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Research on the Optimization of UAV Smoke Screen Interference Bomb Delivery Strategy Based on Spatio - Temporal Motion Model

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Abstract: This paper aims to optimize the coordinated masking tactics of UAV smoke screen interference bombs to maximize the effective masking time against incoming missiles. The research focuses on the strategy deployment problem of UAVs delivering smoke screen interference bombs and gradually constructs and solves a series of mathematical models. First, for the single - UAV - single - bomb scenario under fixed parameters, a spatio - temporal motion trajectory model of the missile, the UAV, and the smoke screen cloud is established. Through the line - of - sight blocking principle, an effective masking duration of 1.38 seconds is calculated, which lays a benchmark for subsequent optimizations. On this basis, the problem is transformed into a multi - dimensional nonlinear optimization problem. The flight angle, speed of the UAV, and the delivery and detonation times of the interference bombs are taken as decision variables, aiming to maximize the effective masking time. By using the genetic algorithm to conduct a global parameter scan under multiple physical and tactical constraints, the effective masking time is significantly increased to 4.58 seconds. Subsequently, to address the complexity of the coordinated delivery of multiple bombs (three interference bombs) by a single UAV, the optimization model framework is extended. More decision variables are introduced, and under the delivery interval constraints, the particle swarm optimization (PSO) algorithm is used to efficiently solve the high - dimensional non - convex problem. Finally, an effective masking duration of a total of 6.25 seconds is achieved.

Keywords: Kinematic model, intelligent optimization algorithm, parameter optimization.

1. Introduction

The continuous evolution of the modern warfare paradigm has made electronic countermeasures an increasingly crucial factor influencing the outcome of battles. Smoke screen interference bombs play an indispensable role as they can effectively block the enemy's photoelectric detection and tracking capabilities[1-2]. To effectively protect key ground targets from the serious threat of air - to - ground missiles, using UAVs to deliver smoke screen interference bombs to form a mask in a specific airspace in front of the target, thereby interfering with enemy missiles and reducing their hit probability, is regarded as a low - cost and high - efficiency protection strategy. Currently, smoke screen interference technology has been able to achieve the fixed - point accurate dispensing and sequential detonation of interference bombs, which lays a solid foundation for further optimizing the interference effect. The core task of this study is to design an optimal UAV smoke screen interference bomb delivery strategy to maximize the effective masking time of the real target. This requires us to accurately determine a series of key parameters such as the flight direction and speed of the UAV, as well as the delivery and detonation points of the smoke screen interference bombs. To systematically solve this complex problem, this paper constructs a hierarchical and progressive mathematical modeling and optimization framework. The study first establishes the spatio - temporal kinematic models of the missile, the UAV, and the smoke screen cloud, and conducts geometric masking judgments based on the line - of - sight blocking principle to evaluate the masking duration under fixed parameters[3-4]. On this basis, the problem is transformed into a multi - dimensional nonlinear optimization problem aimed at maximizing the total effective masking time. The decision variables cover the UAV flight parameters and the timing of interference bomb delivery and detonation, and multiple physical and tactical constraints are comprehensively considered. To efficiently solve optimization problems of different complexities, this paper adopts intelligent optimization algorithms. For example, the genetic algorithm is used for the single - UAV - single - bomb scenario, and it is extended to the single - UAV - multiple - bombs scenario where the particle swarm optimization algorithm (PSO) is used to handle high - dimensional decision - making problems[5]. Through the combination of these models and algorithms, this study aims to provide strong theoretical support and data basis for the practical deployment of UAV smoke screen interference bombs. Table 1 lists the symbols used in this paper. Other symbols appearing in the text will be explained when they occur.

Table 1 Explanation of Symbols

symbol	Symbol description	symbol	Symbol description
t	Time	v	UAV flight speed
J	Total masking time	Δt	Time sampling interval
θ	Flight direction	tm	Delivery time
tn	Detonation time	Np	Particle swarm particle count
vs	Falling speed of the cloud	Nq	Particle iteration count

2. Calculation of the effective masking duration of smoke bombs under initial parameters

2.1. Model establishment

As known from the problem, the initial position of missile M1 is PM(20000, 0, 2000), the initial position of FY1 is PFY1(17800, 0, 1800), the flight speed v = 120 m/s, the time for delivering smoke screen interference bombs after receiving the mission is t1 = 1.5 s, and the detonation interval time is t2 = 3.6 s. It is assumed that the time when M1 is detected as being controlled is the time origin (t = 0), the coordinates of the false target are the three - dimensional coordinate origin (0, 0, 0), the unit vector of the missile's motion speed direction is $\mu_{\mathrm{M}} = \frac{P_M}{|P_M|}$, and the unit vector of the UAV's motion speed direction is $\mu_{\mathrm{F}} = \frac{P_{FY1}}{P_{FY1}}$.

