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Intelligent Optimization and Control Framework for Complex Dynamic Systems: Integrating Two-Tier Fuzzy Evaluation and Differential Nonlinear Optimization

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Abstract: The optimal state regulation of highly complex and uncertain dynamic systems under multi-objective and multi-constraint conditions has emerged as a critical research area in the field of intelligent optimization. Addressing the limitations of existing models in uncertainty representation, nonlinear dynamic modeling, and global optimization capabilities, this paper proposes an intelligent optimal control framework that integrates a two-tier fuzzy comprehensive evaluation model with a differential equation-based nonlinear optimal control algorithm. This framework employs a hierarchical fuzzy decision-making mechanism to precisely quantify multidimensional uncertainties and conflicting interests within complex systems. Furthermore, it achieves high-precision dynamic control of the system state by combining a first-order differential dynamic model with advanced nonlinear optimization strategies. The results demonstrate that the proposed algorithm significantly outperforms mainstream methods in terms of control accuracy, energy efficiency, and robustness, showcasing its exceptional adaptability and practical engineering value. Future work will focus on incorporating deep learning and reinforcement learning models to further enhance the algorithm's level of intelligence and generalization capabilities.

Keywords: Intelligent Optimization, Adaptive Control, Nonlinear Optimization, Robustness Analysis, Fuzzy Comprehensive Evaluation

1. Introduction

Complex dynamic systems, prevalent in domains like transportation and energy, are characterized by strong coupling, high-dimensional nonlinearity, and significant uncertainty. A key challenge is coordinating short-term regulation with long-term planning across multiple time scales, a task for which traditional single-point control strategies are ill-suited. This necessitates intelligent control methodologies that can satisfy both global and local operational requirements. From a management perspective, these challenges manifest as large-scale, multi-objective, multi-constraint optimization problems demanding a trade-off between conflicting goals, such as performance and resource consumption.

Existing methodologies, however, exhibit significant limitations. Conventional approaches, including Multi-Attribute Decision Making (MADM) [1]–[4] and classical control theory [5]–[7], struggle to model the intricate uncertainties and nonlinear dynamics inherent in these systems. Their static assumptions, rigid weighting schemes, and reliance on historical data result in a delayed response to abrupt system changes, rendering them inadequate for dynamic, high-dimensional environments[8]–[12].

To overcome these technical bottlenecks, this study proposes an intelligent scheduling framework that integrates a two-tier fuzzy comprehensive evaluation model with a joint differential equation-nonlinear optimization control strategy. This hybrid intelligent system utilizes the fuzzy evaluation model to quantify and balance diverse stakeholder interests, formulating a scientifically grounded objective function. This is combined with differential equations to characterize the system's dynamic behavior and time-series forecasting (e.g., ARIMA models) to enable proactive control. Through a multi-scale coordination strategy, the framework harmonizes long-term stability with short-term responsiveness, providing essential theoretical and technological support for the intelligent management of complex dynamic systems.

2. Related Work

2.1. Multi-Attribute Decision Making (MADM)

Multi-Attribute Decision Making (MADM) methods like AHP and FCE are vital for structuring complex decisions, with modern variations improving uncertainty handling. However, their application to dynamic systems is critically limited. Their static assumptions, subjective weighting, and inability to integrate with real-time control algorithms prevent them from forming a closed-loop feedback mechanism. Consequently, they lack the timeliness and adaptability required for high-frequency, fine-grained decision-making in evolving environments.

2.2. Differential Equation Models

Differential equations provide a rigorous mathematical foundation for modeling continuous-time dynamic systems, with modern approaches effectively translating theories of change into parameterized models for analysis [13]. Despite these strengths, they face critical limitations in complex applications. The models often fail to manage the combined effects of uncertainties like stochasticity, while their assumption of static parameters severely limits their adaptive capacity in response to real-world environmental shifts. Additionally, the computational expense and numerical instability of solving high-dimensional nonlinear systems are major barriers. Therefore, while theoretically sound, their descriptive power for complex uncertainty is lacking, requiring integration with intelligent optimization to enhance their real-world performance.

