S(x) = S_i(x)$ is a cubic polynomial, therefore, $S''(x)$ is a linear function of x , and $S''(x^{i-1}) = M^{i-1}$, $S''(x^i) = M^i$, $x \in [x^{i-1}, x^i]$.

By integrating twice continuously, we can get

$$S_i = M_{i-1} \frac{(x_i - x)^3}{6h_i} + M_i \frac{(x - x_{i-1})^3}{6h_i} + A_i(x_i - x) + B_i(x - x_{i-1}) \quad (4)$$

Lastly, only need to find these $M_i (i = 0, 1, \dots, n)$ to find the cubic spline interpolation function, and then solve the predicted value for each indicator for the next three years.

Then, by solving the equation system constructed above, the weights of the 11 indicators can be found. Finally, according to the formula, the predicted values of the three indicators, pet quantity, pet market size, and pet household penetration rate, for the next three years are calculated.

3. Applications on other countries

3.1. Data preparation

Since the data of each indicator for each country collected is on an annual basis and the data volume is small, this paper considers integrating the data of the four countries for subsequent analysis.

After searching related literature and materials, the pet industry scale of China, the United States, France, and Germany is relatively large and representative. Therefore, this paper has collected relevant data from these four countries and uses them as representatives to analyze the development of the global pet industry.

3.2. Global Pet Industry Development Analysis

The pet market size of China and the United States has grown rapidly from 33.2 billion and 95.7 billion in 2019 to 45.5 billion and 108 billion in 2023, with a large growth amplitude. The growth amplitude of European countries represented by France and Germany is smaller, but it still shows a trend of increasing year by year; the total output value of pet food in these four countries also shows a trend of increasing year by year. In addition, the expenditure on pet food and medical services in various countries has increased year by year, with China and the United States having larger expenditures, indicating that the pet market development trend is expanding, and the pet economic momentum in East Asia and North America is rapid.

According to the collected data, as shown in the table below, the pet household penetration rate in China and the United States has increased year by year. More and more families are keeping pets, while the pet household penetration rate in European countries such as France and Germany has remained basically unchanged, indicating that the pet industry in European countries has entered a stable stage, and the pet industry in China and the United States is still expanding, and the pet industry has good development prospects.

3.3. Construction and Solution of Forecasting Model Based on Factor Analysis and Multi-step Difference Regression Analysis

This paper has collected data on the number of pets, the number of cats and dogs, the total output value of pet food, the total export value of pet food, the pet market size, pet food expenditure, veterinary service expenditure, and pet household penetration rate in these four countries, with a total of 9 indicators. Due to the large number of indicators and four representative countries, this paper considers using factor analysis to integrate these variables and retain the most important characteristics in the dataset.

1. KMO Test and Bartlett's Sphericity Test.

The KMO test and Bartlett's spherical test are performed on the 9 indicators to determine whether factor analysis is suitable.

The KMO value is $0.639 > 0.6$, and the significance is < 0.001 , indicating that there is a significant difference between the correlation coefficient matrix and the identity matrix, making it suitable for factor analysis.

2. Selection of the Number of Factors.

From the total variance explanation result of SPSS, the cumulative contribution of the first two factors' variance is $89.434\% > 60\%$, so the first 2 factors should be selected.

To more intuitively determine the number of factors, this paper uses SPSS to perform factor analysis on the data of the

9 indicators and draws a scree plot, as shown in the figure below. According to the scree plot, 2 principal factors should be extracted.

3. Determination of Principal Factor Components.

The rotated component matrix obtained from SPSS is shown in the table below, which can determine the components of the selected two principal factors.

It can be seen that the load value of the number of pets overlaps with other factors and needs to be removed. The load values of other factors are high, among which, the first principal factor is composed of the number of pet cats and dogs, the total output value of pet food, veterinary service expenditure, pet market size, and pet food expenditure, denoted as F_1 ; the second principal factor is composed of the total export value of pet food and pet household penetration rate, denoted as F_2 .

The first eight indicators are denoted as $A_i (i = 1, 2, \dots, 8)$, and the scoring formulas for the two principal factors are as follows.

$$F_1 = 0.971A_1 + 0.988A_2 + 0.961A_3 + 0.891A_5 + 0.982A_7 + 0.991A_8 \quad (5)$$

$$F_2 = -0.619A_4 + 0.940A_6 \quad (6)$$

This paper considers calculating the two principal factor scores for the four countries over the past five years and adding them together. Finally, the total scores of the two principal factors for the four countries over the past five years are added together to represent the development of the global pet market size, because the fluctuation of the pet market size is positively correlated with the demand for pet food, so it is considered to use this indicator's growth rate as the standard to predict the global demand for pet food for the next three years.

4. Conclusions

This paper presents a multi-step combined model to analyze and predict the development of the pet industry. The paper categorized pets into cats, dogs, and others, and uses cubic spline interpolation and multiple linear regression to predict the pet population, market size, and household penetration rate for the next three years in China. The results show stable growth trends in these indicators. Then we analyzed the global pet industry by collecting data from China, the US, Germany, and France. Factor analysis and multi-step difference regression are applied to predict global pet food demand. The model extracts two main factors and forecasts an annual growth rate in global pet food demand over the next

three years. The paper provides valuable insights into the development trends of the pet industry and offers a basis for formulating strategies for sustainable development in this sector.

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