Establish the motion equation of the missile.

$$\mathrm{M}(\mathrm{t}) = \mathrm{PM0} + 300\mathrm{t}\mu\mathbf{M}, \mathrm{t} \geq 0 \quad (1)$$

Establish the motion equation of the unmanned aerial vehicle (UAV):

$$\mathrm{F}(\mathrm{t}) = \mathrm{PFY1} + 120\mathrm{t}\mu\mathbf{F} \quad (2)$$

According to the assumption, the detonation moment of the smoke screen interference bomb is T1 = t1 + t2 = 5.1 s, and the delivery position is Pt1 = F(t1). Taking the upward direction as the positive direction, the free - fall equation of the interference bomb is derived from the free - fall formula:

$$Z(t) = \mathrm{P_t} + 120(t - t_1)\mu_{\mathrm{F}} + (0,0,-\frac{1}{2}g(t-t_1)^2) \quad (3)$$

Then, the position of the smoke screen interference bomb at the moment of detonation is

$$Z(T_1) = P_{t_1} + 120(T_1 - t_1)\mu_{\mathbf{F}} + (0,0,-\frac{1}{2}g(T_1 - t_1))^2 \quad (4)$$

After detonation, the position equation of the uniformly - descending straight - line movement of the cloud is:

$$\mathrm{D}(\mathrm{t}) = \mathrm{Z}(\mathrm{T1}) + (0, 0, -\mathrm{vs}(\mathrm{t} - \mathrm{T1})) \quad (5)$$

According to the problem, the smoke screen within a 10 - m range of the cloud center is effective smoke screen, and its spherical equation is:

$$\mathrm{S}(\mathrm{t}) = \{\mathrm{x} | \mathrm{x} - \mathrm{D}(\mathrm{t}) \leq \mathrm{r}\} \quad (6)$$

The straight - line equation of the line segment connecting the missile and the real target can be listed as:

$$\mathrm{l}(\mathrm{s},\mathrm{t}) = (0,200,0) + \mathrm{s}(\mathrm{M}(\mathrm{t}) - (0,200,0)) \quad (7)$$

At time t, the missile is masked if and only if there exists s ∈ [0, 1] such that the distance between the vector of the line segment connecting the missile and the real target and the center of the cloud is less than or equal to the radius. The position parameter of the closest point on the line segment (the line segment connecting the missile and the real target) of the center point D(t) of the cloud is:

$$s(t) = \frac{(D(t)-(0,200,0))\times(M(t)-(0,200,0))}{(M(t)-(0,200,0))^2} \quad (8)$$

Masking the real target is equivalent to the inequality $r^2 \geq d(t)$. Therefore, the distance between the missile and the center of the cloud:

$$\mathrm{d}(\mathrm{t}) = ((0,200,0)\,\mathrm{s}(\mathrm{t})(\mathrm{M}(\mathrm{t}) - (0,200,0) - \mathrm{D}(\mathrm{t}))^2 \quad (9)$$

By solving the above equations simultaneously, we will obtain the effective masking duration T.

2.2. Solution of the model

First, we conduct a line - of - sight intersection judgment. For each moment t (with a step size of 0.01 s to ensure accuracy), we check whether the line segment from the "missile position to the key point P of the real target" intersects with the "smoke screen sphere". This is achieved through solving a quadratic equation[6-7].

Expanding the sphere equation yields a quadratic equation:

$$\mathrm{as2} + \mathrm{bs} + \mathrm{c} = 0 \quad (10)$$

Where a = ((0, 200, 0) − D(t))2, b = 2 × (D(t) − D(T)) × ((0, 200, 0) − D(t)), c = (D(t) − D(T))2 − r2.

If the discriminant $\Delta = b^2 - 4ac > 0$ and there exists s ∈ [0, 1] that satisfies the equation, the current moment t is determined as "effective masking".