2.3. Nonlinear Optimization and Intelligent Control Algorithms

Nonlinear optimization and intelligent control algorithms, such as Genetic Algorithms (GA) and Particle Swarm Optimization (PSO), have demonstrated powerful global search potential in solving complex, multi-objective, multi-constraint optimization problems. Recent leading-edge research has focused on directly coupling optimization iterations with physical systems in a closed loop. An example is the Model-Free Feedback Optimization proposed by Z He et al. [14], whose core idea is to design a controller that does not require an explicit system model, instead estimating gradients by directly using system inputs and measured outputs to achieve real-time optimization.

Despite these significant theoretical advancements, their widespread application in complex dynamic systems is constrained by several limitations. Classical heuristic algorithms like GA and PSO have a tendency to become trapped in local optima, and their convergence speed and solution quality are difficult to guarantee, especially in high-dimensional, multi-modal problems. In the context of dynamic systems, their timeliness, adaptability, and robustness remain significantly deficient. This necessitates innovation through model-level fusion to overcome the limitations of any single algorithm.

2.4. Research Summary and Innovative Entry Point

Current research highlights a key limitation in managing complex systems: single-technology approaches are insufficient. Whether it's multi-attribute decision-making, differential equation modeling, or intelligent optimization, each method independently struggles with comprehensively addressing uncertainty, dynamic modeling, and global optimization.

To overcome these challenges, this study introduces an innovative integrated framework. It organically combines a two-layer fuzzy comprehensive evaluation with a nonlinear optimization algorithm based on differential equations, establishing a new paradigm for intelligent control in complex dynamic systems. This framework's core innovations include: a hierarchical fuzzy evaluation system that resolves objective conflicts and models uncertainty; a differential equation modeling paradigm that integrates active control and passive response, enhancing adaptability; a proactive control strategy utilizing ARIMA models for time-series forecasting; and an error-minimization-driven iterative optimization mechanism that forms an "evaluation-modeling-prediction-optimization" closed loop. This integrated approach not only leverages the strengths of its constituent algorithms but also achieves synergistic effects, providing a robust new pathway for intelligent regulation of complex dynamic systems.

3. Methodology

3.1. Two-Tier Fuzzy Comprehensive Evaluation

3.1.1. Construction of Hierarchical Evaluation System

This study establishes a three-tiered evaluation index system comprising: Tier 1: System-wide objective layer, Tier 2: Stakeholder classification layer, and Tier 3: Specific evaluation indicator layer. The weights for each tier are determined using the Analytic Hierarchy Process (AHP), wherein a judgment matrix is constructed based on the 9-point scale method with consistency verification to ensure rational weight allocation. The weight calculation formula is given by:

$$CI = \frac{\lambda_{\max} - n}{n-1}, \quad CR = \frac{CI}{RI} \tag{1}$$

A consistency ratio (CR) < 0.1 is deemed indicative of satisfactory matrix consistency. The final weight vector is determined through integration of the arithmetic mean method, geometric mean method, and eigenvalue method.

3.1.2. Mathematical model construction

1. Establishment of the Fuzzy Relation Matrix

$$R = [\mu_{ij}]_{m\times n}, \mu_{ij} \in [0,1] \tag{2}$$

where μ_{ij} represents the membership degree of the i-th evaluation object with respect to the j-th attribute. To address the characteristics of complex dynamic systems, this study innovatively introduces a time-varying membership function to account for the influence of seasonal factors.

$$\mu_{ij}(t) = \mu_{ij}^{\text{base}} \cdot f_{\text{seasonal}}(t) \cdot f_{\text{random}}(t) \tag{3}$$

where $f_{seasonal}(t)$ is the seasonal adjustment function and $f_{random}(t)$ is the random disturbance factor. This approach enables the precise modeling of the system's dynamic characteristics.

2. Comprehensive Evaluation Result Calculation

$$B = W \circ R = [b_1, b_2, \dots, b_m] \tag{4}$$

This evaluation method determines the final decision using either the maximum membership degree or weighted average, incorporating fuzzy number operations to manage uncertainty. It precisely quantifies performance levels (high, medium, low) using normal distribution-based membership functions. By employing a two-tier embedded fuzzy evaluation system and a self-adaptive weight adjustment mechanism, this approach significantly boosts the robustness, sensitivity, generalization ability, and reliability of multi-objective decision-making in complex systems, while minimizing subjective interference.