By traversing the time range (with a step size of 0.01 s), all continuous intervals are extracted, and the total duration is calculated:

Step 1: Identify continuous masking intervals. For example, when t ∈ [ta, tb] and s(t) ≡ 1, the duration of this interval is: tb − ta.

Step 2: Calculate the total effective masking duration, that is, the sum of the durations of all continuous intervals. After substituting specific values, the final result obtained is 1.38 seconds.

Use MATLAB to visualize the missile, the target, and the smoke screen interference bomb. Through the following figure, we can clearly see that the missile is effectively masked at 8.06 s and the effective masking ends at 9.44 s.Line - of - sight masking effect of the smoke screen on missile M1 is shown in figure 1.

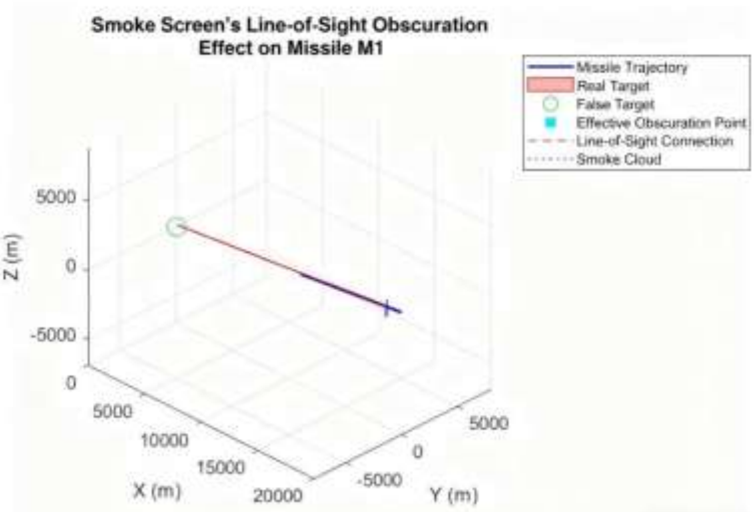

Figure 1 Line - of - sight masking effect of the smoke screen on missile M1

Figure 2 shows the graph of the effective masking state of the smoke over time obtained by using the masking indication function, where 1 represents effective masking and 0 represents ineffective masking. Through this graph, it can be seen that the value of the function is 1 when t is between 8.06 s and 9.44 s, clearly presenting the start, duration, and end stages of the effective masking and intuitively reflecting the effective masking duration of the smoke on M1[8-9].

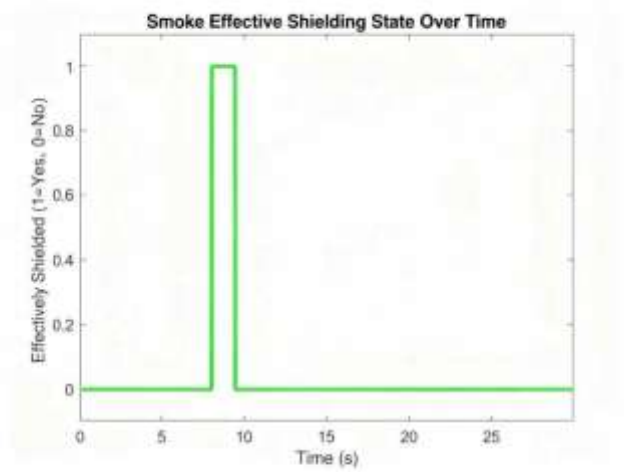

Figure 2 Variation of the effective masking state of the smoke with time

3. Optimization of maximum masking time for a single UAV with a single smoke bomb

3.1. Model Establishment

Based on the given conditions in the problem, the initial coordinates of the first unmanned aerial vehicle (UAV) PFY1 are (17800, 0, 1800). The speed range of the UAV is restricted to |v| ∈ [70, 140], and the flight angle is µFY1 = (cos θ, sin θ, 0)[10].