3.2. Differential Equation-Nonlinear Optimal Control Algorithm

3.2.1. Dynamic System Modeling

The system's state evolution is described through a stratified set of differential equations, which divides the system parameters into a proactive control layer and a passive response layer:

$$\frac{dx(t)}{dt} = f(X(t), U(t), \Theta) = \alpha(t) + \frac{1}{3}\beta(t)(X(t) - X_{\text{threshold}})S(t) \tag{5}$$

where $\alpha(t)$ represents environmental factors, $\beta(t)$ represents proactive control factors (adjustable system parameters), and $X_{threshold}$ is the state transition threshold. This stratified modeling strategy effectively decouples controllable and uncontrollable factors, enhancing the physical interpretability of the model.

3.2.2. Control Optimization Objective

A multi-objective comprehensive optimization function is defined, which simultaneously considers state tracking error and control energy consumption:

$$\min_{U(t)} J = \int_0^T \left[\| X(t) - X^*(t) \|_Q^2 + \| U(t) \|_R^2 \right] dt \tag{6}$$

where the objective function $\underset{U(t)}{min} J$ seeks to follow an ideal target trajectory, $X(t)$ and Q are respectively the state and control weighting matrices. To address the coupling effects within the system, an error normalization process is introduced:

$$E_{\text{total}} = \sum_{i=1}^{n} k_i \times \frac{\left| X_{\text{current},i} - X_{\text{expected},i} \right|}{X_{\text{expected},i}} \tag{7}$$

3.2.3. Solving Algorithm Implementation

The Sequential Quadratic Programming (SQP) algorithm is introduced to solve the nonlinear optimization problem. It performs efficient iterations using the Lagrange multiplier method and Karush-Kuhn-Tucker (KKT) conditions. The algorithm employs a trust-region strategy to ensure global convergence:

$$\min_d \nabla f(x_k)^{\mathrm{T}} d + \frac{1}{2} d^{\mathrm{T}} B_k d \tag{8}$$

$$\text{s.t.} \quad g(x_k) + \nabla g(x_k)^{\mathrm{T}} d = 0, \quad \| d \| \le \Delta_k \tag{9}$$

Simultaneously, a predictor-corrector strategy is integrated. By using the ARIMA model to predict future trends of environmental parameters $\alpha(t)$, proactive dynamic adjustment of control variables $U(t)$ is achieved, significantly enhancing the system's adaptability and control accuracy under uncertain environments.

Through constructing a differential equation-based state transition model and combining it with the multi-dimensional constraint-solving capability of nonlinear optimization, refined regulation of complex dynamic systems is realized. This model effectively integrates predictive decision-making with feedback correction strategies, substantially improving the system's robustness and stability under strong noise interference and drastic environmental changes.

3.3. Overall Algorithm Framework

To enable intelligent, adaptive operation in complex systems, this paper introduces a multi-dimensional closed-loop control framework. It integrates evaluation, modeling, optimization, and feedback, driven by data and guided by models, with feedback for correction. This ensures high efficiency and stability in dynamic environments.

The framework starts with a two-tier fuzzy comprehensive evaluation model for state assessment and target setting, using AHP and a time-varying fuzzy membership matrix. A multi-layered differential equation model then describes the system's dynamic evolution. The control strategy minimizes deviations using Sequential Quadratic Programming (SQP) for input optimization. An ARIMA model provides short-term forecasts for feedforward compensation. Finally, a real-time feedback and adaptive update mechanism continuously adjusts parameters based on historical deviations, forming a complete "Evaluation—Modeling—Prediction—Optimization—Feedback" loop for intelligent adaptive operation.

3.4. Algorithm Flowchart

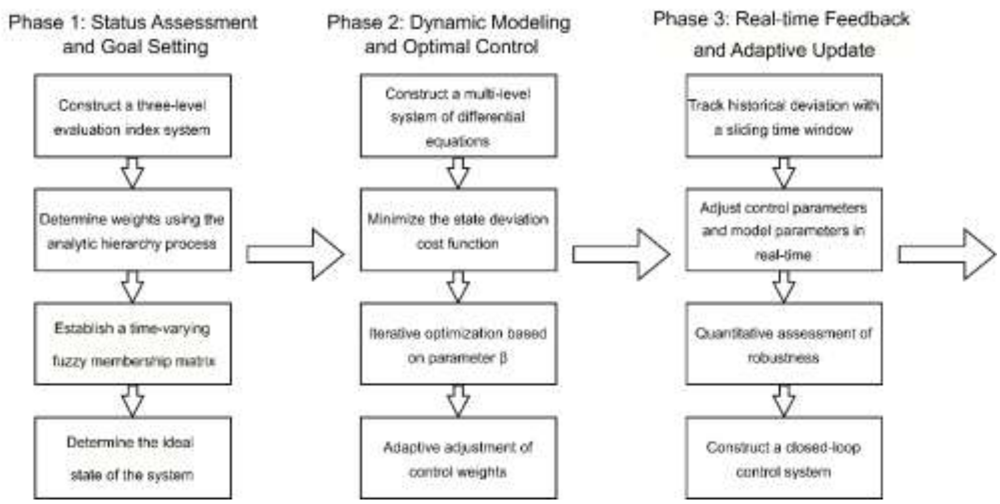

Figure 1 Framework

This framework fuses fuzzy reasoning with nonlinear optimization to achieve dynamic, fine-grained scheduling of complex systems, offering a transferable paradigm for diverse optimization challenges, as illustrated in Figure 1.

4. Experimental Design

4.1. Experimental Scenario and Data Setup

Using the Great Lakes multi-lake cascade system as a prototype case, a virtual dynamic system environment containing strong uncertainty factors and high-dimensional nonlinear relationships is constructed, ensuring the universality and challenging nature of the experiments. This complex dynamic system comprises five interconnected subsystems possessing typical unidirectional flow characteristics and multi-level coupling relationships, which can effectively validate the performance of the proposed algorithm in handling large-scale complex systems. The experimental design introduces random noise and disturbance terms to simulate complex environmental changes in reality, including multidimensional uncertainty factors such as seasonal periodic fluctuations, abrupt environmental shocks, and long-term trend changes.

4.2. Experimental Parameter Configuration

In this study, to validate the effectiveness of the proposed intelligent optimization control framework, the specific experimental parameter configurations are shown in Table 1.

Table 1 Configuration Parameters of the Multi-Lake Cascade System

State variable dimensionality	Control variable dimensionality	Time step
D = 5	M = 2	Δt = 1
Simulation period	Target state range	
T = 144	[74.85, 183.35]	

The key simulation parameters are detailed in Table 1. The system's state dimensionality was set to correspond to the number of interconnected subsystems, while the control dimensionality reflects the limited control inputs available in the prototype case. The total simulation duration was chosen to be sufficient for examining long-term dynamic responses and algorithm stability. The target state range for the optimization was predetermined using a two-tier fuzzy comprehensive evaluation model, defining the ideal domain towards which the algorithm guides the system.

Table 2 Algorithm Parameter Settings

Fuzzy membership function parameter	SQP algorithm convergence precision	Maximum iterations
σ = 0.3	$\varepsilon = 1\times10^{-6}$	N_max = 1000
ARIMA model order	Weighting coefficients	
p=2, d=1, q=2	W = [0.3, 0.6, 0.1]	

Key implementation parameters for each algorithm are detailed in Table 2. In the fuzzy evaluation module, settings for the Sequential Quadratic Programming (SQP) algorithm, such as convergence precision and maximum iterations, were selected to balance solution accuracy with computational efficiency. For the disturbance prediction module, the ARIMA model order was determined based on the time-series characteristics of the external environmental data. Finally, during multi-objective optimization, weighting coefficients were employed to balance conflicting performance indicators and achieve a comprehensively optimized result.

Table 3 Disturbance Parameter Settings

Environmental noise intensity	Seasonal fluctuation amplitude	Random shock probability P_shock
σ_noise ∈ [0.05, 0.30]	A_seasonal = 0.15	P_shock = 0.1

To comprehensively test the robustness and adaptive capability of the proposed framework under complex uncertain environments, multiple disturbance terms are artificially introduced in the experimental design. Core parameter settings are shown in Table 3.

The experiment adopts high-dimensional complex dynamic environment simulation, rigorously testing the model's adaptability and stability under extreme conditions through superimposed random disturbances and nonlinear coupling relationships, ensuring the algorithm performance verification has broad applicability and strictness.