Assuming the delivery time is tm and the detonation time is tn, then the detonation moment is T2 = tm + tn. A decision - making model is established to solve the problem. First, a decision vector a = (θ, v, tm, tn) is established, and then an objective function is constructed:

$$\max d\ J(a) = \int_{T_2}^{T_2+20} \Pi\,(d(t;a) \le r)dt \qquad (11)$$

The constraint conditions of this function are:

$$\begin{cases} t_{p,i+1} - t_{p,i} \ge 1 \\ |v| \in [70,140] \\ \text{The height h remains constant} \\ \text{The smoke screen decoy will be effective as a cover within a } 10-\text{meter radius} \\ v_s \equiv 3m/s \\ \text{The flight direction of the drone is definitely} \end{cases} \qquad (12)$$

Similar to Problem 1, the masking equation of the cloud can be written as:

$$\mathrm{S(t)\ =\ \{x|x\ -\ D(t)\ \le\ r\}} \qquad (13)$$

Construct the expression for the total effective masking time:

$$J(a) = \sum_k \Gamma\,(d(t_k;a) \le r), \quad \epsilon t \qquad (14)$$

According to the established model, the grid search method is then used to solve the strategy.

3.2. Model Solution

We first determine the solution method. Since the objective function J(α) is a non - linear function and there are constraints on the decision variables, namely θ ∈ [0, 2π], v ∈ [70, 140], tm ∈ [0, 15], and tn ∈ [0.5, 10], traditional analytical methods are difficult to use for solving the problem. Therefore, the genetic algorithm is adopted for numerical optimization. The specific steps are as follows:

Then, construct the fitness function. The genetic algorithm needs to transform the maximization problem into a minimization problem. Therefore, the fitness function is defined as the negative value of the objective function: fitness(α) = −J(α). During the algorithm iteration process, the maximization of J(α) is achieved by minimizing fitness(α).

Secondly, set the algorithm parameters. Combining the complexity of the problem and the solving efficiency, the parameters of the genetic algorithm are determined: Genetic Algorithm Parameter is shown in table 2.

Table 2 Genetic Algorithm Parameter Table

Parameter Name	Value (Range)	Parameter Name	Value (Range)
Population Size	80	Maximum Number of Iterations	40
θ	[0,2π]	v	[70,140]
tm	[70,140]	tn	[0.5,10]

Subsequently, the algorithm iteration is carried out. Initially, 80 decision vectors α_1, α_2, ..., α_{80} that satisfy the boundary constraints are randomly generated for the initial generation. For each α, the corresponding delivery point, detonation point, smoke screen cloud trajectory, and missile trajectory are calculated. The J(α) is calculated through the effective masking determination model, and then the fitness fitness(α) is obtained. High - quality individuals are selected according to the fitness, and a new generation population is generated through crossover and mutation. The iteration is repeated 40 times, and finally, it converges to the optimal decision vector α*.

Finally, the optimal solution is verified. For the optimal decision vector $\alpha = (\theta, V^*, t_m^*, t_n^*)$ output by the algorithm, the missile positions and smoke screen cloud positions in the entire time series (t ∈ [0, 70] s, with a step size of 0.01 s) are recalculated. The effective masking state is determined at each moment, the effective masking intervals are extracted, and the total duration is calculated to verify the validity of the solution.

The following figure shows the time chart of the missile being interfered with. The best fitness drops rapidly in the early stage and then tends to be stable, while the average fitness gradually decreases from a relatively high value. In the later stage, both of them stabilize at a level close to - 4.5. Overall, it reflects the process that the fitness is continuously optimized and tends to be stable as the number of generations increases. The best fitness is - 4.57, and the average fitness is - 4.5555. It indicates that active parameter adjustment has increased the effective masking duration of the missile to 4.57 s. Time Chart of Missile Interference is shown in figure 3.