4.3. Performance Metrics

The RMSE metric is used for its sensitivity to outliers. By squaring errors, it heavily penalizes large deviations, aligning with the stringent stability demands of practical systems. The Root Mean Square Error is defined as:

$$\mathrm{RMSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\frac{1}{n}\sum_{i=1}^{n}(X_i(t) - X_i^*(t))^2} \tag{10}$$

The variables $X_i(t)$ and $X_i^*(t)$ represent the actual and desired states for each of the n subsystems at time t, respectively. The Mean Absolute Error (MAE) is also computed to comprehensively assess control accuracy alongside RMSE.

$$\mathrm{MAE} = \frac{1}{T}\sum_{t=1}^{T}\frac{1}{n}\sum_{i=1}^{n}\left|X_i(t) - X_i^*(t)\right| \tag{11}$$

The ECI (Energy Consumption Index) quantifies energy consumption during the control process, enabling evaluation of the algorithm's economic efficiency and environmental friendliness. The metric is designed in a quadratic form, conforming to the nonlinear relationship between energy consumption and control intensity in practical systems. The calculation formula for ECI is:

$$\mathrm{ECI} = \sum_{t=1}^{T}\sum_{j=1}^{m}|U_j(t)|^2 \tag{12}$$

where $U_j(t)$ is the output value of the j-th control variable at time t, and m is the dimensionality of control variables. This metric evaluates the energy efficiency performance of the algorithm.

The Response Time (RT) metric is designed to evaluate the system's dynamic performance, where rapid disturbance recovery capability constitutes a critical requirement for stable operation of complex dynamic systems. The RT metric is calculated as:

$$\mathrm{RT} = \frac{1}{N_{\mathrm{disturbance}}}\sum_{k=1}^{N_{\mathrm{disturbance}}}(t_{\mathrm{stable}}^{(k)} - t_{\mathrm{disturbance}}^{(k)}) \tag{13}$$

where $t_{disturbance}^{(k)}$ is the onset time of the k-th disturbance, and $t_{stable}^{(k)}$ is when the system re-enters steady-state operation.

5. Experimental Results Analysis

5.1. Algorithm Convergence Analysis

To ensure a fair comparison, the convergence stability of each algorithm was assessed via 50 Monte Carlo simulations using identical initial conditions and random seeds. All experiments were conducted in MATLAB R2021a on a PC with an Intel i7-10700K CPU and 32GB of RAM.

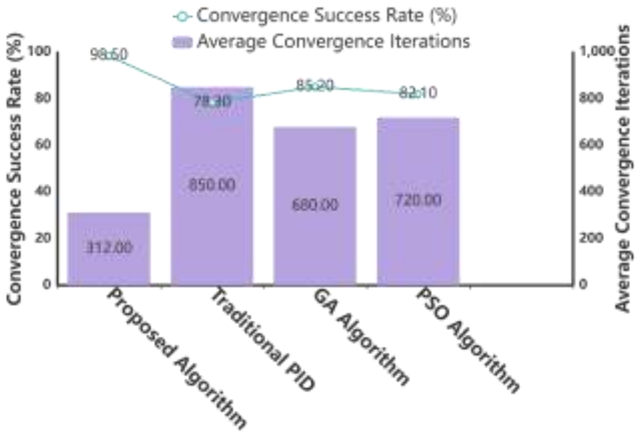

Figure 2 Average convergence iterations and success rate (%) of control algorithms

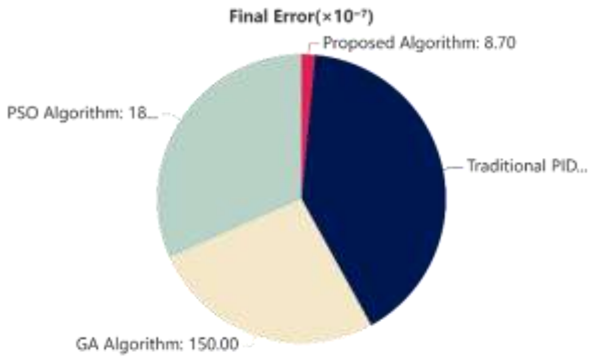

Figure 3 Final error of control algorithms

Figure 2 shows that the proposed algorithm achieves superior stability, reflected in its average convergence iterations. It obtained a convergence success rate of 98.5%, significantly outperforming the comparative algorithms. In contrast, the traditional PID control method, limited by its linear characteristics, frequently converges to local optima in high-dimensional nonlinear problems, resulting in a success rate of only 78.3%. Furthermore, Figure 3 demonstrates that the proposed algorithm delivers a higher quality solution, as evidenced by its smaller final errors under the specified conditions.