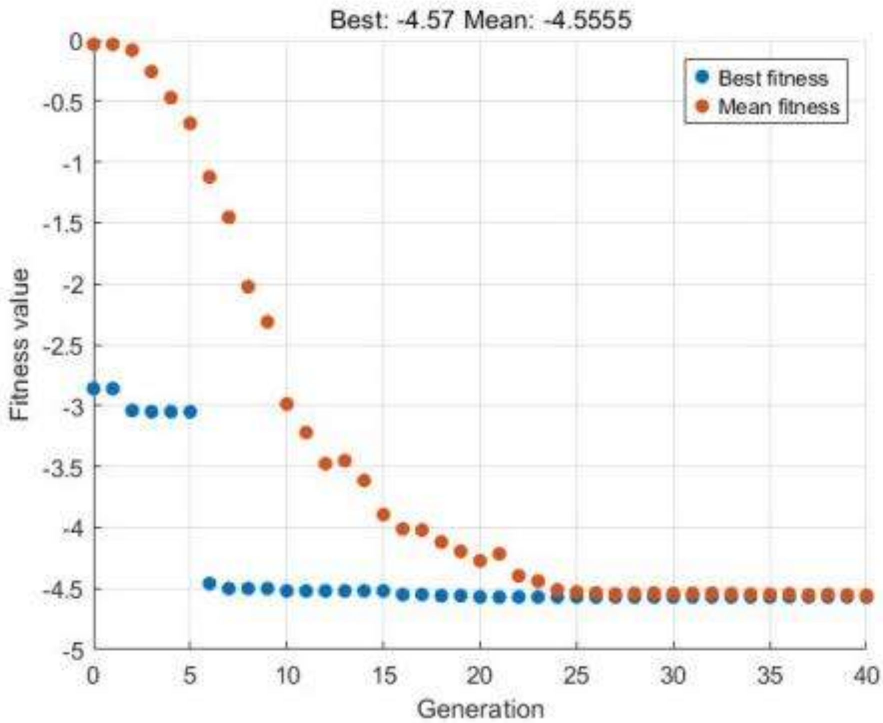

Figure 3 Time Chart of Missile Interference

4. Optimization of Cooperative Delivery Strategy for a Single UAV with Multiple Smoke Bombs

4.1. Model Establishment

The deployment strategy of multiple smoke screen bombs needs to take into account the flight trajectory and speed of the missile, as well as the position of the real target, and carry out precise coordination in both spatial and temporal dimensions. Considering that missile M1 flies at a constant speed in a straight line and its flight speed is much higher than the sinking speed of the smoke screen cloud, the optimal strategy should be to deploy smoke screen bombs at different key positions along the missile's flight path. Under the condition of a single UAV with multiple bombs, we aim to propose the optimal deployment strategy for smoke screen bombs by establishing an optimization model.

We number the smoke screen bombs as i = 1, 2, 3. Let the flight angle of the UAV be θ and its flight speed be v. The deployment time of the smoke screen bombs is recorded as tp, i, and the detonation time is recorded as tq, i. Therefore, the detonation moment is T2 = tp, i + tq, i.

We establish the following objective function:

$$\max d\, J(a) = \sum_{i=1}^{3} \int_{T_2}^{T_2+20} \Pi\,(d(t_i; a) \le r)\, dt_i \tag{15}$$

The constraint conditions of this function are:

$$\begin{cases} t_{p,i+1} - t_{p,i} \geq 1 \\ |v| \in [70,140] \\ \text{The height h remains constant} \\ \text{The smoke screen decoy will be effective as a cover within a } 10 - \text{meter radius} \\ v_s \equiv 3m/s \\ \text{The flight direction of the drone is definitely} \end{cases} \tag{16}$$

The solution to this optimization model will provide the optimal flight direction and speed of UAV FY1, as well as the optimal deployment times and delay times of the three smoke screen bombs, thereby achieving the maximum cumulative effective masking time for missile M1.

4.2. Model solving

We established a complete optimization model for the scenario where the UAV FY1 deploys 3 smoke jamming bombs to jam the missile M1.

First, basic parameters are set: the gravitational acceleration is 9.8 m/s², the sinking speed of the smoke cloud is 3 m/s, the effective radius is 10 m, and the duration is 20 s. The initial position of missile M1 is (20000, 0, 2000), and it flies towards the decoy at a speed of 300 m/s, while the real target is a cylinder located at (0, 200, 0) with a radius of 7 meters and a height of 10 meters.

To find the optimal deployment strategy, we adopted the Particle Swarm Optimization (PSO) algorithm. "Similar to the genetic algorithm, PSO is a population-based optimization tool. The system is initialized as a set of random solutions and searches for the optimal value through iteration. However, unlike the genetic algorithm which uses crossover and mutation operations, particles (potential solutions) search in the solution space by following the optimal particles." [4] We set 50 particles and 100 iterations, where each particle represents a 12-dimensional decision variable group including the UAV's flight direction angle, flight speed, and the deployment time and detonation time of the three smoke jamming bombs. In the optimization process, we specifically considered physical constraints: the flight speed must be between 70-140 m/s, and the detonation time must be at least 0.1 seconds later than the deployment time to ensure the practical feasibility of the scheme.

In each iteration, the algorithm evaluates the quality of the scheme by calculating the fitness value of each particle (i.e., the total time that the missile is effectively shielded). The fitness calculation adopts the time discretization method [5], simulating the missile's flight trajectory and the dynamics of the smoke cloud with a step size of 0.01 seconds. For each time point, we determine whether the missile is within the effective range of the smoke (the distance from the center of the cloud is less than 10 meters) and calculate the total effective time by scanning the continuous shielding intervals.

After optimization iterations, we obtained the optimal parameter combination for FY1: a flight direction angle of 9.88 degrees (0.17 radians) and a flight speed of 90.68 m/s.

The following figure is a visualization of the comparison of the individual effective shielding time of each smoke jamming bomb. It can be seen from the figure that the individual effective shielding time of smoke jamming bomb 1 is 2.95 s, that of smoke jamming bomb 2 is 3.51 s, and that of smoke jamming bomb 3 is 0.00 s. The red dashed line marks the total effective time as 6.2 s, intuitively presenting the individual shielding effect and total duration of each jamming bomb. result is shown in figure 4 and table 3.

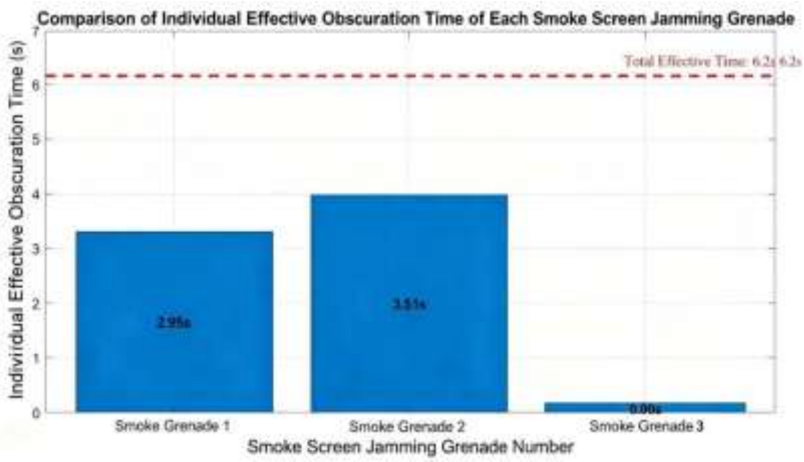

Figure 4 Comparison of individual effective shielding time of each smoke jamming bomb

Table 3 Single-aircraft multi-bomb deployment strategy

Number of smoke jamming bomb	x-coordinate of the release point of the smoke jamming bomb (m)	y-coordinate of the release point of the smoke jamming bomb (m)
1	17800.56	0.14
2	17810.21	2.61
3	18927.82	288.73

5. Conclusions

This study focuses on the optimal deployment of the coordinated shielding tactics of UAV smoke jamming bombs, aiming to establish a comprehensive model combining dynamic kinematics and geometric judgment to maximize the effective shielding time against incoming missiles. First, under the initial parameter settings, the model accurately calculated the effective shielding duration for missile M1 when UAV FY1 flies at a speed of 120 m/s, releases the smoke jamming bomb after 1.5 seconds, and detonates it after 3.6 seconds. By establishing the spatiotemporal trajectory models of the missile, UAV, and smoke cloud, and conducting geometric judgment based on the line-of-sight blocking principle, it is finally concluded that the benchmark shielding duration in this scenario is 1.38 seconds. Subsequently, the study further explored the optimization in the scenario of a single aircraft with a single bomb. A four-dimensional decision vector including the UAV's flight direction, speed, and the release and detonation times of the jamming bomb was constructed, and a nonlinear optimization model was established to maximize the effective shielding time. After considering constraints such as the range of flight speed and constant altitude, the genetic algorithm was used for global parameter scanning and optimization, which significantly increased the effective shielding time to 4.58 seconds, fully demonstrating the potential of active optimization strategies. Finally, in the complex scenario of coordinated deployment of multiple bombs by a single aircraft, the model was extended to a single UAV deploying three jamming bombs. This problem introduced more decision variables such as the independent release and detonation times of each jamming bomb, and under the constraint that the deployment interval is not less than 1 second, an optimization model was constructed to maximize the total effective shielding time under the combined action of the three jamming bombs. For this high-dimensional non-convex problem, the study adopted the Particle Swarm Optimization (PSO) algorithm for efficient solution, and finally achieved a total effective shielding duration of 6.25 seconds, highlighting the significant advantages of multi-bomb coordination.

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