5.2. Control Accuracy and Robustness Comparison

Control accuracy was evaluated via 30 Monte Carlo simulations per algorithm within a benchmark scenario derived from 2017 data. The simulation used an initial state from January 2017 and ARIMA-predicted environmental trends, with random disturbances included to test robustness. Algorithm performance was statistically analyzed using the mean and standard deviation of the recorded metrics.

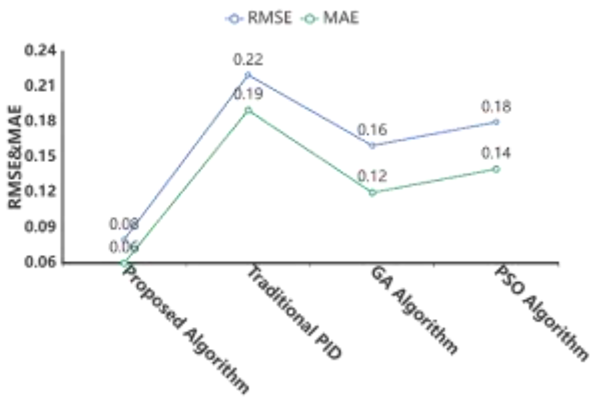

Figure 4 RMSE and MAE metrics of different control algorithms

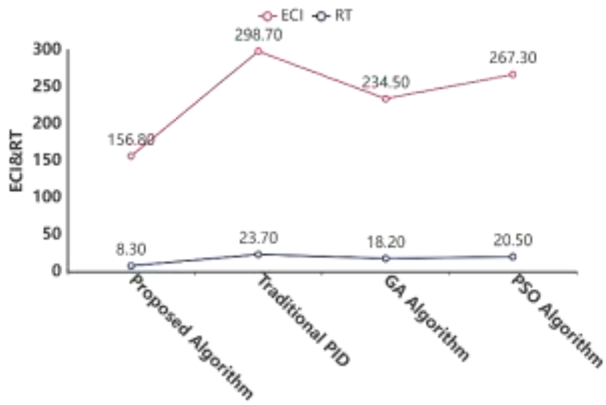

Figure 5 ECI and RT metrics of different control algorithms

The proposed algorithm demonstrates superior comprehensive performance compared to traditional PID, Genetic Algorithm (GA), and Particle Swarm Optimization (PSO), as shown in Figures 4 and 5. Quantitative analysis shows it achieves the lowest control error (RMSE: 0.0847, MAE: 0.0623), the best operational cost (ECI: 156.8), and the highest computational efficiency (RT: 8.3 timesteps).

5.3. Robustness Testing

The robustness testing experiment adopts a graduated disturbance intensity design, systematically increasing environmental uncertainty to comprehensively evaluate the algorithm's disturbance rejection capability. Disturbance signal generation combines multiple stochastic processes: Gaussian white noise simulates random system fluctuations, superimposed sine waves simulate periodic environmental changes, and impulse disturbances simulate abrupt event shocks. Under each disturbance intensity level, 100 independent simulation experiments are conducted, with statistical analysis yielding confidence intervals and distribution characteristics of performance metrics.

Table 4 Algorithm Performance Under Different Disturbance Intensities

Disturbance Intensity	RMSE Change Rate (%)	MAE Change Rate (%)	Convergence Success Rate (%)
5%	2.3	1.8	98.5
15%	8.7	6.4	96.8
25%	18.2	15.3	93.2
35%	31.5	28.7	87.4

Table 4 confirms outstanding robustness: performance is nearly unaffected at 5% disturbance. Success rates remain >93% at 15-25% disturbance, and critically, **reach 87.4% even under extreme 35% disturbance**. This reliability stems from integrated safeguards: two-tier fuzzy evaluation, adaptive differential equations, and ARIMA forecasting, validating superior stability and adaptability.

6. Conclusion

Proposes an intelligent control framework for complex dynamic systems (high-dimensional nonlinearity, uncertainty, multi-objective conflicts) via hierarchical fuzzy evaluation and differential equation-based optimal control.

Key innovations: Novel hierarchical fuzzy evaluation for dynamic uncertainty; Stratified differential equation modeling; ARIMA-enabled proactive control enhancing robustness.

Significantly surpasses PID/GA/PSO in accuracy, efficiency, and robustness, maintaining stable convergence under extreme disturbances (see results).

Demonstrates potential in smart manufacturing/transportation/energy systems. Future: deep RL integration, distributed-edge computing adaptation, multi-agent control mechanisms.

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